Study of The Section Width To Thickness Ratio of Cross Section Thin-walled Hollow Pier

Chengjun YAN
College of Civil Engineering, Chongqing Jiaotong University, Chongqing, 400074, CHINA

Abstract: The formula of the constant section of buckling load was derived via A.H.KopoóoB's method, which consider. The influence of gravity. The expression of the formula was simplified by unified. Thin-wall hollow pier was composed by thin plate. According to the destructional forms that thin-wall hollow pier occurs preferentially overall buckling, the stress of overall stability is less than the local critical stress as a control condition, deduced the critical thickness ratio (thickness) formula of compression wall, which the pier should meet. Verified the formula by using finite element model. The results show that it is feasible to calculate the ultimate thickness under known plate width by using the formula in this paper.

Keywords: Thin-wall hollow pier; Local stability; Limit width-to-thickness ratio; Critical stress

1. Introduction

With the continuous development of China's transportation industry, a large number of highways, high-speed railway construction to the development of high pier bridge has brought historic opportunities, but also faces many problems. In order to save the cost, reduce the amount of material used for bridge pier, reduce the weight and the stress of the foundation, the high pier is often designed as a thin wall hollow structure. Both at home and abroad for high pier stability problems have done a lot of research, but more emphasis on high pier characteristic value, ultimate bearing capacity and stability analysis of the impact of construction defects on[1~4], but relatively little research on the local buckling analysis of thin-walled hollow pier. The hollow pier can be regarded as a thin wall component which is composed of a thin plate. In addition to the overall stability of the pier, the local stability of the thin plate should also be considered. Peng Yuancheng[5] for the Longtan River Bridge 178m hollow thin-walled pier, deduced under different constraint conditions to satisfy the local stability of thin-walled concrete minimum thickness ratio.

In this paper, the formula for calculating the critical load of the central pressure bar is derived by using the method of the. According to the thin-wall hollow pier occurs preferentially overall collapse failure form of failure, the overall stability of pier wall stress is less than the critical stress as a control condition, deduced the pier should meet the critical compression wall thickness ratio (critical thickness) formula, computation and finite element analysis, verify the correctness of the formula.

2. Critical Load of Center Pressure rod

Aiming at the problem of uniform center buckling of struts, the domestic and foreign scholars have conducted a study of systematically[6~7], derived the calculation formula of critical load for different boundary conditions and under different loading conditions. In Table 1, the critical load of the member under uniform load and concentrated load is obtained under the boundary conditions of the upper and the lower ends of the upper and lower ends of the lower consolidation, which are under the two boundary conditions.

Table 1. The instability load under different loads

boundary condition	Load	
	Uniformly distributed load	concentrated load
Lower end consolidation ,Upper end free	$(ql)_{cr} = \frac{7.837EI}{l^2}$	$P_{cr} = \pi^2 \frac{EI}{(2l)^2}$
Lower end consolidation ,Upper end hinge	$(ql)_{cr} = \frac{52.511EI}{l^2}$	$P_{cr} = \pi^2 \frac{EI}{(0.7l)^2}$

When the uniform load and concentrated load are combined, the corresponding critical load can be solved by Bessel function:

$$P_{cr} = \frac{\pi^2}{4}\left[\left(\frac{4mU}{\pi}\right)^{\frac{2}{3}} - m\right]\frac{EI}{l^2} \tag{1}$$

Form in:

$m = ql / P_E$ - The ratio of dead weight to Euler load;

U - The root of Bessel equation, Bessel function look-up table can be obtained.

The critical load can also be calculated by using A.H.KopoóoB's[8] method.

A.H.KopoóoB in the calculation of cable-stayed bridge tower stability when a simple approximation algorithm, the calculation diagram shown in Figure 1. Suppose n concentrated loads P_1, P_2, ... P_n are respectively applied to the height h_1, h_2, ... h_n, and if these loads increase by β_j times, the bars will be unstable. If these load reduction $(h_i/h)^2$ times and all moved to the top of the bar, so that the sum of the role of the top of the bar in the same critical load, there are:

$$P_j = \left[P_1\left(\frac{h_1}{h}\right)^2 + P_2\left(\frac{h_2}{h}\right)^2 + \ldots + P_n\left(\frac{h_n}{h}\right)^2\right]\beta_j = \frac{\pi^2 EI}{4h^2}$$

Where h is the tower height and EI is the tower transverse stiffness. From this we can deduce the stability coefficient: $\beta_j = \dfrac{\pi^2 EI}{4h^2\sum_{i=1}^{n} P_i (h_i/h)^2}$.

