Deducing Formula of Critical Load of Thin Wall Hollow Pier In Construction Stage

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Abstract: According to the calculation formula of critical load of thin-walled hollow pier in the construction stage of the derivation of Bessel differential equation, and using the ANSYS finite element model is established to verify the formula, numerical results show that using the formula to calculate the construction stage stability of thin-walled hollow pier is feasible, and has high precision.

Keywords: Thin-Wall Hollow Pier;Central Pressure Par;Bessel Equation;Critical Load

1. Introduction

High pier and long span bridge is a kind of structural form which is often used in mountain highway and high speed railway crossing gorge. Because of the limitation of the terrain condition and the span of the bridge structure, the height of the pier is increasing. In order to reduce the amount of material used for bridge piers, reduce the weight of the pier and the stress of the foundation, the high pier is usually designed as a thin wall hollow structure. A lot of researches have been carried out at home and abroad on the stability of high piers, but more emphasis on the influence of the characteristics of high pier, the ultimate bearing capacity and construction defects on the stability[1~6]. Most of the studies are the thin-walled hollow pier is simplified as prismatic members studied, while ignoring the influence of the weight bar. In order to meet the actual engineering situation, in the derivation of the formula, we should consider the influence of gravity.

This article from the equilibrium differential equation of rod of the Bessel equation in the center bar under the action of gravity instability load is deduced; using chi lobov method to deduce the weight and concentrated load and the center of critical load calculation formula, and the formula expression of unified treatment. The correctness of the formula is verified by numerical examples and finite element analysis.

2. Basic Assumption

The thin-walled hollow pier in the height range of cross-section form.

It ignores the influence of the superstructure rigidity on the critical load, the upper part of the weight of the structure of cantilever construction stage to load the effect of P on the pier top center position.

The assumption of pile caps and piers connected together. In order to simplify the analysis, the influence of pier bottom pile is ignored.

It does not consider the effect on the wind load and temperature gradient.

The center bar meet other assumptions, which ignore the construction quality defects, the axis deflection initial defect.

3. Deduced of Critical Load Based on Bessel Equation

In the process of thin-walled hollow pier construction, the overall stability of thin-walled hollow pier is divided into the stability of the self stability and the maximum cantilever state in the construction stage. The main pier construction stage dead load and construction load, the maximum cantilever state, which can convert the weight of girder and the hanging basket construction load for the role in the vertical force on the pier top, the overall stability of the thin-walled hollow pier can be simplified as the center of the compression bar stability calculation, as shown in figure 1.

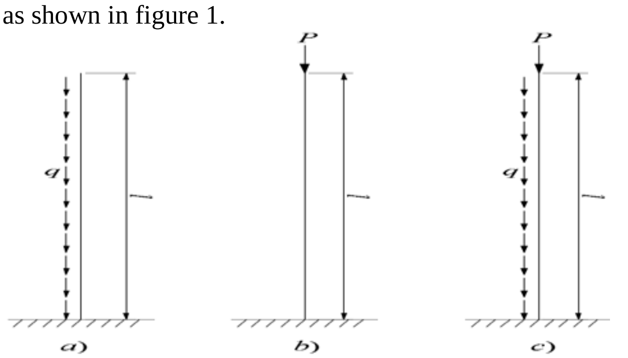

Figure 1. Simplified Graph of Bar

Under the distributed load, the calculation of instability is shown in figure 2.

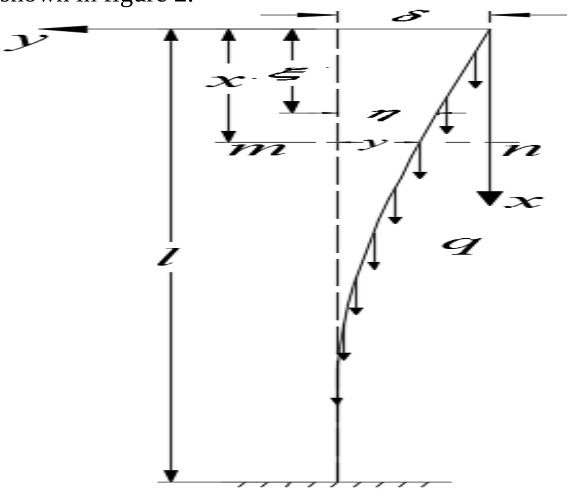

Figure 2. Figure of Instability of Rod

Under the action of vertical uniform load, the differential equation of the deflection curve:

$$EI\frac{d^2y}{dx^2}=\int_0^x q\left(\eta-y\right)d\xi \tag{1}$$

Derivativing for equation 1 on both sides:

$$EI\frac{d^3y}{dx^3}=-qx\frac{dy}{dx} \tag{1}$$

Order $t=\frac{2}{3}\sqrt{\frac{q}{EI}x^3}$, there are:

$$\frac{dy}{dx}=-\left(\frac{q}{EI}\right)^{\frac{1}{3}}\left(\frac{3}{2}t\right)^{\frac{1}{3}}\frac{dy}{dt}$$

$$\frac{d^2y}{dx^2}=\left(\frac{q}{EI}\right)^{\frac{2}{3}}\left[\frac{1}{2}\left(\frac{3}{2}t\right)^{-\frac{1}{3}}\frac{dy}{dt}+\left(\frac{3}{2}t\right)^{\frac{2}{3}}\frac{d^2y}{dt^2}\right]$$

$$\frac{d^3y}{dx^3}=\frac{q}{EI}\left[\left(\frac{3}{2}t\right)\frac{d^3y}{dt^3}+\frac{3}{2}\frac{d^2y}{dt^2}-\frac{1}{4}\left(\frac{3}{2}t\right)^{-1}\frac{dy}{dt}\right]$$

