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A Novel Low-dimensional Modeling Method for Control of Unknown Nonlinear Distributed Spatial Processes

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Abstract: Data-based low-dimensional modeling for control design of nonlinear distributed spatial processes is necessary because there are usually some unknown uncertainties in first-principles modeling. In this paper, a novel low-dimensional modeling method is proposed for nonlinear distributed spatial processes. New discrete basis functions are generated according to linear combination of empirical eigen functions from empirical orthogonal function analysis(EOF). A low-dimensional model is identified by traditional identification techniques for the corresponding temporal dynamics. Thus, the nonlinear spatio-temporal dynamics of unknown distributed spatial processes can be reconstructed by synthesizing new discrete basis functions and the obtained low-dimensional model. The numerical simulations show that the proposed method has evidently better performance than that empirical eigen functions based modeling.

Keywords: Nonlinear Distributed Spatial Processes; Low-dimensional Modeling; Empirical Orthogonal Function Analysis; Linear Combination; Identification

1. Introduction

Theirinfinite-dimensionalspatio-temporalcouplingandcomplexnonlinearbehaviorof distributed spatial processes make modeling, system analysis, numerical simulation and control design very difficult. In practice, a low-dimensional model results in the feasible implementation for control of the nonlinear distributed spatial processes. When the first-principles-based partial-differential-equation (PDE) model of distributed spatial processes is known, there are many approaches to model reduction and control problems[1,2].Traditional methods such as finite-difference method (FDM) and finite-element method (FEM) can be easily applied to discretization of the PDEs, and lead to high-order ordinary differential equations (ODE).Under some conditions, a low-order ODE model also may be possible obtained by using Galerkin method, collocation method and the approximate inertial manifold method [2,3].However, the PDE modelo f distributed spatial processes is often unknown in many situations because of incomplete process knowledge; thus, data-based spatio-temporal modeling from the input and output data has to be used. Recently, the identification of nonlinear distributed spatial processes has been studied widely[4].

With the difference in spatial information processing, the spatio-temporal dynamical identification for distributed spatial processes can be classified as local and global approaches currently. The local method assumes that the local dynamics is determined by the neighborhood of the identified spatial location. Utilizing the measurements at small spatio-temporal regions, local models can be established, based on the identification theory of the lattice dynamical system [5]. The dimension of the obtained model is high because it is determined by the number of spatial locations.

Alternatively, the idea of the global approach comes from the Fourier series expansion[6]. A spatio-temporal variable can be expressed by an infinite number of basis functions $\{\varphi_i(x)\}_{i=1}^{\infty}$: $Z(x,t)=\sum_{i=1}^{\infty}\varphi_i(x)z_i(t)$, where $\varphi_i(x)(i=1,\cdots,\infty)$ represents the spatial frequencies from low to high order and $z_i(t)(i=1,\cdots,\infty)$ denotes the corresponding temporal coefficients (states).Once the spatial basis functions are selected, the corresponding states can be determined by projecting the spatio-temporal data onto the spatial basis functions. Many traditional approaches, such as the nonlinear state-space model[6], the nonlinear auto regressive with exogenous input(NARX)model[7,8], the Volterra model[9],the Wiener model[10], the Hammerstein models[11], Neural Network[12] and the least-squares support-vector-machine(LS-SVM)[13], are used to model the input-state dynamics. However, modelingaccuracyandefficiencyishighlydependentonthechoiceofbasisfunctionssuchasfinite-elementbases[7], Fourier series[6], Legendre polynomials, Jacobi polynomials, and Chebyshev polynomials[14].

In particular, empirical orthogonal function analysis (EOF), or proper orthogonal decomposition (POD), is a popular approach to find the principal spatial structures and reducing the dimension of the data. The amount of variance of a system represented by the leading empirical eigen functions is often taken as an indication of the quality of a reduced model using those first several empirical eigen functions. However, past studies [15,16], have pointed out that empirical eigen functions-based models can have difficulties reproducing behavior dominated by irregular transitions between different dynamical states. For this reason, there are many nonlinear method[8] to transform the high-dimensional spatio-temporal data into a low-dimensional time domain. However, these approach need more computation cost for the low-dimensional modeling of nonlinear distributed spatial processes.