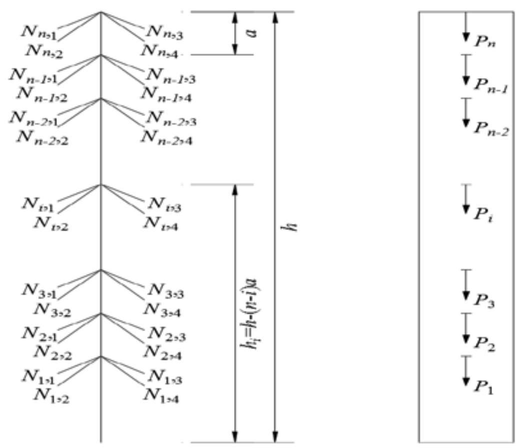

Figure 1. Cable force distribution and tower stability calculation

In this paper, the self-weight load ql, which is evenly distributed along the length of the bar, is equivalent to a concentrated force P_1 acting on the tip of the bar. As shown in Fig. 2, the concentrated force exerted on the top of the bar is P_2.

Order $P_1 = \psi ql$, P_1 as the equivalent load of self-weight, which reflects the instability state of the rod's weight. Therefore, P_1 has the same stability coefficient as the self-weight uniform load ql, i.e.: $P_1\lambda_q = P_E$, $(ql)\lambda_q = (ql)_{cr}$ .So there: $P_E = (\psi ql).\lambda_q = \psi (ql)_{cr}$, ψ can be further expressed as $\psi = \dfrac{P_E}{(ql)_{cr}}$.□

When the equivalent concentrated forces P_1 and P_2 act simultaneously to the bar instability, the critical load of P_2 is Pcr, which is obtained by KopoóoB's method: $(P_1 + P_2)\lambda_{cr} = P_E$.

In the formula, λ_{cr} is the number of stability coefficients, and the same value should be used for multiple loads. When P_1 and P_2 at the same time acting on the bar, P_1 is the equivalent weight of self-weight, the size of known and constant action on the bar, the critical load is only the critical value of P_2. Therefore, the critical load parameter λ_{cr} of P_1 and P_2 should be different. There are:

$$P_{cr2} = P_2\lambda_{cr} = P_E - P_1\lambda_{cr}^{\ q}$$

The safety reserve of the deadweight should take the minimum value, so $\lambda_{cr}^{\ q} = 1$, so there are:

$$P_{cr} = P_E - \psi ql \qquad (6)$$

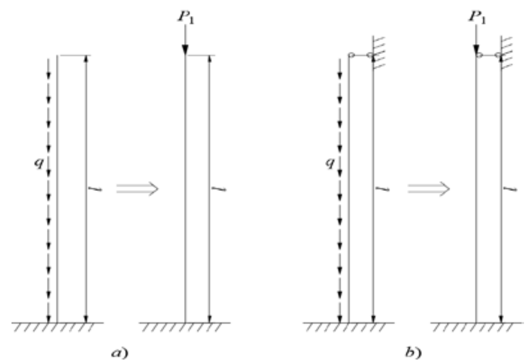

Figure 2. Load Equivalent Schematic

Unified forms of expression, do the following transformation:

Let $m = ql/P_E$, into equation (2) are: $P_{cr} = (1-\psi m)P_E$, Order $P_E = \dfrac{\pi^2 EI}{(\mu l)^2} = \dfrac{\pi^2}{\mu^2}\dfrac{EI}{l^2}$, the critical load can be expressed as:

$$P_{cr} = (1-\psi m)\frac{\pi^2}{\mu^2}\frac{EI}{l^2} \qquad (3)$$

The values of the parameters in 错误！未找到引用源。 are shown in Table 2.

Table 2. Critical load calculation parameters

parameters	Lower end consolidation ,Upper end free	Lower end consolidation ,Upper end hinge
ψ	0.314	0.383
μ	2	0.7

Let $\beta = (1-\psi m)\dfrac{\pi^2}{\mu^2}$, the critical load be expressed as: $P_{cr} = \beta \dfrac{EI}{l^2}$.

