Specific Analysis of the Basic Theory and Calculation Method of the Alignment of Suspension Bridge Cable

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Abstract: About the theoretical analysis and calculation method for the alignment of main cable of suspension bridge, many domestic and foreign researchers have conducted related researches. However, no specific derivation and interpretation of these researches has been provided in common textbooks of bridge engineering, and some researchers only know the result without understanding the process. In accordance with the differential equation of main cable, the parabolic equation of suspension cable under the uniformly distributed load along the span length and the suspension cable equation under the uniformly distributed load along the cable length can be derived. In addition, we also compare which equation has more accurate actual alignment through specific examples.

Keywords: Suspension bridges; Main cable; Parabola equation; Catenary equation; Numerical calculation analysis

1. Introduction

The cable is the main load bearing member of suspension bridge, and the accuracy of cable alignment is critical to the structural parameters of finished bridge, which is also the difficult point during the learning process. Based on current researches[1-2], this paper conducts specific theoretical calculation and analysis of the alignment under the uniformly distributed load along the span length and cable length, in this way to help the beginners to better understand this part.

2. Parabolic Calculation Method under the Uniformly Distributed Load Along the Span Length

When the suspension bridge has a small span, it can be calculated by adopting the elastic theory, and the elastic theory for suspension bridge calculation makes the following assumptions[3]:

1) The main cable is ideal and flexible, which is not under tension, and it is not under bending moment either. The dead load with horizontal even distribution makes the main cable present the geometrical shape of quadratic parabola, and the dead load is carried by the main cable.

2) The flexural rigidity of beam EI is constant along the beam length.

3) The densely arranged cables should be considered forming “film”, and it is assumed that under any circumstance, the main cable is under no stress, and the cable length maintains the same.

Calculation of parabola alignment of main cable: with the theoretical top of main cable left tower as the starting point of coordinate and vertical coordinate as y, the downward direction is positive.

For the coordinate orientation of main cable, see(Figure 1~ figure 2).

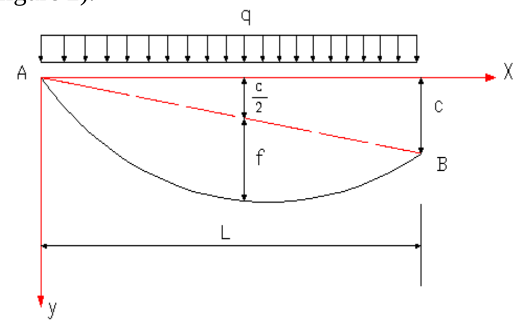

Figure 1. Along the span uniform load calculation diagram

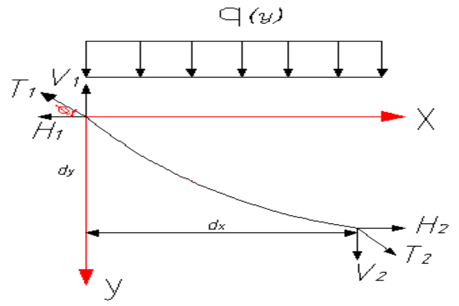

Figure 2. Cable diagram element

Assume the curve form of main cable is y=y(x), the tension of main cable is T, and the horizontal component of tension is H. In accordance with the balance condition of infinitesimal element, we can obtain:

$$\sum x = 0 \quad H1 = H2 = H \tag{1}$$

$$\sum y = 0 \quad V1= q(y) \cdot dx + V2 \tag{2}$$

In accordance with the infinitesimal element, we know:

$$dv1 + v1= v2 \tag{3}$$

$$V1= H\cdot \tan\varphi= H\cdot \frac{dy}{dx} \tag{4}$$

By combining (1), (2), (3) and (4), we can obtain the differential equation of main cable:

$$H\cdot\frac{d^2y}{dx^2}+q(y)=0 \tag{5}$$

When the main cable of suspension bridge is only under the vertical load, the tension at any point of main cable has equal horizontal component. Vertical load is evenly distributed along the span, q(y)=q, There are:

$$H\cdot\frac{d^2y}{dx^2}+q=0 \tag{6}$$

$$\frac{d^2y}{dx^2}=-\frac{q}{H} \tag{7}$$

Through two integrations of Formula (7), we can obtain:

$$y=-\frac{qx^2}{2H}+C1?x+C2 \tag{8}$$

By introducing the boundary condition: x = 0, y = 0; when x =L, y = c, we can obtain:

$C1=\frac{L}{C}+\frac{qL}{2H}$; C2=0

By substituting it into Formula (8), we can obtain the parabolic equation:

$$y=\frac{qx}{2H}\cdot(L-x)+\frac{C\cdot x}{L} \tag{9}$$

In Formula (9), q refers to the vertical load uniformly distributed load along the span; L is the calculated span; f is the midspan sag; C refers to the theoretical elevation difference between the two tower tops on two banks.

The horizontal component H of parabolic equation derived from the above formula is still unknown, and by substituting any point on the main cable into Formula (9), we can obtain H. The midspan sag f is known.

Take $x=\frac{L}{2}, y=\frac{L}{2}+f$; into Formula (9), we can obtain the horizontal force :

$$H=(qL^2)/8f \tag{10}$$

By combining (9) and (10), we can obtain the main cable of parabolic equation:

$$y=\frac{4f}{L^2}\cdot x(L-x)+\frac{C?x}{L} \tag{11}$$

If the top on both sides at the same height, C=0, Main cable of the parabolic equation:

$$y=\frac{4f}{L^2}\cdot x(L-x) \tag{12}$$

The micro-segment of cable length is $ds=\sqrt{dx^2+dy^2}$, and the cable length with stress at any point is S: S= $\int_0^x\sqrt{1+(\frac{dy}{dx})^2}$;

When x=L can be obtained: S= $\int_0^L\sqrt{1+(\frac{dy}{dx})^2}$ (13)

Using series expansion:

$$\sqrt{1+(\frac{dy}{dx})^2}=1+\frac{1}{2}(\frac{dy}{dx})^2-\frac{1}{8}(\frac{dy}{dx})^4+\frac{1}{16}(\frac{dy}{dx})^8-\ldots\ldots;$$

$$\frac{dy}{dx}=\frac{4f(L-2x)}{L^2}+\frac{C}{L} \tag{14}$$

The ratio of height to span suspension bridge between 1/9-1/12, So take the points of the first two ~ three have precise enough. a cross-cable in cable-length:

$$S=\int_0^L[1+(\frac{dy}{dx})^2]?dx \tag{15}$$

Through analysis of the micro-segment of main cable, we can obtain the elongation of main cable under the finished bridge state:

$$\Delta S=\frac{H}{EA}\int_0^L[1+(\frac{dy}{dx})^2]?dx \tag{16}$$

By substituting Formula (14) into Formula (16), we can obtain: $\Delta S=\frac{HL}{EA}\cdot(1+\frac{16}{3}?k^2)+\frac{c^2}{L^2}$ (17)

In equation (17): $k=\frac{f}{L}$;

E-- elastic modulus of main cables;

A-- area of the main cable,

So, bridge main cables under non-stress length:

$$S0 = S-\Delta S \tag{18}$$

Considering the impact of temperature on the elongation of main cable, assume the temperature difference is Δt, and the expansion coefficient of main cable is α, we can obtain:

$$S0=(S-\Delta S)?(1+t?\Delta t\cdot\alpha) \tag{19}$$

3. Catenary Calculation Method under the Uniformly Distributed Load Along the Main Cable Length

Under the free cable state, the alignment of main cable is catenary, and the basic assumptions [4-6]:

1) The cable works within the elastic scope, which satisfies Hooke's law.

2) The main cable is ideal and flexible, which is only under tension, and the bending moment of main cable can be ignored.

Calculation of main cable catenary: take a segment of free suspended cable with no elongation between the main cable hanging poles, its vertical coordinate is y, the downward direction is positive, and the weight of unit cable length is q ,see (Figure 3~ Figure 4).