The present study derives a new low-dimensional modeling method is proposed for unknown nonlinear distributed spatial processes. The spatio-temporal output of the system is measured at a finite number of spatial locations, and the input is finite-dimensional temporal variable with certain spatial distributions. New discrete basis functions are generated according to linear combination of empirical eigen functions from EOF. The low-dimensional model is obtained by identification of traditional identification techniques for the corresponding temporal dynamics. Thus, the nonlinear spatio-temporal dynamics of distributed spatial processes can be reconstructed by synthesizing new empirical basis functions and dynamics of the obtained low-dimensional model. The numerical simulations show that the proposed method has evidently better performance than that empirical eigen functions based modeling.

2. Empirical Eigen Functions based Modeling

This study focuses on the global method of the spatio-temporal model identification for distributed spatial processes as shown in Figure 1.Generallyspeaking,this class of modeling approach follows three steps: selection of basis functions, time/space variable separation, and identification by traditional system identification technique for the input-state dynamics.

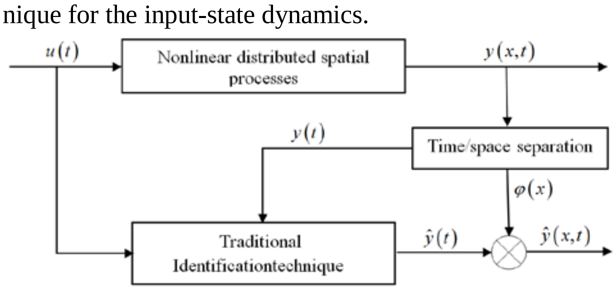

Figure 1. Variables separation based modeling for nonlinear PDEs

Consider the nonlinear distributed spatial processes in Figure 1. With $u(t) \in R^m$ as the temporal input and $y(x,t) \in R$ as the spatiotemporal output, where $x \in \Omega$ is the spatial variable, Ω is the spatial domain, and $t \in [0,\infty)$ is the time variable. For simplicity, suppose that the distributed spatial processes is controlled by m actuators with implemented temporal signal $u(t)$ and a certain spatial distribution. The output is measured at the M spatial locations $x_1, x_2, \cdots, x_M$. In this modeling problem to identify a proper spatio-temporal model from the input $\{u(t)\}_{t=1}^{L}$ and output $\{y(x_i,t)\}_{i=1,t=1}^{M,L}$, where L is the total time duration. In order to derive new empirical basis functions for model reduction of distributed spatial processes, EOF is used for time/space separation from measured spatio-temporal output to obtain the empirical eigen functions and the corresponding temporal coefficients.

For simplicity, as sume that the process output $\{y(x_i,t)\}_{i=1,t=1}^{M,L}$ (called snapshots), is uniformly sampled in time and space. Define the inner product, norm, and ensemble average as $[f(x), g(x)] = \int_{\Omega} f(x)g(x)dx$, $\|f(x)\| = [f(x), f(x)]^{1/2}$, and $< f(x,t) >= (1/L)\sum_{t=1}^{L} f(x,t)$.

Motivated by Fourier series, the spatiotemporal variable $y(x,t)$ can be expanded on to an infinite number of orthonormal spatial basis functions $\{\varphi_i(x)\}_{t=1}^{\infty}$ with temporal coefficients $\{y(t)\}_{t=1}^{\infty}$

$$y(x,t) = \sum_{i=1}^{\infty} y_i(t)\varphi_i(x) \tag{1}$$

Because of the spatial basis functions are orthonormal, the temporal coefficients can be computed from the following equation.

$$y_i(t) = [\varphi_i(x), y(x,t)], \quad i = 1,2,\cdots,\infty \tag{2}$$

In practice, the expression has to be truncation to a finite dimension.

$$y_M(x,t) = \sum_{i=1}^{M} y_i(t)\varphi_i(x) \tag{3}$$

The $y_M(x,t)$ denote the M -order approximation.

