

# Slope Stability based on Grey Prediction Model

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**Abstract:** For the factors affecting the stability of subgrade slope has the complexity and uncertainty, this article described the grey theory firstly, and forecast the slope stability coefficient through the establishment of grey model, get the estimates of the next stability coefficient of slope detection. Finally, we analysis and compare the estimates thought the application example, and find it meet the actual situation. The topic of this thesis is to consider the application of grey theory to soil slope stability analysis, slope engineering systems is considered to belong to the grey system category. Starting from grey GM(1,1) modeling ideas, introduces the basic principles, basic content and calculation methods of grey system theory. Application grey system theory to study the soil slope stability just starting out, though displacement monitoring data of a slope which located at the Bi-Wei highway, establishment two GM(1,1)model which one is during the slope support, the other one is after completion of slope support.

**Keywords:** The slope stability; Grey model; The stability coefficient

## 1. Introduction

The analysis of slope stability is the core issue of slope engineering research and it also the hot topics of geotechnical engineering study[1]. By the development in the recent years, theory and research methods continue to improve, the study of slope stability entered into an unprecedented stage.

In recent years, due to the understanding of uncertainty in slope engineering, the method of slope stability analysis developed from deterministic analysis to uncertainty analysis. Meanwhile, because of the complexity of the slope engineering, slope stability evaluation is also developed from single method to comprehensive evaluation analysis. Usually the stability safety coefficient is considered as a standard of judgment in the stability of subgrade slope degree. The factors affecting the stability of slope are many, with the complexity and uncertainty. And the grey theory is the applied mathematics subject that the researched information of clear, parts of unclear with the phenomenon of uncertainty[2]. In slope engineering problems we face are mostly analysis method is not comprehensive, information limited and difficult to response of the various parameters in geotechnical engineering. So the slope engineering problems can be thought of as part of partial information known, unknown. Here, in this paper, through the establishment of grey model to forecast the slope stability co-efficient, is a research of reliability1~2.

The China collapse, landslide distribution is shown as Figure 1. The Landslide hazard map1 is shown as Figure 2. The Landslide hazard map2 is shown as Figure 3.

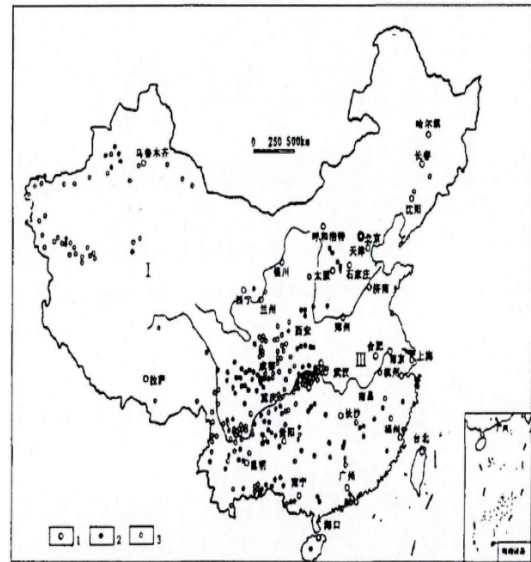


Figure 1. China collapse, landslide distribution



Figure 2. Landslide hazard map1



Figure 3. Landslide hazard map2

## 2. Establishment of Grey Model

At present, China is at rapid economic development and the western development period, 70% of the total area of the country is land, plateaus and hills, many infrastructures related to slope governance<sup>[3]</sup>. The actual slope stability and reasonable evaluation of the state, have a significant impact on the project success and people's lives and property. Reasonably reliable for slope stability analysis and evaluation is very important<sup>[4]~[7]</sup>. Grey relational analysis of slope stability analysis has obvious advantages, it is also an important part grey theory. In analyzing the factors that affect slope stability, the stability of the factors that influence the degree of sequence on soil slope, the actual project can catch primary problems; propose targeted measures. Describes the theoretical requirements associated sequence of grey methods. Summarized the factors affecting slope stability generalize, and focus on the importance of a particular sequence factors with a detail slope project<sup>[8]</sup>. Through the grey relational analysis reason of slope destabilization, put forward the corresponding engineering control measures. This article was first introduced grey correlation analysis of soil slope engineering analogy applied to 15 known slope as a reference. Deduced 5 study slope the stability steady or not<sup>[9]</sup>. Prediction results compliance with actual Rey model prediction is to process the system of raw data, explore its intrinsic attributes, and grasp its evolution rule, is a kind of quantitative prediction 3~4.

