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Incremental Approach for Updating Knowledge in Dynamic Data

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Abstract: Dynamic data from real applications often occurs when new attributes or objects are inserted or old ones are removed. In the framework of rough set theory, set approximation is one type of important knowledge which need to be updated from time to time for further data reduction and analysis. Some incremental methods have been proposed either for the variations in attribute set or in the object set. In this paper, we combine the two dynamic situations and give the principles and corresponding algorithms to incrementally updating approximations. The experimental results and analysis on five data sets from UCI show that the incremental approach outperforms the traditional non-incremental method especially in the dynamic situation of removing attributes.

Keywords: Dynamic information systems; Incremental approach; Set approximations

1. Introduction

Rough set theory (RST) proposed by Pawlark is a relatively new mathematical tool to deal with inconsistency and ambiguity information [1]. In RST, uncertain concepts are described by two crisp sets: lower approximation and upper approximation. Based on the set approximations, knowledge reduction and decision rules extraction can be further implemented. As a simple and effective information processing tool, RST has been widely applied to data mining [2, 3] and decision making [4, 5]. To deal with preference-ordered information, Greco et al. firstly developed the framework of dominance relation-based rough set approach (DRSA) where dominance relations substitute equivalence relations to describe the preference orders of attribute values [6-8].

However, these rough set approaches assume that the involved data sets are static. In real applications, the collected data often update from time to time. The variations mainly include the insertion/removal of attributes/objects. When any change occurs, traditional RST is very computational intensive to update the dominance classes, set approximations, attribute reductions, and then decision rules. In RST and its extensions, efficient incremental approaches have been proposed to deal with the variation of attributes sets or object sets. According to the types of involved variations, the studies can be classified into three groups which are conducted under the variations of the attribute set [9-13], the object set [14-25] and the attribute values [26-29], respectively.

Computing set approximations is a necessary step for further knowledge reduction and decision-making in rough sets. The above mentioned studies consider the variations of attribute set and object set separately, and they are effective when either the attribute set or the object set evolves over time in information systems. However, the variations often happen in both of them. In this paper, focusing on multi-criteria classification problems [30-31], we develop an incremental approach for updating approximations of DRSA under the variations of both attributes and objects. We consider two types of dynamic environments by combing the insertion/removal of attributes/objects. In each of the dynamic environment, the updating rules of approximations are presented and experimental results are given to show the merits of our proposed incremental approach.

2. Preliminaries

As prior knowledge, some basic concepts in dominance-based rough set are briefly introduced in this section, including target information system, dominance (dominated) class, and upper/lower set approximations.

Definition 1 (Target information system) A 4-tuple $S=(U,A,V,f)$ is called as a target information system, where U is a non-empty set of objects ; A is a non-empty set of attributes. $A=C\cup d$, where C is a set of conditional attributes and d is a decision attribute. V is the set of attribute values, and $f:U\times A\rightarrow V$, assigning each attribute of each object with a value in V .

Definition 2 (Dominance/dominated relation) Let $S=(U,A,V,f)$ be a target information system, $B\subseteq A$, we denote:

$$R_B^{\le} = \{(x_i, x_j) \in U \times U : f(x_i, a) \le f(x_j, a), \forall a \in B\} \quad (1)$$

$$R_B^{\ge} = \{(x_i, x_j) \in U \times U : f(x_i, a) \ge f(x_j, a), \forall a \in B\} \quad (2)$$

$R_B^{\le}$, $R_B^{\ge}$ are the dominance and dominated relations of the information system, respectively.

Definition 3 (Dominance matrix) For a given information system S, $\forall a \in C$, $x_i, x_j \in U$, we use m_{ij}^a to describe the relation between objects x_i and x_j:

$$m_{ij}^a = \begin{cases} 1: f(x_i, a) \ge f(x_j, a) \\ 0: f(x_i, a) < f(x_j, a) \end{cases} \quad (3)$$

where $m_{ij}^a = 1$ means that x_i dominates x_j with respect to attribute a; while $m_{ij}^a = 0$ implies x_j dominates x_i with respect to a. Then define the matrix

$M^a = [m_{ij}^a]_{|U|\times|U|} = \begin{bmatrix} m_{1,1}^a & \cdots & m_{1,|U|}^a \\ \vdots & \ddots & \vdots \\ m_{|U|,1}^a & \cdots & m_{|U|,|U|}^a \end{bmatrix}$ as the dominance matrix w.r.t attribute a.