The result of derivation into equation (2) to simplify:

$$\frac{d^3y}{dt^3}+\frac{1}{t}\frac{d^2y}{dt^2}+\left(1-\frac{1}{9t^2}\right)\frac{dy}{dt}=0 \tag{3}$$

Order $u=\frac{dy}{dt}$,type (3) simplified as:

Type (4) for the Bessel differential equation of order 1/3, the general Bessel function is used to represent for:

$$u=C_1J_{\frac{1}{3}}\left(t\right)+C_2J_{-\frac{1}{3}}\left(t\right) \tag{2}$$

Among them:

$$J_{\frac{1}{3}}\left(t\right)=\sum_{n=0}^{\infty}\left(-1\right)^n\frac{1}{n!\Gamma\left(n+\frac{1}{3}+1\right)}\left(\frac{t}{2}\right)^{2n+\frac{1}{3}}$$

$$J_{-\frac{1}{3}}\left(t\right)=\sum_{n=0}^{\infty}\left(-1\right)^n\frac{1}{n!\Gamma\left(n-\frac{1}{3}+1\right)}\left(\frac{t}{2}\right)^{2n-\frac{1}{3}}$$

Are 1/3 order and -1/3 order Bessel function, C1, C2 for the integral constant.

Free conditions from the top of the rod: $\left(\frac{d^2y}{dx^2}\right)_{x=0}$ =0 ,When x=0, t=0; can be expressed as:

$$\left(\frac{1}{3}t^{-\frac{1}{3}}u+t^{\frac{2}{3}}\frac{du}{dt}\right)_{t=0}=0 \tag{3}$$

The type (5) into (6) C1=0.

$$u=C_2\sum_{n=0}^{\infty}\left(-1\right)^n\frac{1}{n!\Gamma\left(n-\frac{1}{3}+1\right)}\left(\frac{t}{2}\right)^{2n-\frac{1}{3}} \tag{4}$$

The bar at the bottom of the consolidation: $\left(\frac{dy}{dy}\right)_{x=l}=0$ .According to the aforementioned conditions: derivation: u=0, $t=\frac{2}{3}\sqrt{\frac{ql^3}{EI}}$.

The -1/3 order Bessel function to meet the minimum t listed in table u=0 the value of tmin=1.866, i.e.: $\frac{2}{3}\sqrt{\frac{ql^3}{EI}}=1.86$.

$$\left(ql\right)_{cr}=\frac{7.837EI}{l^2} \tag{5}$$

4. Example Verification

In order to verify the correctness of the formula, the finite element model was established by ANSYS. The pier section is shown in figure 3. Section parameters are: b=6.5m, c=3.5m, t=0.6m, tc=0.6m, pier height of l1=50m, l2=80m, l3=100m three different high pier check. The pier concrete material is C40, the elastic modulus is E=3.25×104MPa, Poisson's ratio is□=0.2, the density is □=2500Kg/m
section parameters and material density, the weak axial inertia moment of the pier section is I=17.85m4, and the self weight load is q=264000kN/m.

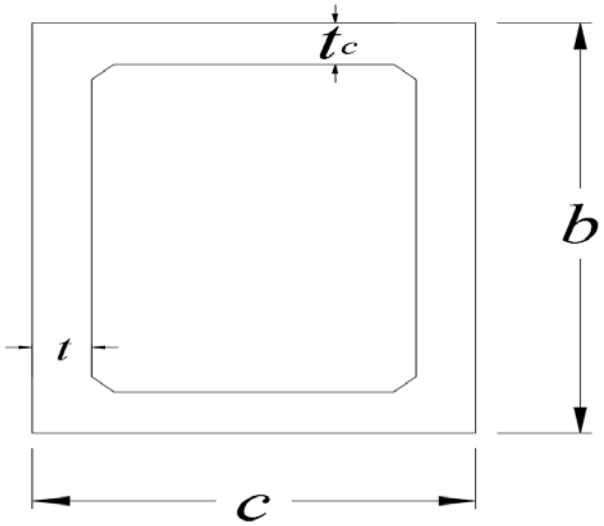

Figure 3. Pier Section

Table 1. Stability Coefficient of Bridge Pier

l/m	Formula value	Finite element results	error /%
50	5.948	5.858	1.51
80	2.273	2.261	0.56
100	1.415	1.413	0.12

According to the calculated analysis, results can be obtained by the equivalent of the dead weight of the pier is feasible using chi lobov method, finite element consolidation bottom top free calculated results are in good agreement with the values of the formula, the maximum error is 1.51%, and with the increase of high pier L, error gradually decreases.

5. Epilogue

According to the calculation formula of stability of thin-walled hollow pier in the construction stage of the Bessel differential equation is deduced, and the use of ANSYS to establish the finite element model simulation analysis, the results show that the formula (8) calculation of the construction stage of thin-walled hollow pier is feasible, and has high precision.

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