Massive MIMO Precoding Method based on GaBP Algorithm

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Abstract: Massive multiple input multiple output (MIMO) is one of the key technologies of the fifth generation (5thGeneration, 5G) mobile communication network. In a massive MIMO system, precoding processing technology can not only effectively eliminate multi-user interference, greatly increasing the system capacity, but also simplify the receiver design and solve the problem of power consumption and volume at the user-side [1].Minimum mean square error (MMSE) precoding requires matrix inversion, which leads to high computational complexity [2-4].This paper analyzes the biased MMSE precoding that pre-processes the transmission signal through a simplified iterative way, including jacobi method (JM)[5], conjugate gradient (CG)[6], neumann series expansion (NS) method, this paper proposes a precoding algorithm based on gaussian belief propagation (GaBP). Simulation results and complexity analysis show that with low computational complexity, the bit error rate (BER) performance of the GaBP precoding algorithm is significantly better than the traditional JM, CG, and NS method, and faster convergence rate. This shows that MMSE precoding based on GaBP algorithm can have lower total hardware implementation complexity and latency.

Keywords: Massive MIMO; Precoding; MMSE; GaBP

1. Introduction

Massive MIMO is considered to be one of the key technologies for 5G communication. The massive MIMO system is a base station (BS) with hundreds of antennas to serve many single antenna users. Compared with the traditional MIMO system, the massive MIMO system has higher spectrum efficiency and energy of the communication system.

However, massive MIMO systems be confronted with many challenging problems in actual use, one of which is to find a low-complexity and high-performance precoding scheme. Since the number of antennas and the number of users are much larger than traditional MIMO system, the use of non-linear precoding will not be possible due to high computational complexity. The channel matrix of the massive MIMO system is orthogonal, and the MMSE precoding scheme can achieve near-optimal performance, but the MMSE precoding scheme involves matrix inversion, which has high computational complexity when the number of antennas is huge (O^3).

There are several methods based on secondary calculations to obtain approximate solutions for matrix inversion. For example, JC, CG, NS and so on. These methods have lower computational complexity than MMSE precoding to obtain an approximate solution of the matrix inverse, but they will reduce the BER performance. The JC and CG methods use an iterative calculation method, and a smaller number of iterations can effectively reduce the computational complexity, but it will reduce the BER performance. A large number of iterations can achieve good BER performance, but the computational complexity may reach or exceed the MMSE method. The CG method converges faster and has better performance than the JC method. The NS method uses series expansion to reduce the computational complexity of matrix inversion, and the BER performance is poor. This paper proposes a low-complexity precoding method based on GaBP. The channel matrix of the massive MIMO system is a sparse matrix, and the message between the sending node and the receiving node is approximately Gaussian, so the GaBP algorithm can be used to reduce the computational complexity of matrix inversion.

2. System Model

Wireless communication is affected by parameters such as time delay, frequency shift, and space correlation, resulting in a very complicated signal propagation process. The massive MIMO system is equipped with multiple antennas at both the transmitting end and the receiving end. The simplified transmission model of the point-to-point MIMO system is shown in Figure 1.
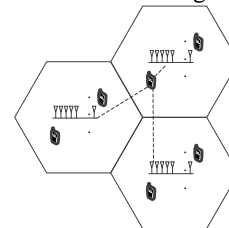

Figure 1. Simple model of massive MIMO system

2.1. System model

For a single-cell multi-user downlink massive MIMO system, the number of base station transmitting antennas is M, and the number of single antenna receiving users is N, where M is much larger than N. Modulate the original information bit stream, obtain the original signal vector $b \in C^{N \times 1}$, and the precoding matrix is set to $W \in C^{M \times N}$. After precoding the original signal vector b, the expected transmission signal is $S \in C^{M \times 1}$, then the received signal $y \in C^{N \times 1}$:

$$y = HWb + n \qquad (1)$$

Where $H \in C^{N \times M}$ is the downlink channel matrix, its fading is flat Rayleigh fading. n is additive white Gaussian noise with a mean value of 0 and a variance of N_0. Assume that the base station has known channel state information, and the sub-channels are independent identically distributed, $H \sim CN(0,1)$. The signal-to-noise ratio of the downlink transmission is $SNR = M \cdot Es / N_0$, and Es is the average energy of the transmitted signal. The precoding process is shown in Figure 2.
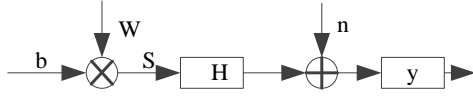

Figure 2. Downlink precoding system

2.2. MMSE precoding

According to the MMSE detection theory, the precoding signal can be expressed as:

$$Wb = H^H (HH^H + \sigma^2 I)^{-1} b \qquad (2)$$

Where $\sigma^2 = N_0 / P$, where P is the transmitted signal power.