The main problem of using EOF for time/space separation is computing the most characteristic spatial structure $\{\varphi_i(x)\}_{i=1}^{M}$ among the spatio-temporal output $\{y(x_i,t)\}_{i=1,t=1}^{M,L}$. This typical structure can be found by minimizing the objective function

$$\min_{\varphi_i(x)} \left\langle \left\| y(x,t) - y_M(x,t) \right\|^2 \right\rangle \\ subject\ to\ (\varphi_i, \varphi_i) = 1, i = 1, \cdots, M \tag{4}$$

The orthogonal constraint is imposed to ensure that the function $\varphi_i(x)$ is unique. The Lagrangian function alcorresponding to this constrained optimization problem is

$$J = \left\langle \left\| y(x,t) - y_M(x,t) \right\|^2 \right\rangle + \sum_{i=1}^{M} \lambda_i \left[(\varphi_i, \varphi_i) - 1 \right] \quad (5)$$

And the necessary condition of the solution can be obtained as

$$\int_\Omega D(x,\zeta)\varphi_i(x)\,d\zeta = \lambda_i \varphi_i(x), (\varphi_i,\varphi_i) = 1,\ i = 1,\cdots,M \quad (6)$$

Where $D(x,\zeta) = < y(x,t)y(\zeta,t) >$ is the spatial two-point correlation functions. $\varphi_i(x)$ is the ith eigen function, and the λ_i is the corresponding eigen function eigen value. Given that the covariance matrix D is symmetric and positive definite, its eigen values λ_i are real, and its eigenvectors $\varphi_i(x), i = 1,2,\cdots,M$ form an orthogonal set. Since the data are always discrete in space, one must solve numerically the integral Eq.(6). Discretizing the integral equation gives a $M \times M$ matrix eigen value problem. Thus, at most M eigen functions at M sampling spatial locations can be obtained.

The maximum number of nonzero eigen values is $N = \min(M, L)$. We arrange the eigen values $\lambda_1 > \lambda_2 > \cdots > \lambda_N$ and the corresponding eigen functions $\varphi_1(x), \varphi_2(x), \cdots, \varphi_N(x)$, in order of the magnitude of the eigen values. Each eigen function has an energy percentage which depends on the associated eigen values of the eigen functions

$$F_k = \frac{\lambda_k}{F} \quad (7)$$

where $F = \sum_{i=1}^{N} \lambda_i$ denotes the sum of the eigen values of the covariance matrix. Assuming that the eigen values are sorted in descending order, the eigen functions are ordered from most to least energetic.

In general, an expansion in terms of only the first few temporal coefficients

$$y_N(x,t) = \sum_{i=1}^{N} y_i(t)\varphi_i(x) \quad (8)$$

can be used to represent the dominant dynamics of non linear PDEs.

3. The Calculation of New Discrete Basis Functions

The new empirical basis functions are derived by the linear combinations from initial empirical eigen functions as follows $(n < N)$. Each new empirical basis function is a linear combination of initial empirical eigen functions, which can be given as follows:

$$\phi_i = \sum_{j=1}^{N} S_{ji}\phi_j,\ i = 1,2,\cdots,n \quad (9)$$

which can be rewritten as

$$\{\phi_1(x), \phi_2(x), \cdots, \phi_n(x)\} = \{\varphi_1(x), \varphi_2(x), \cdots, \varphi_N(x)\} S \quad (10)$$

where $\{\phi_1(x), \phi_2(x), \cdots, \phi_n(x)\}$ and $\{\varphi_1(x), \varphi_2(x), \cdots, \varphi_N(x)\}$ denote the new empirical basis functions and initial empirical eigen functions respectively, S denote the matrix of coefficients.

The calculations of coefficient matrix are very crucial that heavily influences the performance of new empirical basis functions based modeling. In this subsection, an algorithm to obtain the coefficient matrix by balancing of empirical gramians [17,18] is present. The approach calculates empirical gramians from the corresponding temporal data of the initial empirical eigen functions with different excited temporal signals. In the identification of nonlinear distributed spatial processes, the excited signals can be imposed on the distributed spatial processes and the corresponding spatial-temporal output can be measured. Thus, the corresponding temporal data can be derived using a time/space separation based on initial empirical eigen functions. These gramians are then balanced by the same procedure as is used for linear systems. The balancing transformation is used within a Galerkin projection in order to transform the empirical gramians into balanced form.