The corresponding critical stress:

$$\sigma = \frac{P_{cr}}{A} = \beta\frac{EI}{l^2 A} = \beta\frac{E}{(l/i)^2} = \beta\frac{E}{\lambda_1^2} \quad (7)$$

In the formula:

$i = \sqrt{I/A}$ - radius of gyration of section;

$\lambda_1 = l/i$ - nominal slenderness ratio of components.

In actual engineering, the thin-wall hollow pier is concrete piers, after the material enters the elastic-plastic stage, the modulus of elasticity in the formula should be modified, and the modulus of elasticity E is replaced by the tangent modulus E_t. Let $\tau = E_t / E$, the critical stress of the elastic-plastic phase is:

$$\sigma_{cr} = \beta\frac{E\tau}{\lambda_1^2} \quad (5)$$

3. Limit Width-thickness Ratio Formula of Thin-wall Hollow Pier

Compared with the solid piers, thin-walled hollow piers can obtain larger cross-sections with less material to resist the moment of inertia, and give full play to the mechanical properties of the piers to meet the overall stability and rigidity requirements. Thin-walled hollow pier has both the whole problem and the local stability of the wall plate. The local stability of the wall plate can be guaranteed by limiting the plate-to-plate thickness ratio, with the local buckling allowable stress higher than the overall stability stress.

The local stability of the thin-walled hollow pier wall plate can be obtained by buckling the elastic thin plate under uniformly distributed pressure. Longitudinal pressure, the plate from the plane position and buckling resulting in local instability. Only one direction buckling of the compressed plate is shown in Figure 3.

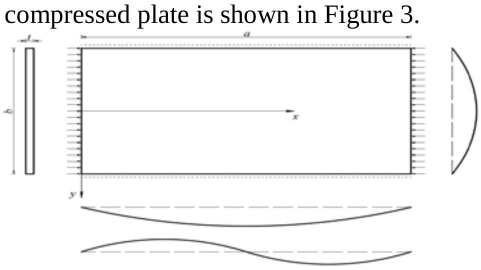

Figure 3. Buckling of thin plate

The elastic buckling of thin plate under uniform pressure is:

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + \sigma_x t\frac{\partial^2 w}{\partial x^2} = 0 \quad (8)$$

In the formula:

$D = \dfrac{Et^3}{12(1-v^2)}$ - flexural stiffness per unit width;

v——Poisson's ratio;

E——elastic modulus;

W——board surface deflection.

For the boundary conditions of four edges simply supported, the buckling deflection of the plate can be expressed by the two-stage series of (7)

$$w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn} \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \quad (7)$$

Where m, n = 1, 2, 3, ... are the number of half-waves along the x-axis and y-axis when the plate is buckled.

The deflection curve into the equation (6), finishing elastic buckling critical load can be obtained as follows:

$$\sigma_c = \frac{\pi^2 E t^2}{12(1-v^2)b^2}\left(\frac{mb}{a} + \frac{an^2}{mb}\right)^2 \quad (8)$$

In the formula, $\alpha = a/b$, $k = \left(\dfrac{m}{\alpha} + \dfrac{\alpha}{m}\right)^2$, the formula (8) is further expressed as:

$$\sigma_c = k\frac{\pi^2 E}{12\left(1-v^2\right)}\left(\frac{t}{b}\right)^2 \quad (9)$$

It can be proved that the minimum value of k with respect to the natural number field m is 4[9].

$$\sigma_c = \frac{4\pi^2 E}{12\left(1-v^2\right)}\left(\frac{t}{b}\right)^2 \quad (10)$$

Concrete wall structure generally occurs elastoplastic instability, when calculating the critical stress of the entire cross section of the pier, and consider the elastoplastic concrete, formula (10) is rewritten as[9]:

$$\sigma_c = k\frac{\pi^2 E\sqrt{\tau}}{12\left(1-v^2\right)}\left(\frac{t}{b}\right)^2 \quad (9)$$

In the formula:

$\tau = E_t / E$ - ratio of material tangent modulus to elastic modulus;

$k = \left(2 + \dfrac{2}{10\zeta + 3}\right)^2$ - warping coefficient;

