

Semi-analytical Method for Stress Intensity Factor of Pipelines with MSD Structure

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Abstract: Multiple site damage (MSD) is a common form of damage in the service of nuclear pipeline. The typical structure is multiple circumferential internal surface cracks of the pipeline. In order to evaluate the reliability of the nuclear pipeline and discuss the damage tolerance of the pipeline with multiple cracks, a semi-analytical method for stress intensity factor (SIF) of pipelines with MSD structure is proposed. The SIF of multiple cracks are studied by using crack distance, length and depth as single factor variable. The results show that the SIF of crack is influenced by the distance, length and depth of the adjacent crack. Comparing with the finite element analysis results, the feasibility of this method is verified.

Keywords: Pipeline; MSD; Stress intensity factor; Compounding method

1. Introduction

With the increasing of service life of nuclear power plants, nuclear accidents happen frequently. As an important part of nuclear power equipment, the reliability assessment of safety life of nuclear pipeline has attracted extensive attention of the industry [1]. In the past, the research object is only limited to single crack in pipeline, and the research on multiple crack problem is a little rough. However, the multiple site damage (MSD) structure of the inner surface of the pipe is a typical damage form [2, 3] in actual processing or use, as shown in Fig.1, where the ABC and A'B'C' areas are surface crack shapes.

The commonly used methods to solve MSD stress intensity factor (SIF) are: boundary element method [4], crack growth method and crack closure method [5], fi-

nite element method [6]. The surface crack belongs to the category of non-through crack, which is difficult to discuss. However, due to its complexity, the surface multiple crack problem cannot get accurate analytical solution in most cases. At present, three-dimensional finite element solution is considered as the most accurate SIF solution for surface cracks because it consists well with the experimental results. However, the three-dimensional finite element mesh is very dense, and the calculation work is quite heavy. In this paper, a semi-analytical method for solving the stress intensity factor of MSD structure of pipeline is proposed, and the actual change law of the SIF in the direction of crack depth (point A) under the action of multiple cracks in nuclear pipeline is studied.

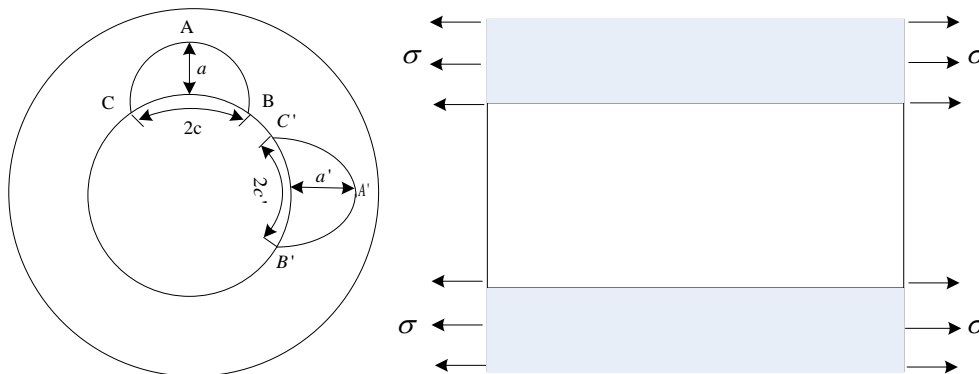


Figure 1. MSD Structure and loading mode of inner surface of nuclear pipeline

2. Compounding Method

2.1. Basic principle

The compounding method [7] is an approximate calculation method of MSD stress intensity factor based on the principle of constraint substitution and superposition of theoretical mechanics. Under N boundaries far away from each other, the interaction of each boundary is omitted according to the local effect, and the stress intensity factor of the research object can be obtained. The common multiplicative compounding solution in engineering is to multiply the correction coefficients to solve the stress intensity factor in complex cases, which can be expressed as

$$K = K_0 f = K_0 \prod_{i=1}^N f_i \quad (1)$$

Where K_0 is the SIF of the basic situation, K_i is the SIF of the i th situation, N is the number of simple situations which be divided into, f_i is the SIF correction factor of the i th simple situation.

2.2. Calculation function of SIF of MSD structure constructed by compounding method

Taking the nuclear pipeline as an example, which subjects to the axial tensile stress σ . The basic situation of MSD problem on the internal surface of the pipeline is a semi-infinite body with a single semi-elliptical crack, which subjects to the tensile load σ in the z direction, as shown in Figure 2.