Here $|*|$ denotes the number of elements in set $*$.

Based on the notion of dominance matrix, dominance class can be defined. For a subset of attributes $P \subseteq C$, $\forall a \in P$, define $f_{ij}^P = \sum_{a \in P} m_{ij}^a$ to denote the relationship between x_i and x_j. If $f_{ij}^P = |P|$, then x_i is in the dominance class of x_j with respect to P.

Definition 4 (Dominance/Dominated class) For an object $x_i \in U$, its dominance/dominated classes are denoted by $D_P^+(x_i)$ and $D_P^-(x_i)$, respectively.

$$D_P^+(x_i) = \{x_j \in U : m_{ji}^P = |P|\} \quad (4)$$

$$D_P^-(x_i) = \{x_j \in U : m_{ij}^P = |P|\} \quad (5)$$

Definition 5 (Lower/Upper approximations) For any subset of objects $X \subseteq U$, define the upper and lower approximations of X with respect to dominance relation as follows:

$$\underline{R}^{\le}(X) = \{x_i \in U : D_P^+(x_i) \subseteq X\} \quad (6)$$

$$\overline{R}^{\le}(X) = \{x_i \in U : D_P^+(x_i) \cap X \ne \varnothing\} \quad (7)$$

which are respectively called the lower and upper approximation of X with respect to dominance relation R.

3. The Principles of Updating Lower and Upper Approximations

In this section, we discuss two types of dynamic environments which are often encountered in information systems, including the combinations of inserting/deleting attributes and objects. In each dynamic situation, the previous information will be reused to update the set approximations of a given subset of objects $X \subseteq U$, and therefore the computation time could be saved a lot. For convenience, in following sections, we denote the updated universe, the given subset of objects, lower and upper set approximations by U', X', $\underline{R}^{\le}(X')$, $\overline{R}^{\le}(X')$, respectively.

The basic idea is to firstly separate the changes of the attribute set from those of the object set, and update dominance classes after the change of attribute set. Then based on the updated dominance classes, the change of object set is considered. Finally, the lower and upper approximations are updated incorporating both the changes of attribute set and object set. The updating rules in each situation are given with proofs as follows.

3.1. Insertion of attributes and an object

Firstly, consider the change of attribute set. When some attributes (denoted by a subset of attributes Q) are added into the original attribute set P, the dominance classes become smaller. This can be described by the following Lemma 1: Let $P \subseteq C, Q \subseteq C$ and $P \cap Q = \varnothing$. For $\forall x \in U$,

$$D_{P\cup Q}^+(x) \subseteq D_P^+(x),\ D_{P\cup Q}^-(x) \subseteq D_P^-(x) \quad (8)$$

The dominance classes are updated based on the concept of dominance matrix. Here we suppose that the original dominance matrix on attribute set P has been given, it is only needed to compute the matrix on Q (which is usually smaller than P), and then new dominance classes can be computed.

Secondly, consider the change of the object set. Suppose one object x^+ is added and then the new universe is $U' = U \cup \{x^+\}$. The dominance classes should be updated again as follows.

Lemma 2: After an object x^+ is inserted into the information system, based on the results of Lemma 1, the dominance classes can be updated as:

$$D'^{+}_{P\cup Q}(x) = \begin{cases} D_{P\cup Q}^+(x) \cup \{x^+\} & x \in D_{P\cup Q}^-(x^+) \\ D_{P\cup Q}^+(x) & else \end{cases} \quad (9)$$

Finally, according to the relation between x^+ and the given subset $X \subseteq U$, we have the following propositions.