Let $T^N = \{T_1, T_2, \cdots, T_r\}$ be a set of r orthogonal $N \times N$ matrices, where r denotes the number of matrices for excitation/perturbation directions; Let $M^s = \{c_1, c_2, \cdots, c_s\}$ be a set of s positive constants, where s denotes the number of different excitation/perturbation sizes for each direction; Let $E^P = \{e_1, e_2, \cdots, e_P\}$ be P standard unit vectors in R^P, where P denotes the number of inputs to the system (9) for Definition 1 and Definition2.Given a function $v(t)$, define the mean $\bar{v}(t)$ by

$$\bar{v}(t) = \lim_{T\to\infty} \frac{1}{T} \int_0^T v(t)dt \quad (11)$$

Definition 1: Discrete Empirical Control lability Gramian: Let T^N, M^s and E^P be given sets as described above, where N is the number of states. For system (7), the discrete empirical controllability gramianis defined by

$$\hat{W}_C = \sum_{l=1}^{r}\sum_{m=1}^{s}\sum_{i=1}^{p} \frac{1}{rsc_m^2} \sum_{k=0}^{Ntim} \Phi_k^{ilm} \Delta t_k \quad (12)$$

Where $\Phi_k^{ilm} \in \Re^{N\times N}$ is given by

$$\Phi_k^{ilm} = \left(x_k^{ilm} - \bar{x}^{ilm}\right)\left(x_k^{ilm} - \bar{x}^{ilm}\right)^T \quad (13)$$

and x_k^{ilm} is the corresponding temporal variable at the step k corresponding to the input $u(k)=c_m T_l e_i \delta(k)$ at the certain locations, $Ntim$ is the number of step of the sampling for distributed spatial processes.

Definition 2:Discrete Empirical Observability Gramian

Let T^N, M^s and E^P be given sets as described above. For system (7), the discrete empirical observabilitygramianis defined by

$$\hat{W}_O=\sum_{l=1}^{r}\sum_{m=1}^{s}\frac{1}{rsc_m^2}\sum_{k=0}^{Ntim}T_l\Psi^{lm}T_l^T\Delta t_k \tag{14}$$

Where $\Psi_k^{lm}\in\Re^{N\times N}$ is given by

$$\Psi_{k\ ij}^{lm}=\left(y_k^{ilm}-\bar{y}^{ilm}\right)^T\left(y_k^{jlm}-\bar{y}^{jlm}\right) \tag{15}$$

and y_k^{ilm} is the temporal output variableon the measured locations at the step k corresponding to the initial condition $x_0=c_m T_l e_i$, $Ntim$ is the number of step of the sampling for distributed spatial processes.

The empirical control lability gramian and empirical observability gramian are computable generalization of controllability gramian and observability gramian to nonlinear systems, which can be calculated from the process data. A simple numerical technique for balancing the empirical gramians $\hat{W}_c$ and $\hat{W}_O$ is as follows [17]. First, apply the Cholesky factorization [19] to $\hat{W}_O$ so that $\hat{W}_O=ZZ^T$, with Z lower triangular with non-negative diagonal entries. Let $U\Sigma^2U^T$ be a eigenvalue decomposition of $Z^T\hat{W}cZ$, and let $\bar{S}=\Sigma^{1/2}U^TZ^{-1}$. Then $\bar{S}\hat{W}o\bar{S}^T=(\bar{S}^{-1})^T\hat{W}c\bar{S}^{-1}=\Sigma$

The columns of $\bar{S}$ may be thought of as giving the modes of the system associated with the Hankel singular values in Σ. To derive a superior set of new spatial basis functions, the first n columns of matrix $\bar{S}$ of balancing of the empirical gramians is selected to be a $N\times n$ spatial basis functions transformation matrix. Using the MATLAB style colon notation, transformation matrix $S=\bar{S}(:,1:n)$.