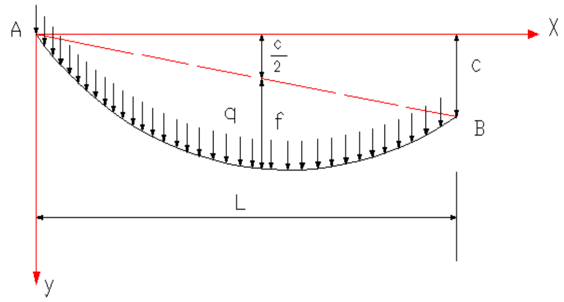

Figure 3. Along the cable lengths are calculated load

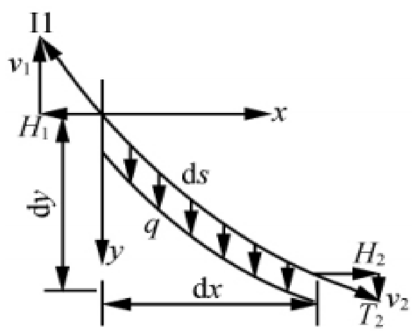

Figure 4. Cable diagram under micro-gravity

When the weight of the main cable of suspension bridge, its dead load distributed evenly along the cable length, cable lengths uniform load is q, there are:

$$q{\cdot}ds = q(y)\cdot dx \qquad (20)$$

The micro-segment of cable length is: $ds=\sqrt{dx^2+dy^2}$, Take into the (20) can be drawn:

$$q(y)=q?\sqrt{1+(\frac{dy}{dx})^2} \qquad (21)$$

By substituting Formula (21) into Formula (5), we can obtain:

$$H\cdot\frac{d^2y}{dx^2}+q\cdot\sqrt{1+(\frac{dy}{dx})^2}=0 \qquad (22)$$

Through two integrations of Formula (22), and by introducing the boundary condition: x = 0, y = 0; when x =L, y = c, we can obtain:

$$y=\frac{H}{q}[\cosh(\alpha)-\cosh(\frac{2\beta x}{L}-\alpha)] \qquad (23)$$

Of which : $\alpha=\sinh^{-1}[\frac{\beta(c/L)}{\sinh\sinh\beta}]+\beta$, $\beta=\frac{qL}{2H}$,

The micro-segment of cable length is: $ds=\sqrt{dx^2+dy^2}$, and the cable length with stress at any point is S: $S=\int_0^x\sqrt{1+(\frac{dy}{dx})^2}$; in accordance with Formula (23), we can obtain:

$$\frac{dy}{dx}=-\sin h(\frac{2\beta x}{L}-\alpha) \qquad (24)$$

By substituting Formula (24) into Formula (13), we can obtain the cable length with stress:

$$S=\frac{2H}{q}\cdot\sin h(\beta)\cos h(\alpha-\beta) \qquad (25)$$

Through analysis of the micro-segment of main cable, we can obtain the elongation of main cable under the finished bridge state: by substituting Formula (24) into Formula (16), we can obtain:

$$\Delta S=\frac{H^2S^2}{EAL^2} \qquad (26)$$

So, under the free cable state, the no-stress cable length of main cable is: S0 = S－ΔS

Considering the impact of temperature on the elongation of main cable, assume the temperature difference is Δt, and the expansion coefficient of main cable is α, we can obtain:

$$S0=（S－\Delta S)?(1+t?\Delta t\cdot\alpha)$$

$$V1=H\cdot sh\alpha;V2=\frac{q}{2}(c\cdot cth\beta-1)$$

4. Segment Catenary Calculation method for the Main Cable Alignment of Actual Finished Bridge

The main cable of suspension bridge is the catenary equation, and its actual alignment is segment catenary. Not considering the change in cross sectional area of main cable before and after the deformation, the weights of stiffening girder, sling and cable clamp all bear on corresponding location of main cable in the form of equivalent concentrated force Pi. [7-9](Figure5).

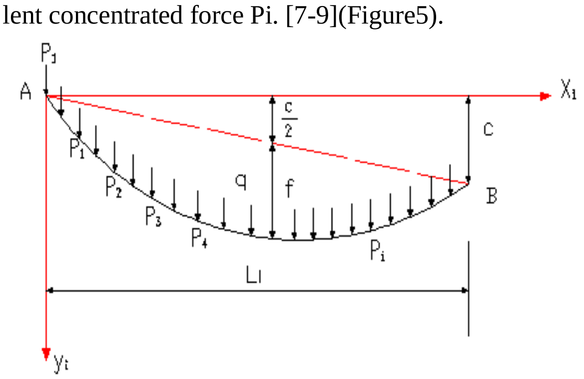

Figure 5. Mechanical model of linear suspension bridge

The suspended cable must be continuous at the segment of any two suspenders. The concentrated force divided the suspended cable into n segments, and the alignment equation of ith segment of main cable is:

$$yi = \frac{2H}{q}[\cos h(\alpha i) - \cos h(\frac{2\beta ixi}{Li} - \alpha i)] \quad (27)$$