Proposition 1: If $x^+ \notin X'$, i.e., $X' = X$, then:

$$\underline{R}^{\le}(X') = \underline{R}^{\le}(X) \cup \{x \in X - \underline{R}^{\le}(X) : D'^{+}_{P\cup Q}(x) \subseteq X\} - \{x \in \underline{R}^{\le}(X) : x^+ \in D'^{+}_{P\cup Q}(x)\}; \quad (10)$$

$$\overline{R}^{\le}(X') = \overline{R}^{\le}(X) \cup \{x^+ : D_{P\cup Q}^+(x^+) \cap X \ne \varnothing\} - \{x \in \overline{R}^{\le}(X) : D'^{+}_{P\cup Q}(x) \cap X = \varnothing\}. \quad (11)$$

Proof: The proposition can be proved based on Lemma 1 and 2. The details are omitted.

Proposition 2: If $x^{+} \in X'$, i.e., $X' = X \cup \{x^{+}\}$, we have

$$\underline{R}^{\leq}(X') = \underline{R}^{\leq}(X) \cup \left\{x \in X' - \underline{R}^{\leq}(X) : D'^{+}_{P\cup Q}(x) \subseteq X'\right\}; \quad (12)$$

$$\overline{R}^{\leq}(X') = \overline{R}^{\leq}(X) \cup \left\{x \in U' - \overline{R}^{\leq}(X) : x^{+} \in D'^{+}_{P\cup Q}(x)\right\} - \left\{x \in \overline{R}^{\leq}(X) : D'^{+}_{P\cup Q}(x) \cap X' = \varnothing\right\}. \quad (13)$$

Proof: Omitted.

3.2. Removal of attributes and an object

When both of attributes and an object are removed, we firstly delete the corresponding dominance matrixes of the removed attributes, and then easily update the dominance matrix as M^{P-Q}. Secondly, consider the removal of an object x^{-}. The corresponding row and column of x^{-} are deleted from M^{P-Q}, and then the dominance class $D^{+}_{P-Q}(x)$ of each $x \in U'$ is updated. Finally, the set approximations are computed by the following propositions. Due to the similar ideas to previous proofs and the limit length of this paper, we omit the proofs in this subsection.

Proposition 7: If $x^{-} \notin X$, i.e., $X' = X$, we have

$$\underline{R}^{\leq}(X') = \underline{R}^{\leq}(X) - \left\{x \in \underline{R}^{\leq}(X) : D^{+}_{P-Q}(x) \not\subset X\right\} \quad (14)$$

$$\overline{R}^{\leq}(X') = \overline{R}^{\leq}(X) \cup \left\{x \in U' - \overline{R}^{\leq}(X) : D^{+}_{P-Q}(x) \cap X \neq \varnothing\right\} - \left\{x^{-} : x^{-} \in \overline{R}^{\leq}(X)\right\} \quad (15)$$

Proof: Omitted.

Proposition 8: If $x^{-} \in X$, i.e., $X' = X - \{x^{-}\}$, then

$$\underline{R}^{\leq}(X') = \underline{R}^{\leq}(X) - \left\{x \in \underline{R}^{\leq}(X) : D^{+}_{P-Q}(x) \not\subset X'\right\} - \left\{x^{-} : x^{-} \in \underline{R}^{\leq}(X)\right\} \quad (16)$$

$$\overline{R}^{\leq}(X') = \overline{R}^{\leq}(X) \cup \left\{x \in U' - \overline{R}^{\leq}(X) : D^{+}_{P-Q}(x) \cap X' \neq \varnothing\right\} - \left\{x \in \overline{R}^{\leq}(X) :\ D^{+}_{P-Q}(x) \cap X = \{x^{-}\}\right\} - \left\{x^{-}\right\} \quad (17)$$

Proof: Omitted.

4. Incremental Updating Algorithms

Based on the updating principles in Section 3, we provide the incremental algorithms to implement the proposed approach, which are called Incremental algorithms 1-2.

Incremental algorithm 1:

Based on proposition 1-2, the incremental algorithm is given when inserting both attributes and an object.

