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Advanced Modeling and Optimal Temperature Control Strategy for the Bathtub

Juan YANG, Linling DUAN
College of Civil Engineering, Chongqing Jiaotong University, Chongqing, CHINA

Abstract: The modeling goal is to instruct the person in bath to maintain a constant water temperature simultaneously minimizing the water consumption. Three sub-models: the heat-transfer model for determining the optimal shape of bathtub, the time-space-based temperature field model to seek the distribution of the temperature field, a minimum water flow optimization model are applied to output the best strategy. In model one, the innovative "line-plane-space" approach is invoked to describe the process of heat dissipation inside the bathtub system. For various shape corresponds with different dissipation rate, the optimal shape of bathtub determined is oval. In model two, we simplify the process of water flowing from the faucet heating the tub into the problem of point heat source heating medium. Then the unsteady heat conduction differential equation is given via Fourier transform and finally apply the difference method and multigrid method to obtain the distribution of temperature field. Model three is the optimization model targeting the minimum water flow with the constraint condition guaranteeing the even water temperature. By involving the global search algorithm, the optimal water flow is output as 0.0028 m/s^2 .

Keywords: Heat-transfer; Time-space-based temperature field; Minimum water flow optimization

1. Introduction

Although a spa-style tub with heated water features is a luxury enjoy, most modern bathtubs we use today are ordinary ones that have overflow and waste drains and may have taps mounted on them. Since we need repeatedly let some of the cooled water out and refresh the tub with warm water, if we can practically instruct people to keep an evenly maintained temperature throughout the bathing via our study, it makes sense for a significant improvement to our life quality.

We are required to build a mathematical model to provide a optimal strategy for the bather to keep an oven bath temperature. We decompose the problem into three sub-problems Determining the optimal shape of bathtub, Seeking the distribution of the temperature field. Constructing the minimum water flow optimization model.

2. Model One

The Heat-Transfer Model to Determine the Optimal Shape of Bathtub

We analyze that the reason why the bathtub water gets colder is the heat transfer. Heat transfer is the exchange of thermal energy between physical systems, depending on the temperature and pressure, by dissipating heat[1]. Our main consideration is that the shape of bathtub will impact a lot on the heat transfer process. Thus, if we can first determine the shape of bathtub that goes against heat dissipation, it will simplify our later work.

We initially intended to define the "optimal" from two aspects:1) the "heat transfer rate" and 2) the comfort degree of human in bath. Then we could make a comprehensive evaluation via an AHP model. Unfortunately, we failed to get valid data support to acquire the comfort degree feedback from bathtub users. However, we can see that the impact of comfort degree can be ignored compared with the main need to keep an even temperature.

Thus, we invoke an innovative accept "heat transfer rate" as the only criterion to determine the optimal shape. It refers to the amount of heat in unit time, which we denote as χ . The smaller the value of the heat transfer rate χ , the better the shape of bathtub will be.

we first introduce some Terminologies as follows:

Thermal conduction: The transfer of internal energy by microscopic diffusion and collisions of particles or quasi-particles within a body or between contiguous bodies [2].

Convective heat transfer: the transfer of heat from one place to another by the movement of fluids [3].

Thermal radiation: electromagnetic radiation generated by the thermal motionof charged particles in matter[4].

According to the heat-transfer principle, three fundamental ways for heat to transfer are conduction, convection and radiation. We can exclude the thermal radiation for

its slight impact and complex process based on a previous article [5].As we assume that the bathtub is isotropic, the heat flux inside the tub won't impact the non-uniformity. And for this specific problem, we must add another aspect: water evaporation off the surface.
Thus, we further classify these three approaches for heat-transfer into two types. The thermal conduction is as the internal while the convective heat transfer and water evaporation are integrate as the external approach for heat dissipation. We illustrate our analysis in the flow chart Fig.1 below.

2.1. The heat loss from thermal conduction

Our modeling process is developed from local to the whole based on the Fourier's Law .we show our Line-Plane-Space procedure in Fig.2.