Catenary geometry can be assured that the paragraph about the node value $\boldsymbol{\alpha}$ and the vertical force V:

$$\alpha(i) = a\sinh(V_{i,1}/H) \quad (28)$$

$$\beta i = \frac{qLi}{2H} \quad (29)$$

$$\alpha(i+1)\text{-}2\beta(i+1) = a\sinh(V_{i,2}/H) \quad (30)$$

$$V_{i,1} = H\sinh(\alpha) \quad (31)$$

$$V_{i,2} = -H\sinh(2\beta(i+1) - \alpha(i+1)) \quad (32)$$

In the above formula:
V—is the hanging point of vertical force;
H—Is the level of main cables;
Pi—Is the suspender Force;
Vi,1, Vi,2—Paragraph i is the catenary nodes around the vertical force.
Suspended cable at any point by point given, set Pi function in the left end of each section catenary, then hanging around Point section of the vertical force and Pi meet the Force equilibrium equations:

$$Pi = Vi,1 - Vi,2 \quad (33)$$

The catenary left and right ends of the vertical distance:

$$Ci = \frac{H}{q}[\cos h(\alpha i) - \cos h(2\beta i - \alpha i)] \quad (34)$$

From the above formula (27)-(34), We can worke out calculations required by the main cable main cable horizontal force H and the top vertical force V.
Therefore, you need to know rise and two cables of suspension bridge tower height difference can only determine the main cable line. From the above kinds of Calculation of main cable shape can be obtained by numerical iteration. Establish the iterative process is as follows:
(1) Assuming a set of initial values at the Tower level H0 ($\frac{qL^2}{8f}$ and vertical force V0 can be calculated based on the parabola method);
(2) Take H0 and V0 into (27), (28), (29), (32)can be obtained: $\alpha 1, \beta 1, c1, V2,1$;
(3) Take $\alpha 1, \beta 1, c1, V2,1$ into (27)-(34) can be obtained: $Vi+1,1$, αi, βi, ci, $Vi+1,1$;
(4) Use αi, βi, ci, $Vi+1,1$ instead of $\alpha 1, \beta 1, c1, V2,1$;
Repeat (3) steps until all paragraphs on the catenary to complete. Use the ci value of the hanging point's coordinates can be obtained.
(5) If the results meet the convergence conditions $\sum_{i=1}^{m} ci = f$ and $\sum_{i=1}^{n} ci = C$ Then, calculate the end, Otherwise amended H0 ($H_0 = H_0 + \Delta H$) V0 Repeat steps 2-4 until you meet the convergence criteria.

5. Calculation Example

For the parabolic and catenary calculation methods of suspended cable, corresponding iterative computation program is written for the above iterative process, it is verified with a suspension bridge in the mountainous area as the example, its main span is 208m, and its tower top elevation different is 0.
In a suspension bridge in Yunnan Province, the span arrangement is 25m + 208m + 25m, the rise span ratio is 1/11, it has 4 main cables in total, 2 cables on each side, each cable consists of 451 Φ5mm zinc-coated wires, and the dead load intensity on main cable is 1.39kN/m; the transverse spacing on main cable is 7.2 meters, and the conversion intensity is 74.195kN/m; the suspended cable uses 19 Φ5mm zinc-coated parallel wires to bond into cable, and the cable spacing is 4m,the distance between the cable bent tower center line to the near suspended cable is 4m, and the cross-sectional area of single main cable is 0.0177m2; the bridge midspan consists of 102 suspended cables in total, with 102 suspension centers.
The above analytical method and the MIDAS finite element software can be used to conduct alignment analysis of the free cable and finished bridge states of the main cable of this bridge. The coordinates of suspension center X calculated by using the parabolic, catenary and MIDAS methods are basically the same, so this paper will only list the coordinates of y in the following Table 1.