Input: $\forall x \in U$, M^{P}, X, $\overline{R}^{\leq}(X), \underline{R}^{\leq}(X)$, Q, x^{+}

Output: $\overline{R}^{\leq}(X'), \underline{R}^{\leq}(X')$

1. Begin
2. Compute M^{Q}, and update $D^{+}_{P\cup Q}(x)$
3. Compute $D^{+}_{P\cup Q}(x^{+}), D^{-}_{P\cup Q}(x^{+})$
4. For i=1,…,$|U'|$
5. If $\left(x_i \in X' - \underline{R}^{\leq}(X)\right)$ & $\left(D^{+}_{P\cup Q}(x_i) \subseteq X'\right)$ then
6. $\underline{R}^{\leq}(X') \leftarrow \underline{R}^{\leq}(X) \cup \{x_i\}$
7. End
8. If $\left(x_i \in \underline{R}^{\leq}(X)\right)$ & $\left(D^{+}_{P\cup Q}(x_i) \not\subset X'\right)$ then
9. $\underline{R}^{\leq}(X') \leftarrow \underline{R}^{\leq}(X) - \{x_i\}$
10. End
11. If $\left(x_i \in U' - \overline{R}^{\leq}(X)\right)$ & $\left(x^{+} \in D^{+}_{P\cup Q}(x_i)\right)$ then
12. $\overline{R}^{\leq}(X') \leftarrow \overline{R}^{\leq}(X) \cup \{x_i\}$
13. End
14. If $\left(x_i \in \overline{R}^{\leq}(X)\right)$ & $\left(D^{+}_{P\cup Q}(x_i) \cap X' = \varnothing\right)$ then
15. $\overline{R}^{\leq}(X') \leftarrow \overline{R}^{\leq}(X) - \{x_i\}$
16. End
17. End
18. Return $\overline{R}^{\leq}(X'), \underline{R}^{\leq}(X')$
19. End

Incremental algorithm 2:

Based on propositions 7-8, the incremental algorithm 4 is given to update the set approximations when removing some attributes and one object.

Input: $\forall x \in U$, M^{P}, $\overline{R}^{\leq}(X), \underline{R}^{\leq}(X)$, x^{-}, Q

Output: $\overline{R}^{\leq}(X'), \underline{R}^{\leq}(X')$

1. Begin
2. Compute M^{P-Q} and update $D^{+}_{P-Q}(x)$;
3. If $x^{-} \in \underline{R}^{\leq}(X)$ then $\underline{R}^{\leq}(X') \leftarrow \underline{R}^{\leq}(X) - \{x^{-}\}$
4. End
5. If $x^{-} \in \overline{R}^{\leq}(X)$ then $\overline{R}^{\leq}(X') \leftarrow \overline{R}^{\leq}(X) - \{x^{-}\}$
6. End
7. For i=1,…, $|U'|$
8. If $\left(D^{+}_{P-Q}(x_i) \cap X'\right) \neq \varnothing$ then
9. $\overline{R}^{\leq}(X') \leftarrow \overline{R}^{\leq}(X) \cup \{x_i\}$
10. End
11. If $\left(x_i \in \underline{R}^{\leq}(X)\right)$)& $\left(D^{+}_{P-Q}(x_i) \not\subset X'\right)$ then
12. $\underline{R}^{\leq}(X') \leftarrow \underline{R}^{\leq}(X) - \{x_i\}$
13. End

14. If $\left(x_i \in \overline{R^{\leq}}(X)\right) \& \left(D^{+}_{P-Q}(x_j) \cap X'\right) = \varnothing$ then
15. $\overline{R^{\leq}}(X') \leftarrow \overline{R^{\leq}}(X) - \{x_j\}$
16. End
17. End
18. Return $\overline{R^{\leq}}(X'), \underline{R^{\leq}}(X')$
19. End

5. Experimental Results and Analyses

In this section, we use five data sets from UC Irvine Machine Learning Database Repository [32] to validate our proposed incremental methods under variations of both attribute set and object set. The number of objects (i.e., samples), attributes and classes are summarized in Table 1. Notice that in these data sets, only the conditional attribute values have preference orders and therefore the dominance relations should be introduced, while the decision attribute is nominal without order in their values. The decision attribute induces equivalence classes and these classes can form a partition of the universe.