4. New Empirical Basis Functions based Neural Modeling

In the Galerkin method, obtaining an exact analytical description of the low-dimensional ODE systems is impossible because of the unknown nonlinearities in the nonlinear partial differential equations. Therefore, the neural network can be used to identify the long-term dynamical behaviors from the input and corresponding temporal coefficients of new empirical basis functions.

For model identification by the neural network, new empirical basis functions are used to time/space separation for the spatio-temporal output $\{y(x_i,t)\}_{i=1,t=1}^{M,L}$, the corresponding temporal coefficients are calculated using the generalized inverse matrix based on the following Eq.(24).

$$\begin{pmatrix} y(x_1,t_1) & y(x_1,t_2) & \cdots & y(x_1,t_L)\\ y(x_2,t_1) & y(x_2,t_2) & \cdots & y(x_2,t_L)\\ \vdots & \vdots & \vdots & \vdots\\ y(x_m,t_1) & y(x_m,t_2) & \cdots & y(x_m,t_L)\end{pmatrix}=(\phi_1,\phi_2,\cdots,\phi_n)\begin{pmatrix} \bar{y}_1(t_1) & \bar{y}_1(t_2) & \cdots & \bar{y}_1(t_L)\\ \bar{y}_2(t_1) & \bar{y}_2(t_2) & \cdots & \bar{y}_2(t_L)\\ \vdots & \vdots & \cdots & \vdots\\ \bar{y}_n(t_1) & \bar{y}_n(t_2) & \cdots & \bar{y}_n(t_L)\end{pmatrix} \tag{16}$$

Using temporal signal $u(t)$ and the temporal coefficient $\bar{y}(t)$, a feed forward neural network is employed to identify the dynamics.

$$\hat{y}(k+1)=NN\left(\hat{y}(k),u(k)\right) \tag{17}$$

The advantage of the neural networks is its ability to model complex nonlinear relationships without any assumptions on the nature of these relationships. The most often used neural networks include the radial basis function networks, back propagation (BP) neural networks, among others. The present study employs a feed forward BP neural network to construct low-dimensional substitute model for the dominant dynamics. The prediction output of nonlinear distributed spatial processes is obtained by synthesis of temporal predicted output and new empirical basis functions:

$$\hat{y}(x,k)=\sum_{i=1}^{n}\hat{y}_i(k)\phi_i \tag{18}$$

5. Numerical Simulations

Suppose that $y(x,t)$ and $\hat{y}(x,t)$ are the measured output and the predicted output at the M spatial locations $x_1,x_2,\cdots,x_M$ and some sampling time $t_1,t_2,\cdots,t_L$, respectively. For an easy comparison, the root of mean squared error (RMSE) is set up as the performance indexas follows.

$$RMSE=\sqrt{\sum_{i=1}^{M}\sum_{j=1}^{L}\left(y(x_i,t_j)-\hat{y}(x_i,t_j)\right)^2\Big/ML} \tag{19}$$

To evaluate the performance of the proposed kind of empirical basis functions for model reduction, the rescaled Kuramoto-sivashinsky (K-S) equation in one space dimension is considered. The K-S equations is one of a typical partial differential equations, which has first been derived in 1976 by Kuramoto and Tsuzuki as a model equation for interfacial instabilities in the context of angular phase turbulence for a system of a Reaction-diffusion equation that model the Belouzov-Zabotinskii reaction in three space dimensions, and independently, in 1977, by Sivashinsky to model thermal diffusion instabilities observed in laminar Mame fronts in two space dimensions.

$$\frac{\partial T}{\partial t}+4\frac{\partial^4 T}{\partial x^4}+\alpha\left[\frac{\partial^2 T}{\partial x^2}+\frac{1}{2}\left(\frac{\partial T}{\partial x}\right)^2\right]+\sum_{i=1}^{m} b_i\left(x\right)u_i\left(t\right)=0 \quad (20)$$

where $\alpha=84.25$; $b_i(x)=\delta[x+3\pi/4-(i-1)\pi/2]$; $m=4$.