### 2.1. Data testing and processing

In order to guarantee the feasibility of modeling method, we need to do the necessary inspection with known data column. To set the reference data

$$X^{(0)} = (X^{(0)}(1), X^{(0)}(2), \dots, X^{(0)}(n))$$

Calculate the class ratio of sequence

$$\lambda(k) = \frac{X^{(0)}(k-1)}{X^{(0)}(k)} \quad (k = 2, 3, \dots, n)$$

If the entire class ratio falls within the range of  $(e^{-\frac{2}{n+1}}, e^{\frac{2}{n+1}})$ , then series can be used as a data model

and grey prediction. Otherwise take appropriate parameter c transformation

$$y^{(0)}(k) = X^{(0)}(k) + c \quad (k = 1, 2, \dots, n)$$

And make the class ratio meet the requirements.

### 2.2. Modeling

Handling of grey number mainly is to use a kind of data processing method to seek the inherent law of data. By processing the data of the known data series, new data series produced. There are many types of generation, commonly used have accumulation generation, inverse accumulation generation and mean generation.

$$X^{(1)}(k) = \sum_{i=1}^k X^{(0)}(i) \quad (k = 2, 3, \dots, n)$$

, which is the first power accumulation generation of sequence  $X^{(0)}$ .  $X^{(1)} = (X^{(1)}(1), X^{(1)}(2), \dots, X^{(1)}(n))$ , which is called the first power accumulation generation sequence of  $X^{(0)}$ .

Define the  $X^{(1)}(k)$  of grey derivative :

$$d(k) = \frac{X^{(1)}(k) - X^{(1)}(k-1)}{k - (k-1)} = X^{(0)}(k)$$

$$Z^{(1)}(k) = \alpha X^{(1)}(k-1) + (1-\alpha)X^{(1)}(k)$$

$Z^{(1)} = (Z^{(1)}(2), Z^{(1)}(3), \dots, Z^{(1)}(n))$  means generating sequence, generally  $\alpha = 0.5$ .

Then define the grey model of grey differential equation of:

$$d(k) + aZ^{(1)}(k) = b \quad \text{or} \quad X^{(0)}(k) + aZ^{(1)}(k) = b$$

Among them,  $a$  is development system,  $Z^{(1)}(k)$  is called whitening background values,  $b$  is called grey action

### 2.3. Solution of the model

Plug in the original data ,

$$\begin{cases} X^{(0)}(2) + aZ^{(1)}(2) = b \\ X^{(0)}(3) + aZ^{(1)}(3) = b \\ \dots \\ X^{(0)}(n) + aZ^{(1)}(n) = b \end{cases}$$

Make

$$Y = (X^{(0)}(2), X^{(0)}(3), \dots, X^{(0)}(n))^T, \quad u = (a, b)^T$$

$$B = \begin{pmatrix} -Z^{(1)}(2) & 1 \\ -Z^{(1)}(3) & 1 \\ \dots & \dots \\ -Z^{(1)}(n) & 1 \end{pmatrix}$$

By the least squares principle:

$\hat{u} = (\hat{a}, \hat{b}) = (B^T B)^{-1} B^T Y$ , then the equation of state for the model can made.

The corresponding white differential equation to  $GM(1,1)$  grey differential equation is:

$$\frac{dX^{(1)}(t)}{dt} + aX^{(1)}(t) = b$$

Solving the equation:

$$\hat{X}^{(1)}(k+1) = (X^{(0)}(1) - \frac{b}{a})e^{-ak} + \frac{b}{a} \quad (k = 1, 2, \dots, n-1)$$

Then

$$\hat{X}^{(0)}(k+1) = \hat{X}^{(1)}(k+1) - \hat{X}^{(1)}(k)$$

**2.4. Testing the predict value**

Residual test:

Defining:

$$\varepsilon(k) = \frac{X^{(0)}(k) - \hat{X}^{(0)}(k)}{X^{(0)}(k)}$$

If  $\varepsilon(k) < 0.2$ , it is considered that the prediction results meet the general requirements. If  $\varepsilon(k) < 0.1$ , the prediction results is considered to meet the higher requirements.