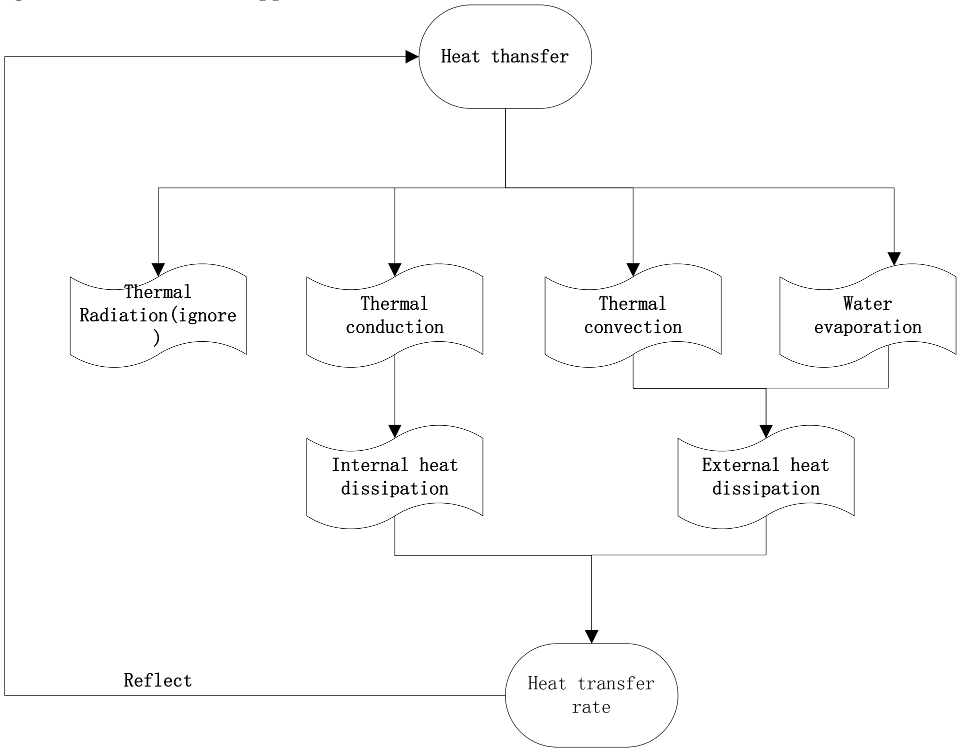

Figure 1. Analysis process of heat transfer

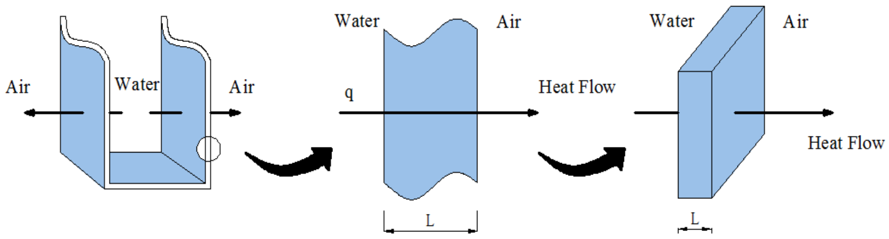

Figure 2. The evolution process based on our Line-Plane-Space thoughts.((a) is the cross section of the tub;(b) is the locally enlarged plane; and(c) is the expanded space)

Step 1: Line
For the bathtub, the thermal conductivity of Water and walls only happens in the vertical direction of the side walls. Thus, the conduction process can be described by the one-dimensional Fourier's differential equation[6]:

$$\rho c_p \frac{\partial T}{\partial t} - \nabla \cdot (k \nabla T) = Q \qquad (1)$$

Where ∇^2 denotes the Laplace operator, C_p denotes the specific gravity, T denotes the temperature of water, ρ denotes the density of water, Q denotes the heat input, and k denotes the flow rate.
Next, we consider the boundary conditions. Boundary conditions are the representation of the thermal energy balance at the bounding surface of the material. They

measure heat exchange interactions between the material and its surroundings in w/m^2 . Common heat exchange mechanisms are convection[7].