Convert solving the precoding signal to solving the inverse of the matrix:

$$Ax = b \qquad (3)$$

Where the matrix $A = HH^H + \sigma^2 I$, and $Wb = H^H x$.

The computational complexity of solving the inverse of matrix A is $O(N^3)$, the increase of user N will increase the computational complexity geometrically. In a massive MIMO system, the rows of the channel matrix H can be considered as full rank, and each row is progressively orthogonal, and the matrix A becomes a diagonally dominant Hermitian positive definite matrix. In this way, the problem of solving A^{-1} can be transformed into a problem of solving a system of linear equations to reduce the computational complexity.

3. Proposedscheme

Linear equations can be represented by an undirected graph, such as MRF(Markov Random Fields). *A* Markov random field undirected graph $G = (V, E)$, V is the set of undirected graph nodes, E is the set of edges, and the vertex elements of MRF are random variables independent of all other variables.

The vertex set V of the undirected graph G corresponds to the variable set $x = \{x_1, x_2, ..., x_N\}^T$ of the linear equation $Ax = b$, and the edge set E corresponds to the non-zero elements of the matrix A. Where N is the number of users of the massive MIMO system.

In this way, from the coefficient matrix A and vector b, the following Gaussian density function can be obtained:

$$p(x) \sim \exp(-\frac{1}{2} x^T A x + b^T x) \qquad (4)$$

The edge potentials function ψ_{ij} and the self potentials function ϕ_i of the undirected graph G:

$$p(x) \propto \prod_{i=1}^{N} \varphi_i(x_i) \prod_{\{i,j\}} \Psi_{ij}(x_i, x_j) \qquad (5)$$

Where

$$\psi_{ij}(x_i, x_j) \triangleq \exp(-\frac{1}{2} x_i A_{ij} x_j) \qquad (6)$$

$$\varphi_i(x_i) = \exp(-\frac{1}{2} A_{ij} x_i^2 + b_i x_i) \qquad (7)$$

Therefore, the GaBP algorithm transforms the problem of solving linear equations into a probabilistic reasoning problem of solving graphs, as shown in Figure 3.
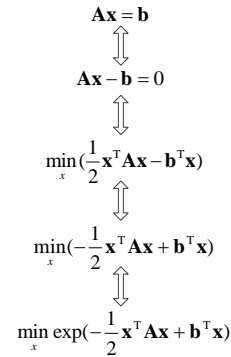

Figure 3. Matrix inversion conversion probability reasoning

The GaBP algorithm updates and iterates through the message exchange between nodes and the update and transfer of external information. The GaBP algorithm passes messages through edges, and is carried out by two calculation rules: "addition and multiplication" and "multiplication". The difference between GaBP algorithm and BP algorithm is that its probability density distribution obeys Gaussian distribution. As shown in Figure 4, it shows the message transfer between node i and neighbor

nodes. Each node has its own potential function ϕ, and the edge between nodes has an edge potential function ψ. The message m_{ij} is propagated between nodes along the edge, so the message transfer only needs to calculate the m_{ij}.
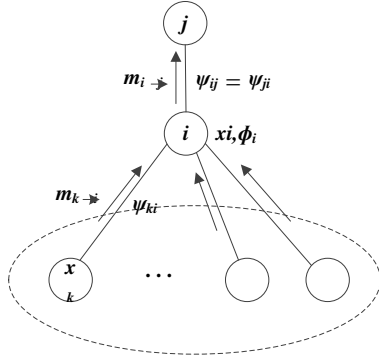

Figure 4. Message passing of node i

Define the following symbols: edge set $\{i, j\}$ includes all non-zero entries of $i > j$ in matrix A, node set $N(i)$ represents all adjacent sets of the i - th node (not including i), $N(i) \setminus j$ means that the node set $N(i)$ does not contain node j.