Table 1. Data sets

Data set	Number of objects	Number of attributes	Number of Class
Hepatitis	155	19	2
Sonar	208	60	2
Ionosphere	351	34	2
Statlog	440	20	2
Climate	540	18	2

In each type of environments, the traditional dominance-based rough set approach and the proposed incremental algorithms 1-2 are implemented in the experiments to compare their performance. All the algorithms are coded by MATLAB7.8 on Windows 7.0 with Inter(R)i5-2400@3.10GHz and 4GB memory.

In our experiments, the inserted or removed attributes are randomly selected from the original attribute set in each data set. The removed object is also randomly selected from the universe of the data set. When inserting an object, we randomly generated a combination of attribute values in the corresponding domain.

In all the experiments, the traditional dominance relation-based rough set approach (DRSA) [13] is used as the non-incremental algorithm. When any variation occurs, DRSA needs to compute all dominance classes and then updates the lower and upper approximations. Our proposed incremental algorithms 1-2 reuse the available information of the original dominance classes and set approximations, and thus reduce the computational cost for updating set approximations in each of the two dynamic situations.

The experimental results are shown in Tables 2-3, where the running time of incremental and non-incremental algorithms is compared, and the unit is second. The enhance ratio is the ratio between the running time of non-incremental algorithm and the incremental algorithm which describe how much the computational time is reduced by the incremental algorithm. Tables 2-3 show the results with insertion/removal of two attributes; while Tables 8-11 show those with the variations of four attributes.

From Tables 2-3, we can observe that the proposed incremental algorithms have obviously reduced the computational cost of non-incremental method. If we compare the enhance ratios, it can be found that different improvement degrees in the computational time are obtained in different dynamic situations. Generally, larger enhance ratios are obtained by incremental algorithms 2 compared with those of incremental algorithms 1. This means that the incremental algorithms are more advantageous in situations when removing attributes. On the other hand, in the same dynamic situation, with increasing number of objects and attributes in data sets, the benefit is more obvious using incremental algorithms. For example, on Climate data which has the largest number of objects, the enhance ratio attains more than 14% (see Table 3). The number of attributes also has obvious impact on the algorithms' efficiency, e.g., since Sonar has more attributes, the improvements of efficiency using incremental algorithms are greater than those of other data sets.

Table 2. Incremental algorithm 1 vs. non-incremental algorithm (insert four attributes and an object)

Data sets	Non-incremental algorithm (s)	Incremental algorithm 1 (s)	Enhance Ratio(%)
Hepatitis	0.2587	0.1060	2.4406
Sonar	1.1009	0.1436	7.6664
Ionosphere	2.6315	0.5340	4.9279
Statlog	3.4837	0.8740	3.9859
Climate	5.1885	1.0628	4.8819
Average	2.5327	0.5408	4.7805

Table 3. Incremental algorithm 4 and non-incremental algorithm (remove four attributes and an object)

Data sets	Non-incremental algorithm (s)	Incremental algorithm 4 (s)	Enhance Ratio(%)
Hepatitis	0.2190	0.0504	4.3452
Sonar	0.8634	0.0524	16.4771
Ionosphere	2.0504	0.1534	13.3664
Statlog	2.7022	0.2236	12.0850
Climate	4.3297	0.2997	14.4468
Average	2.0329	0.1559	12.1441

6. Conclusions and Discussions

In this paper, we have considered two types of dynamic environments which may be encountered in dealing with information or decision systems. In each type of such environments, both the variations of attribute set and object set are incorporated. When any of the variations happens, it is important to update the lower/upper approximations of concepts in the universe to further facilitate updating of reductions and decision rules. By reusing the available information, we have proposed updating rules and corresponding incremental algorithms for the four difference types of dynamic environments. Experimental studies have been done on UCI data sets to show the effectiveness of the proposed algorithms. These algorithms have significantly improved the efficiency of non-incremental algorithm in updating the set approximations. Notice that, except for considering the variations of attribute set, we incorporate the variations of only one object. In fact, if multiple objects are removed or inserted, we can repeatedly use the updating rules after one object is removed or inserted [23].

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