$$Qconv = h(t_2 - t_1) \tag{2}$$

Where h is the heat transfer coefficient(w/m^2) and is the bulk temperature of the surrounding environment.
The differential equation of heat conduction together with the boundary conditions constructs the complete mathematical description of the specific conduction process.
Step 2: Plane
Following step one, we model Figure 2.b as follows:

$$\frac{d^2t}{dx^2} = 0, \quad t = t_1 + (t_2 - t_1)\times(x/L) \tag{3}$$

Where t represents the temperature of the side walls, t_1 represents the air temperature. t_2 is the water temperature, L is the thickness of side walls, and x is the heat coordinate point.
Step three: Space
At last, we consider the thickness and area in Fig2. (c), we can get:

$$\begin{cases} t = t_1 + (t_2 - t_1)\times(x/L), \\ R = L/k, \\ Q = \dfrac{S\times\Delta t}{R}, \end{cases} \tag{4}$$

Where R is the percentage of the heat, and Q is the heat input.

2.2. The heat loss from thermal convection and water evaporation

Thanks to a previous article[8], incorporating each of two components into a single model yields the following equation:

$$\begin{cases} dQ = dQ_0 + L\times dG, \\ dQ_0 = \alpha(t-\phi)dF, \\ dG = \beta_P(P_v^{''} - P_v)dF, \\ \dfrac{\alpha}{\beta} = b = \dfrac{P_0\times C_p}{0.623L}, \end{cases} \tag{5}$$

Where L is the heat of vaporization, ∂ is the thermal diffusivity, t is the water surface temperature(^oC), ϕ is the dry bulb temperature(^oC) F is the vapor contact area(m^2), β_p is the bulk transfer coefficient, P is the atmospheric pressure.

2.3. Ultimate expression of heat transfer rate χ

On the basis of its definition, the heat transfer rate χ is :

$$\varphi = \frac{\sum Q}{t} = \frac{Q_t + Q_0 + Q_G}{t} \tag{6}$$

Where Q_1 is the heat loss through heat conduction, Q_2 is through the convection, Q_3 is through the water evaporation, and t is the dissipation time.
To solve the Equation 6 by MATLAB, we should invoke a shape conduction coefficient A to consider the different surface area. We set the complex oval and circle as $A_1 = 0.8, A_2 = 0.9$ respectively. The model results calculated by MATLAB is shown as Fig.3 and Tab.1.

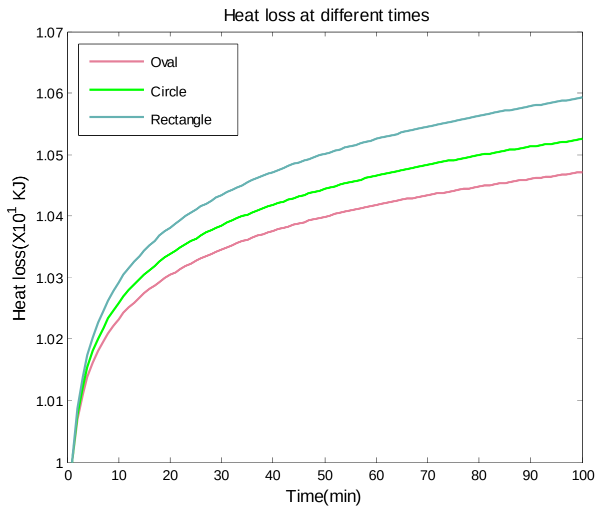

Figure 3. Heat loss process of three types of bathtub

In Figure 3, the purple, green, blue curve represent the heat loss changing process of oval, circle, and rectangle. The oval is the lowest, thus, we can conclude the optimal type of bathtub is oval.

Table 1. The table which contains the result of the different shape

Shape	Area	rate
Rectangular	3.36	0.7521
Oval	A1*3.36	0.7013
Round	A2*3.36	0.7363

3. Model Two

A Time-Space-Based Temperature Field Model
Model two maps the temperature distribution of water in space and time based on the oval shape of bathtub determined by model one.
What is time-space-based? In space: we consider every point of the bathtub water, they construct the space. In time: model two maps the temperature change of every point with time. Temperature field: it refers to the set of temperature distribution of each point in continuous medium. In the three-dimensional Cartesian coordinate, Temperature field can be represented as temperature

$T = f(x, y, z, t)$, and the one that varies with time is called unsteady temperature field.