Table 1. Cable hang point coordinate calculation

Sling numbers	X coordinate	y coordinate(m)			Sling numbers	X coordinate	y coordinate(m)		
		Parabola	Catenary	midas			Parabola	Catenary	midas
1	39.851	1646.173	1646.195	1646.196	27	143.851	1628.719	1628.723	1628.72
2	43.851	1644.801	1644.844	1644.845	28	147.851	1628.803	1628.808	1628.808
3	47.851	1643.486	1643.544	1643.546	29	151.851	1628.943	1628.948	1628.946
4	51.851	1642.227	1642.295	1642.297	30	155.851	1629.138	1629.145	1629.143
5	55.851	1641.024	1641.097	1641.098	31	159.851	1629.39	1629.399	1629.396
6	59.851	1639.877	1639.95	1639.952	32	163.851	1629.697	1629.708	1629.706

Sling numbers	X coordinate	y coordinate(m)			Sling numbers	X coordinate	y coordinate(m)		
		Parabola	Catenary	midas			Parabola	Catenary	midas
7	63.851	1638.786	1638.857	1638.858	33	167.851	1630.061	1630.074	1630.072
8	67.851	1637.751	1637.818	1637.819	34	171.851	1630.48	1630.496	1630.495
9	71.851	1636.772	1636.833	1636.834	35	175.851	1630.955	1630.975	1630.974
10	75.851	1635.849	1635.904	1635.905	36	179.851	1631.487	1631.51	1631.509
11	79.851	1634.982	1635.031	1635.031	37	183.851	1632.074	1632.101	1632.101
12	83.851	1634.171	1634.214	1634.214	38	187.851	1632.717	1632.749	1632.749
13	87.851	1633.416	1633.453	1633.453	39	191.851	1633.416	1633.453	1633.453
14	91.851	1632.717	1632.749	1632.749	40	195.851	1634.171	1634.214	1634.214
15	95.851	1632.074	1632.101	1632.101	41	199.851	1634.982	1635.031	1635.031
16	99.851	1631.487	1631.51	1631.509	42	203.851	1635.849	1635.904	1635.905
17	103.851	1630.955	1630.975	1630.974	43	207.851	1636.772	1636.833	1636.834
18	107.851	1630.48	1630.496	1630.495	44	211.851	1637.751	1637.818	1637.819
19	111.851	1630.061	1630.074	1630.072	45	215.851	1638.786	1638.857	1638.858
20	115.851	1629.697	1629.708	1629.706	46	219.851	1639.877	1639.95	1639.952
21	119.851	1629.39	1629.399	1629.396	47	223.851	1641.024	1641.097	1641.098
22	123.851	1629.138	1629.145	1629.143	48	227.851	1642.227	1642.295	1642.297
23	127.851	1628.943	1628.948	1628.946	49	231.851	1643.486	1643.544	1643.546
24	131.851	1628.803	1628.808	1628.808	50	235.851	1644.801	1644.844	1644.845
25	135.851	1628.719	1628.723	1628.72	51	239.851	1646.173	1646.195	1646.196
26	139.851	1628.691	1628.691	1628.691					

Table 2. State of suspension bridge cable stress-free length (not counting anchorage m)

Location	Parabola	Catenary	midas
Left across	25.9206	26.0368	26.1235
Cross	211.992	212.474	212.475
Right across	25.9204	26.0364	26.1235

In accordance with the above table, we can know that for the alignment of suspended cable of this bridge, the coordinate of suspension center y calculated by using catenary method is basically consistent with the cable length with no stress (not including the anchor span) and the MIDAS result; while the result calculated with the parabolic method has certain error, and the error of suspended cable is within the scope of 4mm～73mm. Therefore, the cable alignment of suspension bridge is closer to the catenary, and in the project, the catenary method should be used to calculate the cable and suspended cable length.

6. Conclusion

(1) The main cable alignment under the uniformly distributed vertical load along the span length and cable length in suspension bridge is specifically derived and analyzed.
(2) Based on the result obtained with the parabolic and catenary methods, the corresponding no-stress cable length, main cable horizontal force and vertical force of main cable can be further derived.
(3) The no-stress main cable length and suspender length should be calculated in accordance with the actual alignment iteration. Through example verification, we find that the catenary method is a practical and accurate analytic calculation method for suspended cable calculation.

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