The Eq.(20) is subject to periodic boundary condition

$$T(x,t)=T\left(x+2\pi,t\right)=0 \quad (21)$$

and temporal inputs

$$[u_1(t),u_2(t),u_3(t),u_4(t)]= [4\cos(t/5),4\sin(t/5),5\cos(t/4),5\sin(t/4)] \quad (22)$$

The initial condition is set to be $\sin x$ .The sampling interval Δt is 0.001s and the simulation time is 0.5s. Due to the infinite-dimensional feature, sufficient sensors should be used to measure the representative spatial features of the distributed parameter system, which depend on the required modeling accuracy. In this case, forty-one sensors uniformly distributed in the space are used for measurement. A noise-free dataset of 500 data is collected from (20).This size of data set used for training may be determined by the system complexity and the desired modelling accuracy. More complex system and higher modelling accuracy may need moredata. A new set of 100 data is collected for testing to compare the performances of two kinds of spatial basis functions. The spatio-temporal output of the K-S equation on testing data can be estimated from the synthesis of the temporal approximate model and empirical basis functions.

In order to demonstrate the modeling performance of the proposed method, the comparisons for the reduced models using two kinds of spatial basis functions on the testing data aregiven. The simulations show that three new empirical basis functions together with a 3-order network are able to denote the dominant spatial-temporal dynamics of the K-S equation. The first three new empirical basis functions and the first three initial empirical eigenfunctions are respectively shown in Figure.2 and 3.