Inspired by an article [9], we modify the author's model to account for the heat loss through the overflow drain in our specific circumstances.

Let Q_0 be the heat emitted by a point source, θ be the added temperature value in any position $P(x, y, z)$, any time t, and the Thermal conductivity α be a constant. We invoke Q_e as the heat loss through an overflow drain. Set the origin at the heat source point, we have $\vec{R} = x\vec{i} + y\vec{j} + z\vec{k}$, $R = \sqrt{x^2 + y^2 + z^2}$.

since the temperature distribution is symmetrical about the origin, the heating function $\theta(\vec{R}, t)$ can be $\theta(R, t)$.

We can have the Equation (7) based on the law of conservation of energy.

$$\frac{Q_0}{C_P \rho} = \int_0^\infty 4\pi R^2 \theta(R, t) dR + \frac{Q_E}{C_P \rho} \quad (7)$$

Where C_p is the specific gravity in Temperature field, ρ is the density of water.

If C_p, ρ and $\vec{R}$ are irrelevant, we conduct the Fourier transform to Equation (7),then the value of added temperature can be solved as follows:

$$\theta = \frac{A}{(4\pi\alpha t)^{3/2}} \exp\left(-\frac{R^2}{4\alpha t}\right) \quad (8)$$

We substitute Equation (8) with Equation (7), then get $A = \frac{Q_0 - Q_e}{C_p \rho}$. Thus, the instantaneous warming formula in Infinite heat conductor can be written as:

$$\theta = \frac{Q_0 - Q_e}{C_P \rho (4\pi\alpha t)^{3/2}} \exp\left(-\frac{x^2 + y^2 + z^2}{4\alpha t}\right) \quad (9)$$

Ultimately, the expression of temperature at any point equals the initial temperature plus the added value, given by:

$$Q_{Ri} = Q_{0i} + \theta \quad (10)$$

Since MATLAB can only solve the two-dimensional model, we conduct lowering the latitude to transfer the four-dimensional temperature field model into the two-dimensional one. These parameters and boundary conditions in Fig.4 and Fig.5 are regarded as the standard state of tests in this paper.