$\zeta = \left(\dfrac{t}{t_c}\right)^3 \dfrac{0.38}{1-\left(tc/t_c b\right)^2}$ - constraint coefficient;

In practical engineering, it is required that the overall instability of thin-walled hollow piers before local destabilization failure, that is, the relationship between local stress and total stress:

$$k\frac{\pi^2 E\sqrt{\tau}}{12\left(1-v^2\right)}\left(\frac{t}{b}\right)^2 \geq \frac{\beta E\tau}{\lambda_1^2} \tag{10}$$

By the formula (12) can be obtained thin-walled hollow pier pressure limit wall thickness ratio as follows:

$$\frac{b}{t} \leq \sqrt{\frac{k\pi^2}{12\left(1-v^2\right)\beta\sqrt{\tau}}}\lambda_1 \tag{11}$$

The concrete material Poisson's ratio $v = 0.2$ into equation (13), there are:

$$\frac{b}{t} \leq 0.295\lambda_1\pi\sqrt{\frac{k}{\beta\sqrt{\tau}}} \tag{12}$$

The formula (14) can be used to calculate the limit thickness of thin-walled piers:

$$t \geq \frac{1}{0.295\lambda_1\pi}\sqrt{\frac{\beta\sqrt{\tau}}{k}}b \tag{13}$$

4. Example Validation

Using the formula (15) to calculate the critical thickness of the thin-walled hollow pier known when the plate width. As the thickness change will affect the radius of the radius of rotation i and the component slenderness ratio
wall thickness t value, the corresponding calculation process shown in Figure 4.

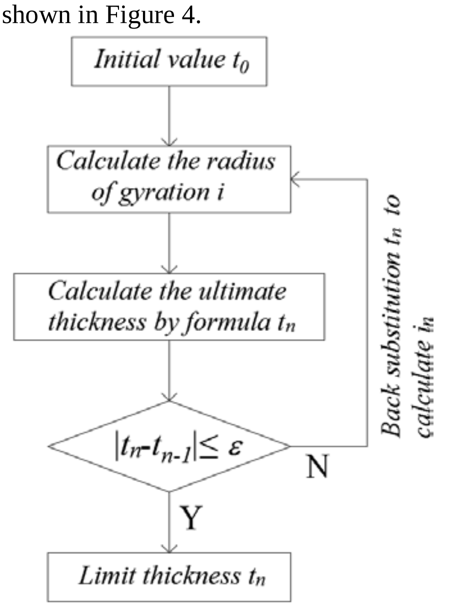

Figure 4. Limit thickness solution flow chart

A hollow thin-walled pier 55m high, C40 concrete, elastic modulus $E = 3.25\times10^4 MPa$, gravity density $\rho = 25kN/m^3$. Pier cross-sectional dimensions of 6.5m × 3.5m, along the bridge to the thickness $t_c = 0.60m$ (pressure plate to ensure the load is mainly spread along the short side, which pressure plate is a unidirectional plate.), Transverse to the wall Thickness t, cross-section shown in Figure 5.

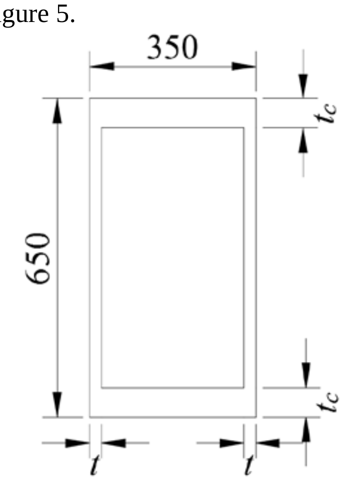

Figure 5. Sectional dimensions of thin - walled hollow piers Unit: cm

The bottom of the consolidation of the bottom of the free end as an example, the only calculation of the bridge pier under the weight of the bridge to the wall thickness t. The first trial to take t = 20cm, ignore the cross-section chamfer effect, there are:

□1, it is necessary (to iterative loop to solve the

$$A = 6.5\times3.5 - (6.5-2\times0.6)(3.5-2\times0.2) = 6.32m^2$$

$$I = \left(6.5\times3.5^3 - (6.5-2\times0.6)(3.5-2\times0.2)^3\right)/12 = 10.066m^4$$

$$i = \sqrt{\frac{I}{A}} = \sqrt{\frac{10.066}{6.32}} = 1.262m,\ \lambda_1 = \frac{l}{i} = \frac{55}{1.262} = 43.582$$

$$\zeta = \left(\frac{0.2}{0.6}\right)^3 \times \frac{0.38}{1-\left(\frac{0.2}{0.6}\times\frac{6.5-0.6}{3.5-0.2}\right)^2} = 0.022$$

$$k = \left(2+\frac{2}{10\times0.022+3}\right)^2 = 6.87,$$

$$\beta = \frac{1}{\psi}\cdot\frac{\pi^2}{\mu^2} = \frac{1}{0.314}\times\frac{\pi^2}{2^2} = 7.837$$

Substituting the above parameters into (15), the thickness t is:

$$t = \frac{1}{0.295\times43.582\times3.14}\sqrt{\frac{7.837\times\sqrt{0.5}}{6.87}}\times5.9 = 0.131m$$

The $t = 0.131m$ as a second spreadsheet initial value, repeat the calculation until the limit thickness t change does not happen until the finally obtained $t = 0.125m$. The calculation shows that with the decrease of the limit thickness t, the value of k is close to 6.97, that is, the thin plate is on both sides of consolidation state. For the simplified calculation, the value of k is 6.97.

In order to analyze the variation rule of the critical thickness of different piers, the height of piers is 65m, 75m, 85m and 95m respectively. The results are shown in Table 3.

To further verify the accuracy of the formula, ANSYS SHELL63 element was used to establish the finite element model, and the critical thickness of each pier height was calculated. The results are listed in Table 3.

Table 3. The results of the free-distributed uniform load on the upper end of the consolidation

l/m	Formula Results /m	Finite Element value results /m	Difference /m
55	0.125	0.130	-0.005
65	0.104	0.109	-0.005
75	0.088	0.094	-0.006
85	0.077	0.082	-0.005
95	0.068	0.073	-0.005

Similarly, at the lower end of the upper end of the consolidation boundary conditions of freedom, self-respect by the piers and pier at the top of the combined effects of concentrated force wall thickness value, the calculated value of finite element results are shown in Table 4.

Table 4. Critical Thickness of Joint Action of Concentrated Load and Uniform Load

l/m	Formula Results /m	Finite Element value results /m	Difference /m
55	0.065	0.072	-0.007
65	0.054	0.060	-0.006
75	0.046	0.052	-0.006
85	0.040	0.046	-0.006
95	0.036	0.041	-0.005

Table 5 shows the consolidation of the lower end of the upper hinge boundary conditions by the piers and piers under the concentration of concentrated force under the results of the calculation.

Table 5. The lower end consolidates the upper end hinged critical thickness

l/m	Formula Results /m	Finite Element value results /m	Difference /m
55	0.205	0.206	-0.001
65	0.171	0.174	-0.003
75	0.146	0.151	-0.005
85	0.127	0.133	-0.006
95	0.112	0.119	-0.007

Comparing the results of Table 3, Table 4 and Table 5, the following conclusions can be drawn: ① As the height of piers increases, the limit thickness decreases, which can be explained by the formula of critical load $P_E = \pi^2 EI/(\mu l)^2$. In the case of section size and material invariance , The critical load decreases with the increase of pier height, and the corresponding calculated stress also decreases, leading to the decrease of the limit thickness. ② The calculated values obtained by the derivation formula are less than the finite element results. The finite element method is solved by unit discretization and simultaneous equations. The accuracy of element mesh affects the result of solving, so the result of finite element analysis is usually larger than the theoretical value. ③ The results of the formula have good agreement with the results of finite element analysis, and with the increase of pier height, the difference tends to decrease. From the above analysis, it can be seen that the critical thickness of the piers in the piers can be estimated by formula (15) at the design stage.

5. Epilogue

The formulas for calculating the critical load with the interaction of the dead weight and the concentrated force are deduced and the expression form of the formula is unified by KopoóoB's method. Based on the condition that the overall stability stress of the pier is less than the critical stress of the slab, the formula of the critical width to thickness ratio (critical thickness) of the piers under pressure is deduced. The rationality of the formula is verified by ANSYS model, which provides a theoretical reference for the design unit to determine the pier wall thickness.

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