The BP algorithm passes messages through edges, and is carried out by two calculation rules: "addition and multiplication" and "multiplication". The message $m_{ij}(x_j)$ indicates that the message is transferred from the node i to the node j on the same edge, and the message $m_{ij}(x_j)$ is expressed as:

$$m_{ij}(x_j) \propto \int_{x_i} \psi_{ij}(x_i, x_j) \varphi_i(x_i) \prod_{k \in N(i) \setminus j} m_{ki}(x_i) dx_i \qquad (8)$$

Use the "multiplication" rule to calculate the edge:

$$p(x_i) = \alpha \varphi(x_i) \prod_{k \in N(i)} m_{ki}(x_i) \qquad (9)$$

Where α is the normalization constant.

The GaBP algorithm process can be divided into three parts: initialization, iteration and solving. The iteration part is carried out by two steps: message accumulation and message update. The algorithm steps are as follows:

A. Initialization

$P_{ii} \triangleq A_{ii}$, $\mu_{ii} \triangleq b_i$; $P_{ij} = 0$, $\mu_{ij} = 0 (i \neq j)$; $P_{ki} = 0$, $\mu_{ki} = 0$

B. Iteration

Message accumulation:

$$\tilde{P}_i = P_{ii} + \sum_{k \in N(i)} P_{ki}, \quad \tilde{\mu}_i = \mu_{ii} + \sum_{k \in N(i)} \mu_{ki}$$

Message update:

$$P_{i \setminus j} = \tilde{P}_i - P_{ji}, \quad \mu_{i \setminus j} = \tilde{\mu}_i - \mu_{ji}$$

$$P_{ij} = -A_{ij}^2 P_{i \setminus j}^{-1}, \quad \mu_{ij} = -P_{i \setminus j}^{-1} A_{ij} \mu_{i \setminus j}$$

C. Solve

$$x_i = \tilde{\mu}_i / \tilde{P}_i$$

4. Numericalresults

In this section, we consider a downlink massive MIMO system with 128 BS antennas and 16 single antenna users. We consider the 64-QAM modulation scheme without channelcoding. Suppose H is already known for the BS and its elements are independent identically distributed with 0 mean unit variance. The *SNR* of the downlink transmission is $SNR = M \cdot E_s / N_0$.

4.1. BER Performance

The simulation in this section will be divided into two cases for comparative analysis: the first case analyzes the performance and convergence speed of the GaBP algorithm; the second case analyzes and compares the performance of the GaBP algorithm with other sub-optimal precoding algorithms.

A. Compare the BER performance of the GaBP algorithm with different iteration times

Figure 5 shows the BER performance of GaBP algorithm under MMSE precoding. The number of iterations is two to five. It can be seen from the figure that the GaBP algorithm has poor convergence in the case of two iterations; in three iterations, its BER performance can reach 3×10^{-4} at 18dB, but its performance is far from MMSE precoding. Obviously; the BER performance of the fourth iteration can reach 5×10^{-5} at 18dB; the fifth iteration of the GaBP algorithm achieves approximate convergence, and its performance is only 1dB worse than that of the MMSE algorithm. However, using the GaBP algorithm to perform sub-optimal precoding can reduce the complexity of MMSE precoding $O(N^3)$ to $O(N^2)$, and can achieve similar MMSE precoding performance after multiple iterations.
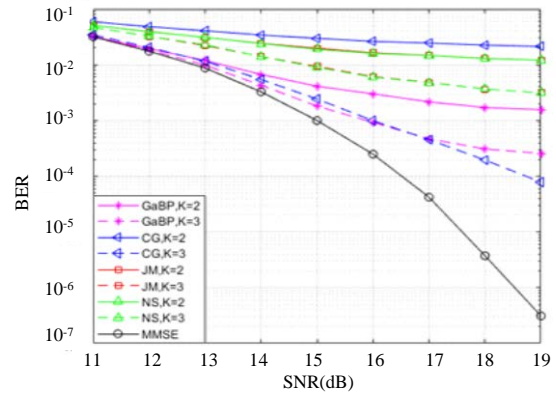

Figure 5. GaBP precoding with different iteration times K
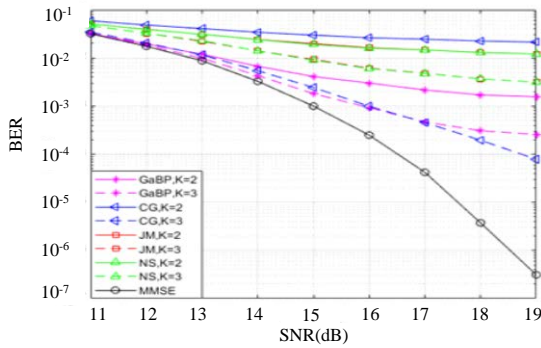
HK.NCCP