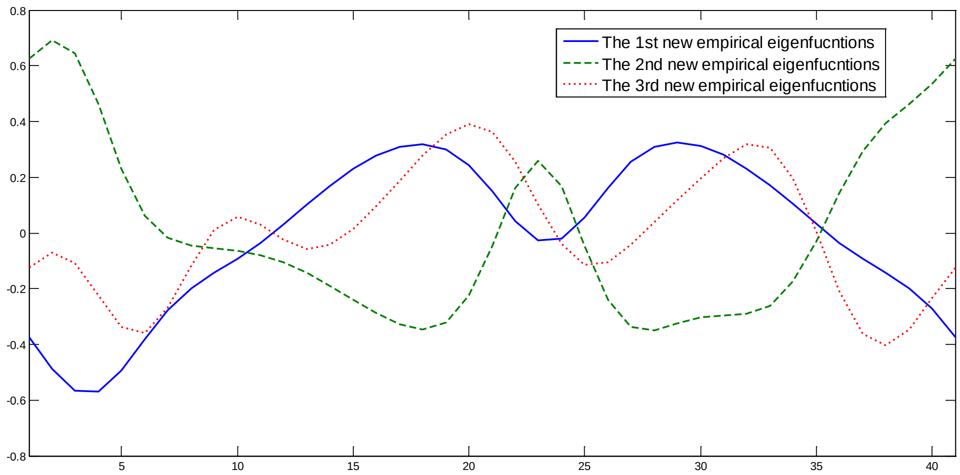

Figure 2. The first three empirical eigenfunctions

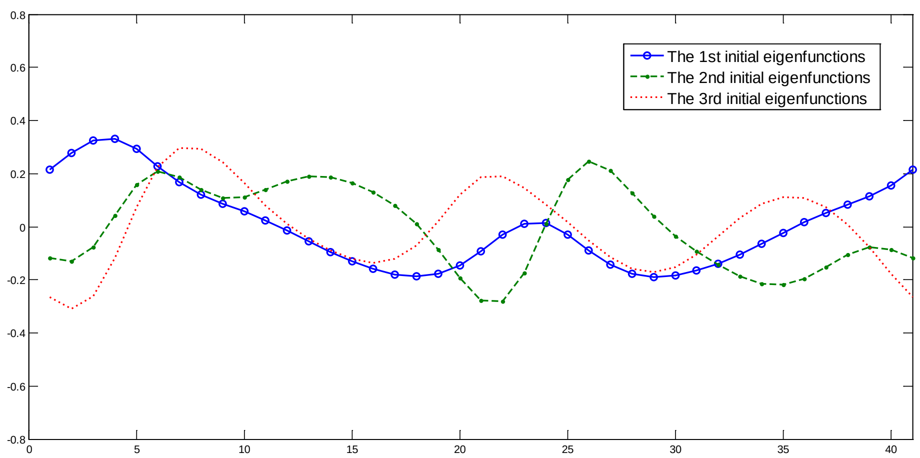

Figure 3. The first three initial empirical eigenfunctions

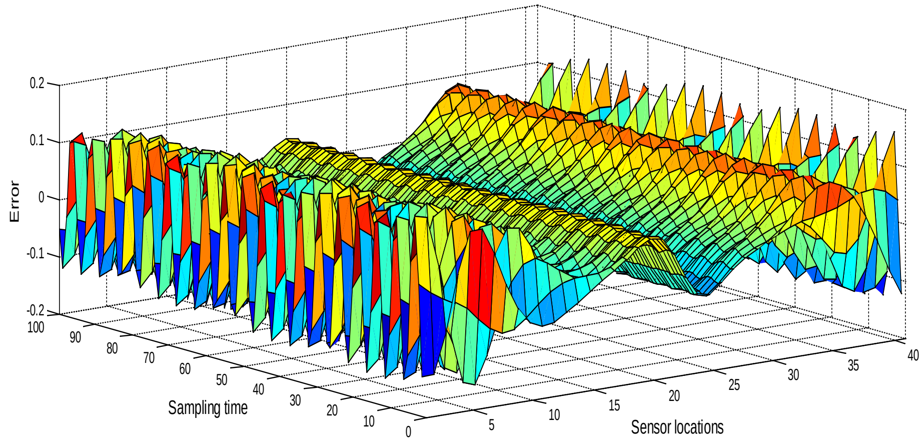

Figure 4. Distributed error based on three new empirical basis functions

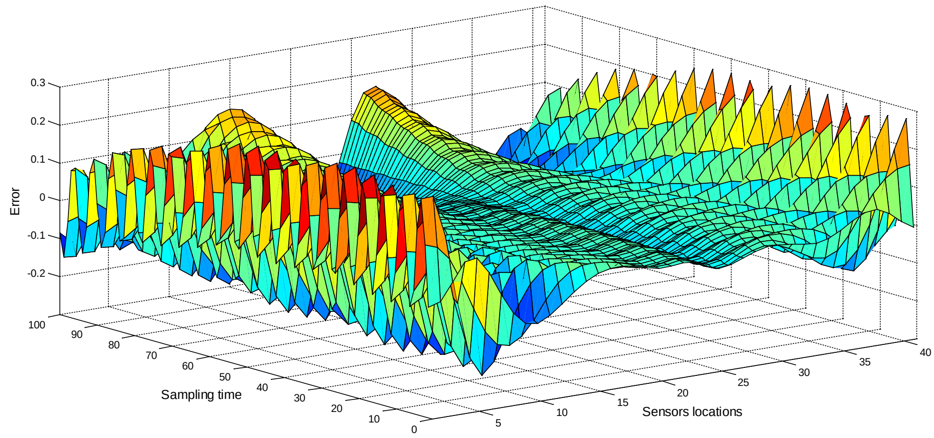

Figure 5. Distributed error based on threeinitial empirical eigenfunctions

Compared with the testing data, the predicted distribution errors based on two kinds of empirical basis functions are shown in Fig. 4 and Fig.5, respectively. And the RMSEs of the approximate model based on the 3 new empirical basis functions and 3 initial empirical eigen functions are 0.0565 and 0.0728, respectively. The results have shown that the performance of new empirical basis functions based modeling for nonlinear distributed spatial processes is superior to that of initial empirical basis functions based modeling with the same order.

6. Conclusions

In this paper, a new low-dimensional modeling method was proposed for data-based modeling of unknown nonlinear distributed spatial processes. New empirical basis functions were generated according to linear combination of empirical eigen functions. A low-dimensional model was identified by traditional identification techniques for the corresponding temporal dynamics. Thus, the nonlinear spatio-temporal dynamics of distributed spatial processes could be reconstructed by synthesizing the new discrete basis functions and the obtained low-dimensional model. The numerical simulations showed that the proposed method has evidently better performance than that empirical eigen functions based modeling directly for unknown distributed spatial processes.

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