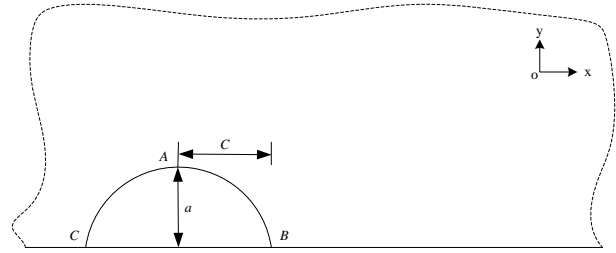


Figure 2. Semi infinite body with single surface crack

The research object is mainly affected by two factors: the pipeline boundary and the adjacent crack. The stress intensity factor of point A of the crack is

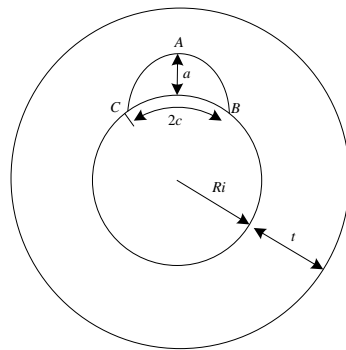
$$K_A = K_{A0} f_p f_l \quad (2)$$

Where K_{A0} is the SIF of the basic situation, f_p is the SIF correction factor of the pipe boundary, f_l is the SIF correction factor of the adjacent crack.

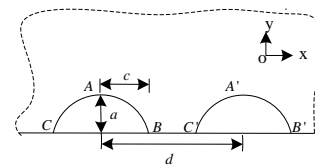
By using the multiplicative compounding solution in reverse, the compounding SIF calculation function is obtained

$$K_A = K_{Ap} f_l \quad (3)$$

Where K_{Ap} is the SIF of single crack on the inner surface of the pipe, f_l is the SIF correction factor of the adjacent crack of MSD semi-infinite body. The structure diagram is shown in Fig.3.



(a) Single crack on the inner surface of the pipe



(b) Coplanar double cracks in the semi-infinite body

Figure 3. Structure of influencing factors on MSD crack of pipeline

2.3. Solution of K_{Ap} and f_l

Referring to the European Manual of stress intensity factors [8], the stress intensity factors of crack in depth direction of pipeline are

$$K_I = \sqrt{\pi a} \left(\sum_{i=0}^3 \sigma_i f_i \left(\frac{a}{t}, \frac{2c}{a}, \frac{R_i}{t} \right) + \sigma_{bg} f_{bg} \left(\frac{a}{t}, \frac{2c}{a}, \frac{R_i}{t} \right) \right) \quad (4)$$

Where $\sigma_i (i=0-3)$ is the normal stress component along the crack surface, σ_{bg} is the axial bending stress,

$f_i\left(\frac{a}{t}, \frac{2c}{a}, \frac{R_i}{t}\right)$ and $f_{bg}\left(\frac{a}{t}, \frac{2c}{a}, \frac{R_i}{t}\right)$ are the corresponding shape correction factor.

By analyzing the crack shape and loading mode, as shown in Fig.3(a), the stress intensity factor of the crack K_I is

$$K_I = \sqrt{\pi a} \left(\sigma_0 f_0 \left(\frac{a}{t}, \frac{2c}{a}, \frac{R_i}{t} \right) \right) \quad (5)$$

The shape correction factor $f_0\left(\frac{a}{t}, \frac{2c}{a}, \frac{R_i}{t}\right)$ can be solved by the corresponding analytical formula or obtained by looking up the table according to the size parameters. By plugging $\sigma_0 = \sigma$ and shape coefficient f_0 , the analytical solution of SIF of cracks can be obtained. The result is the stress intensity factor K_{AP} of the pipe with single surface crack

$$K_{AP} = \sqrt{\pi a} \left(\sigma f_0 \left(\frac{a}{t}, \frac{2c}{a}, \frac{R_i}{t} \right) \right) \quad (6)$$

SIF of coplanar double cracks as Fig.3 (b) shown is solved by the 1/4 node method in the finite element method [9]. The hexahedral 20 node isoparametric element is selected to analyze the semi-infinite body with coplanar surface cracks. By changing the crack center distance d , a series of SIF values of point A at the front edge of the crack are obtained [10]. The normalized SIF value of point A is the SIF correction factor of the adjacent crack f_i . In order to make the solution of the coefficient more universal, the spacing ratio λ is used as the dependent variable of f_i , $\lambda = 2a / d$.