**3. Application Example**

Taking a secondary highway roadbed slope as an example. In recent years, it was inspected periodically every 6 months. Calculation of stability co-efficient were 1.47, 1.42, 1.40, 1.39, 1.36, 1.34, and make sure no intensive on the subgrade slope during this period. Suppose every time testing instruments and equipment are the same, and the testing data with the next year is 1.33, in this case through the grey model to forecast. The retaining wall slope on the left side is shown as Figure 4. The retaining wall slope on the right side is shown as Figure 5.

The surface deformation monitoring with rectangular grid layout, arrangement of four rows along slope toward the direction the surface deformation observation point, 1, 2, 3, 8, 4, 7, a total of 31 points, make the horizontal point parallel to the trend direction, vertical line Parallel to the direction, the horizontal line along the slope to a parallel arrangement. After processing in the slope surface is not found obvious cracks.



Figure 4. Retaining wall slope on the left side



Figure 5. Retaining wall slope on the right side

**3.1 Data testing and processing**

The original number listed:

$$X^{(0)} = \{1.47, 1.42, 1.40, 1.39, 1.36, 1.34\}$$

After testing and processing the data, we can get the class ratio:

$$\lambda = \{0.966, 0.986, 0.993, 0.978, 0.985\}$$

while the feasible region of the class ratio is  $(0.7515, 1.3307)$ , so the data can be used.

**3.2 Calculation of predict results**

After a cumulative

$$X^{(1)} = \{1.47, 2.89, 4.29, 5.68, 7.04, 8.38\}$$

After the mean generation

$$Z^{(1)} = \{2.18, 3.59, 4.985, 6.36, 7.71\}$$

$$B = \begin{pmatrix} -2.18 & 1 \\ -3.59 & 1 \\ -4.985 & 1 \\ -6.36 & 1 \\ -7.71 & 1 \end{pmatrix}$$

$$Y = (1.42, 1.40, 1.39, 1.36, 1.34)^T$$

Estimates can be obtained through the least-square method:

$$\hat{u} = (0.0144, 1.4537)^T$$

Substitute into the white differential equation

$$\text{Then: } \hat{X}^{(0)} = 1.3234$$

### 3.3. Testing the predict value

Residual test:

$$\varepsilon(k) = \frac{1.33 - 1.3234}{1.33} = 0.005 < 0.1$$

That is to say that the predicted results meet the higher requirement.

Just as the results shows, the predicted results with the increase of time gradually decreases, and with the actual test results for the same change trend, and the slope can be thought of in a relatively stable state.

## 4. Conclusions

(1) Grey system theory to predict occupies very important position in the grey theory, the most widely in practical application, The most common. In general, there are four of the gray theory model, is a for slope stability analysis and prediction Concerned with the specific numerical sequence forecast, the established model is a kind of single variable model of exponential function

(2) GM(1,1) model can according to the existing data model with good prediction effect, particularly when the data for the sequence. The monotone increasing and monotone decreasing negative sequences (and negative data can be through the initial value is changed into positive data), in a short time. The model has good applicability and accuracy. When required by the original data does

not meet the level than, can achieve through data processing. Modeling of the data should be has the characteristic such as time interval, when cannot satisfy (most practical process data from monitoring), Linear interpolation processing are available, and then modeling the projections or calculated.

(3) There are a lot of factors influencing the slope stability of subgrade. At the same time with randomness and uncertainty, so it is difficult to conduct qualitative analysis. In this paper, through the establishment of grey prediction model, considering the mutual effect of various influence factors, has a very definite result and reduces the certain quantity. In the design process of the slope, due to the influence factors is more, and don't know what factors between the primary and secondary order. So we can use grey relational analysis of grey theory to study and then can get a good conclusion.

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