Figure 6. Compared with other traditional methods compared with other traditional methods

In Figure 6, the number of iterations $K = 2$, it can be seen that the BER performance of the GaBP algorithm is better than that of the CG, JM, and NS methods, which shows that the GaBP precoding has better BER performance when the number of iterations is low. When the number of iterations $K = 3$, the BER performance of the GaBP algorithm is much better than that of the JM and NS method. When the SNR is less than 17, the BER performance of GaBP is better than that of the CG method. When the SNR is greater than 17, the BER performance of GaBP is better than that of CG. The method is slightly worse. In short, the precoding performance of the GaBP algorithm is better than most traditional methods, and it also has faster convergence and lower BER performance. It can achieve performance close to MMSE precoding.

4.2. Computational complexity

The computational complexity of the algorithm can be evaluated according to the number of adders, multipliers and dividers used [6]. In MMSE precoding, the main computational complexity overhead lies in the multiplier, so this paper uses the number of real multipliers to evaluate the complexity of the algorithm. MMSE precoding itself needs to calculate $A = HH^H + \sigma^2 I$, and other suboptimal precoding algorithms also need to calculate matrix A, so the complexity of this part of the calculation is ignored in the comparison, and only the direct inversion of MMSE precoding is compared with other algorithms. The complexity of the comparison. Where K represents the number of iterations of the algorithm.

As shown in Figure 7, the computational complexity of the JM, GaBP, and CG methods are far lower than that of the MMSE method, and the computational complexity of the NS method is higher than that of the MMSE method when K is greater than 3.
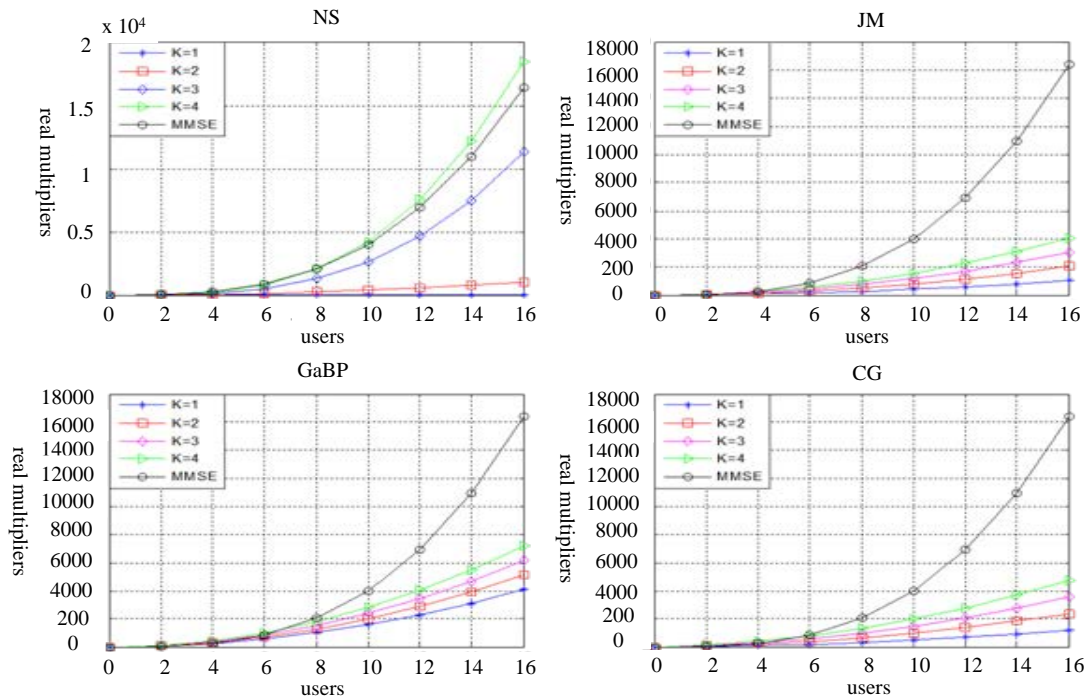

Figure 7. Computational complexity comparison

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