Fig.4 shows the FEM value of the correction coefficient and its fitting curve. By using the polynomial fitting function of Origin software, the finite element fitting solution of the SIF correction factor f_i can be obtained as follows

$$f_i = 1.01729 + 0.01722\lambda + 0.04991\lambda^2 \quad (7)$$

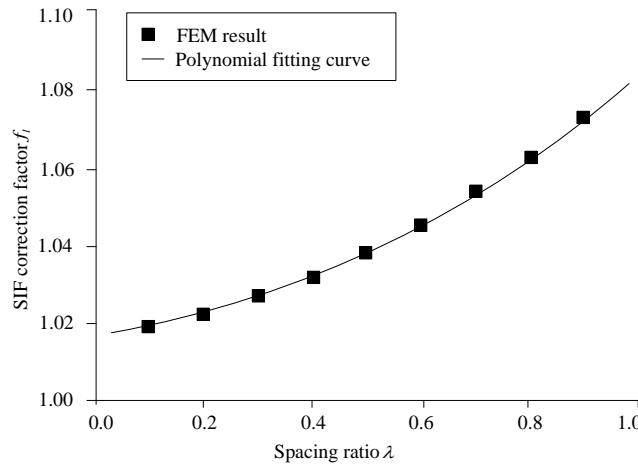


Figure 4. FEM value of correction coefficient and its fitting curve

3. Accuracy Verification of the Method

In order to discuss the calculation results, the analysis is carried out under the condition of load $\sigma = 50MPa$, pipe inner diameter $\sigma = 50MPa$ and pipe wall thickness $t = 10mm$. Select the fixed crack $a = 5mm$, $c = 5mm$, and the adjacent crack $a' = 5mm$, $c' = 5mm$. By chang-

ing the crack spacing l (the arc length between BC' in Fig.1), a series of SIF calculation values are obtained. The solution results are compared with the finite element results in reference [11], as shown in Tab.1. The results show that the calculation results are accurate and reliable, which verifies the accuracy of the semi-analytical method.

Table 1. Comparison of SIF between semi-analytical results and fem results at different distance

L(mm)	KA(MPa·mm ^{1/2})		
	Calculation result		
15	141.3372582	136.1203	3.8326
12	141.7844343	136.6945	3.7235
10	142.1881462	137.2244	3.6172
7	143.052441	138.373	3.3817
5	143.9100406	139.512	3.1524
3	145.1583871	141.1413	2.8461

2	146.0116199	142.3265	2.5892
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4. Analysis of MSD Structure of Inner Surface of Pipeline

In the discussion of method accuracy, it has been proved that the interaction of MSD crack stress intensity factors is related to crack spacing. Under the premise that the length and depth of fixed crack and adjacent crack do not change, calculate the stress intensity factor of crack depth direction (point A) under different crack spacing,

as shown in Fig.5. In the same way, the length and depth of the fixed crack and the adjacent crack are taken as a single variable to calculate the stress intensity factor of point A of the crack, as shown in Fig.6 and Fig.7. In each case, a group of crack pairs will get a data line. To make the calculation results show more information, select five groups of crack pairs to get a series of of data lines.