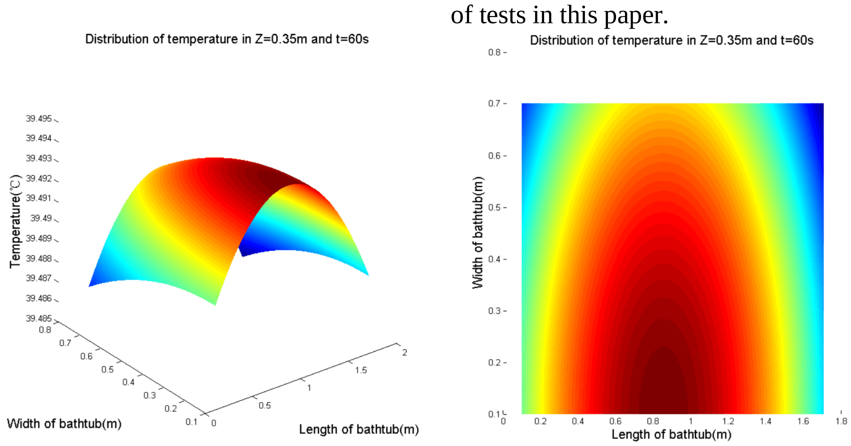

Figure 4. The temperature field distribution of x-y plane

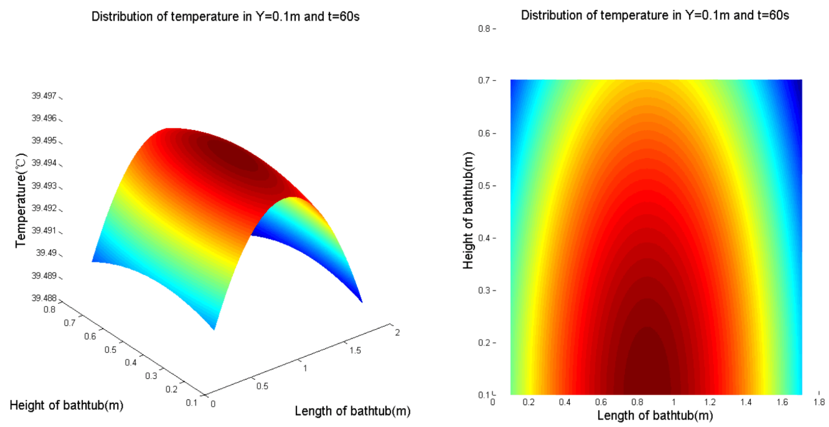

Figure 5. The temperature field distribution of x-z plane

From Fig.4, we can see temperature in x-y plane reaches its peak in pouring source. The closer a point is near the boundary, the lower temperature is, and it is a continuous process.

In Figure 5, we can see that x-z plane shares the same law with x-y plane in Fig.4. Thus, we can say the distribution in three-dimensional space satisfy the law that A hot water pipe inlet temperature maximum, the most distant from the inlet temperature is the lowest, and is a continuous process, that is the temperature field has a continuous movement.

In sum, we can conclude that although there is a continuous change in temperature, but in this oval tub in each area, temperature control in a smaller range, namely, after the hot water into the tub, the heat will quickly spread to the entire region and not stay in one position, namely transient temperature field with motion.

4. Model Three

The Minimum Water Flow Optimization Model

Since the law of temperature distribution has been determined in model two, we can apply it as restrictions on keeping the temperature even and as close as possible to the initial temperature, simultaneously with minimum water consumption as objective function. Thus, the three-fold optimization model achieves its mission to determine the best strategy.

Since the article[10] tells the comfortable temperature for human ranges from $39^{\circ}C$ to $40^{\circ}C$, the constrain condition can be written as:

$$39℃ \le Q_{0i} + \theta \le 40℃ \qquad (11)$$

Where Q_{O_i} is the heats poured into the tub from the faucet, θ is the value of added temperature in model two, and $\theta = \frac{Q_0 - Q_e}{C_p \rho (4\pi\alpha t)^{3/2}} \exp(-\frac{x^2+y^2+z^2}{4\alpha t})$ The relation between heat Q and flow f is given in the article [11] as follows:

$$\begin{cases} Q_0 = f_0 \rho C_p T_0 \\ Q_e = f_e \rho C_p T_e \end{cases}, \qquad (12)$$

Where Q_O represents the heats poured into the tub from the faucet in unit time, Q_e represents the heats taken away through a overflow drain in unit time, f_0 represents the water flow poured into the tub from the faucet in unit time, f_e represents hot water discharge weir flow in a unit time, ρ represents flow density of the water in pipe, C_P represents specific heat capacity of the hot water in pipe, T_0 represents The water temperature hot water pipes.

We modify the equation to account for a significant heat loss when water flows from the faucet to reach the surface. the improved equation is $Q_0 = f_0 \rho C_p T_0 \varphi$, Where φ is the Heat attenuation coefficient.

Thus, the minimum water flow optimization model can be written as:

$$\min\ f_0 = S \times V,$$
$$s.t. \begin{cases} 39℃ \le Q_{0i} + \theta \le 40℃, \\ \theta = \frac{Q_0 - Q_e}{C_p \rho (4\pi\alpha t)^{3/2}} \exp(-\frac{x^2+y^2+z^2}{4\alpha t}), \\ Q_0 = f_0 \rho C_p T_0 \varphi, \\ Q_e = f_e \rho C_p T_e, \\ f_0 = f_e, \end{cases} \qquad (13)$$

Where S is the pipe diameters, V is flow rate, Q_{0i} is the initial value of the temperature at any point, its value equals Q_0. Other symbols are described previously.