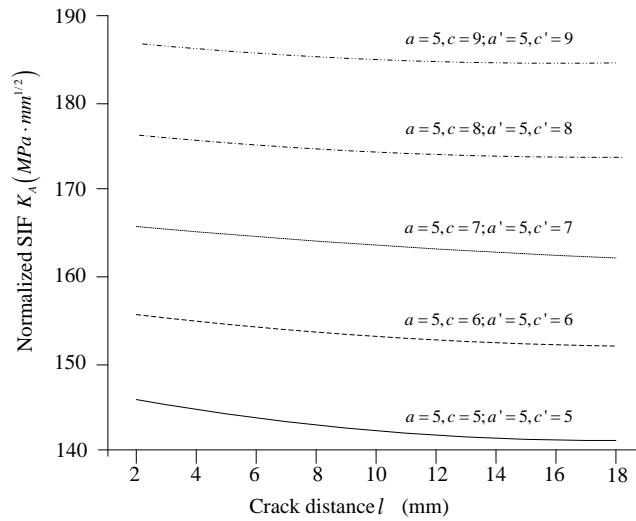


Figure 5. SIF calculation results of msd cracks under different crack distance

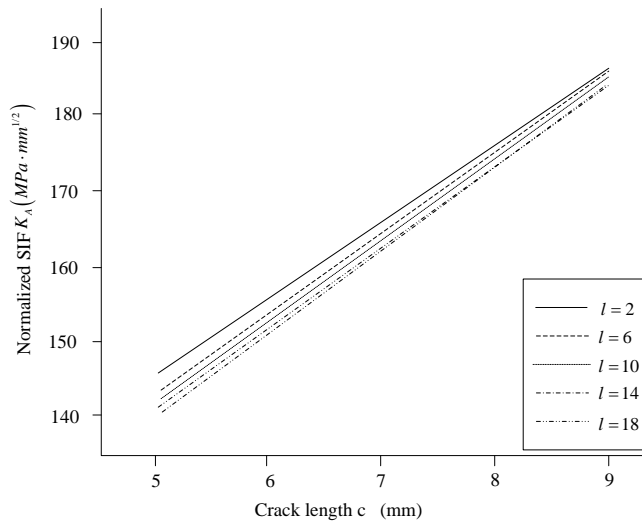


Figure 6. SIF calculation results of msd cracks under different crack length

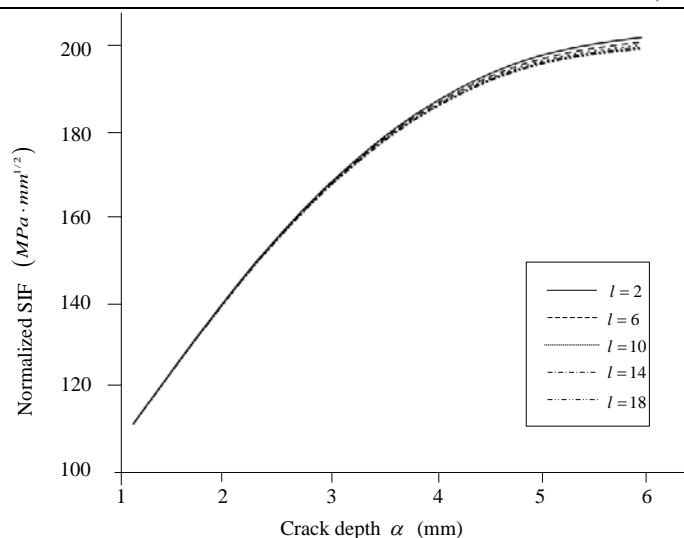


Figure 7. SIF calculation results of msd cracks under different crack depth

Through the analysis of the calculation results, the following conclusions can be drawn.

(1) The stress intensity factors of MSD cracks on the inner surface of pipes will be affected by self cracks and adjacent cracks. The distance, length and depth of the crack will affect MSD cracks.

(2) When the crack distance is a single factor, the SIF in the direction of crack depth decreases with the increase of crack spacing. With the increase of the distance, the influence of the crack itself gradually becomes the dominant factor of SIF, while the influence of the adjacent crack will gradually weaken.

(3) When the crack length is a single factor, the SIF in the direction of crack depth increases with it. At this time, the influence of the crack itself and the adjacent crack will be strengthened, which leads to a linear increase of SIF in the depth direction. This law of change has a certain significance for the further research.

(4) When the crack depth is a single factor, the SIF in the direction of crack depth increases with it. It should be noted that the growth rate of SIF is faster in the early stage of crack growth and slower in the later stage of crack growth. As the crack propagates from the inner surface of the pipe to the outer surface of the pipe along the thickness direction, the stress concentration of the crack is weakened by the arc shape of the outer contour when the crack propagates to the outer surface of the pipe.

5. Conclusion

In this paper, a method is proposed to solve the stress intensity factor of MSD cracks on the inner surface of nuclear pipes. By introducing the compounding method, a complex three-dimensional multiple crack

problem becomes two simpler problems. The SIF of a single crack in a pipe is solved by the analytical method, and the SIF of multiple cracks in a semi-infinite body is solved by the finite element fitting method. Compared with the finite element method, the method is simple in calculation process and reliable in calculation result, which is worth to be popularized in the multiple crack problem of pipeline.

6. Acknowledgment

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