Improved Model

①Considering the shape of a person

For the human body, the impact of the volume of the model we mainly consider a bath of water in thermal conductivity α and specific heat capacity C_p [12].

There are three kinds of body shape: fat, middle, thin . The main difference is that the proportion of the three organizations is not the same, so that people of different thermal conductivity α_p and specific heat capacity C_{pp} is not the same.

We can modify it by invoking:

$$\alpha_p = \sum \alpha_i \times \omega_i \qquad (14)$$

Where α_p is the heat transfer coefficient, α_i is the class, and i represents the heat transfer coefficient organizations.

It's true for human specific heat capacity C_{pp} ,we modify it as:

$$C_{pp} = \sum C_{pi} \times \omega_i \qquad (15)$$

Where C_{pp} is the human specific heat capacity, C_{pi} is the class, i represents the heat transfer coefficient organizations.

②Considering the volume of a person

The model assumes a bathtub is filled with water, when someone taking a bath in the tub, it can be written as :

$V_w + V_p = V_0$ (1)

Where V_w represents the volume of people, V_p represents the volume of water in a bathtub, V_o is the volume of a bathtub.

Applying Equation(16) to human, we can have:

$$\frac{V_p}{V_o}\times\alpha_p+\frac{V_w}{V_o}\times\alpha_w=\alpha_f$$
$$\frac{V_p}{V_o}\times C_{PP}+\frac{V_w}{V_o}\times C_{Pqw}=C_{pf} \quad (17)$$

Where α_w represents water heat transfer coefficient, α_f represents the final bath medium heat transfer coefficient.

③ The improved optimization model

Based on the discussion above, the ultimate single-objective optimization model after we take human into account will be:

$$\min f_o$$
$$s.t.\begin{cases} Q_0=f_0\rho C_pT_0\varphi \\ Q_e=f_e\rho C_pT_e \\ \theta=\dfrac{Q_0\text{-}Q_e}{C_{pf}\rho(4\pi\alpha_f\ t)^{3/2}}\exp(-\dfrac{x^2+y^2+z^2}{4\alpha_f t}) \\ f_0=S\times V \\ f_0=f_e \\ 39℃\le Q_{0i}+\theta\le 40℃ \end{cases} \quad (18)$$

What is determined above by the most primitive single-objective optimization model, the constraints can be seen from the impact of the main results of the model parameters internal and external factors, the initial model, can be considered an internal parameters are fixed, its main source of influence external factors. By changing the external conditions, the optimal solution can be obtained in different situations.

The temperature field under this condition can be shown in Fig 6.

In Fig6. As long as the water temperature at a certain time when the water injection tube, while maintaining a constant temperature boundary conditions, there is an optimal output, the minimum flow of water injection. Therefore we assign to initial water injection tube 70 ℃, the optimal solution for the model output 0.238 m^3/s.

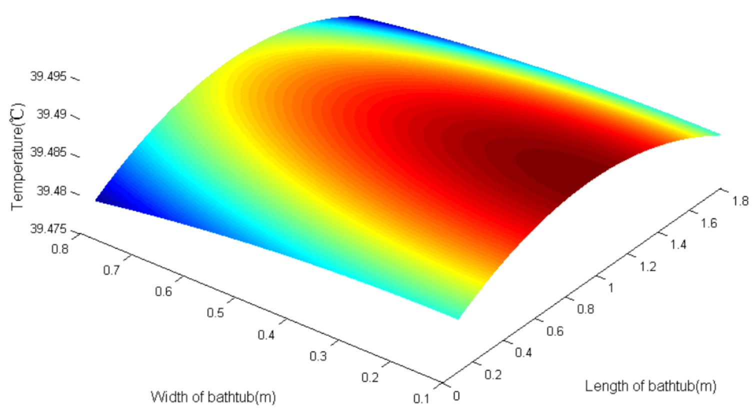

Figure 6. The temperature field distribution of x-y plane

5. Conclusion

Based on the sensitivity analysis in further study, the dependency degree of the model can be ranked as bubbles, shape of the person, shape of the tub, human motion and the volume of the tub.

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