

# Application Research of Algorithm based on Differential Equation

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**Abstract:** According to the defects of calculus equation, this paper puts forward the application of calculus equation algorithm. Based on the model theory of calculus, the paper studies and tests the equation algorithm of calculus, and the result shows that the research can effectively improve the accuracy of image recognition.

**Keywords:** Calculus equation; Algorithm; Model; Image

## 1. Introduction

Image segmentation and image registration are the two main basic tasks in image processing; they are important parts of machine vision, pattern recognition, and other areas. Registration and image segmentation exist from image arising, many researchers have proposed a lot of treatment, but because of the complex content of the image, and user needs vary, each method is often only applicable to a particular image or demand, making these two issues are still research hotspots in the image project [1-3]. In addition, Yezzi proposed registration and segmentation mixture model based on those linkages between registration and segmentation [4].

Mixture method has also become a hot issue in recent image processing. Partial differential equation, stochastic modeling and wavelet constitute a theoretical basis for image processing, compared to the other two models, the theory of partial differential equations has the following outstanding advantages: a) has a strong scalability, a variety of image processing theory can be well integrated under the theoretical framework of the partial differential equations; can be closely connected with multi-scale analysis and provide an efficient algorithm for a variety of image processing problems; has a good theoretical support, applied mathematics, differential geometry and other aspects of the theory results provide mature theoretical basis for new method [5]. The research based on image segmentation of partial differential equation and image registration methods has lasted for a long time and it is fruitful. Image segmentation method based on active contour model and image registration method based on optical flow field and other physical models are typical. This paper based on theory of partial differential equations, talking about specific issues in image segmentation and image registration, presents an improved method and effective solutions.

## 2. Self-adaption Anisotropy Regular Term

Horn model uses the Laplace operator as regular term, and Laplace operator can be decomposed into the diffusion of tangential direction and the normal direction according to local structure of the image.

$$\Delta^2 t = t_{ee} + t_{mm} \quad (1)$$

In it,  $t_{ee}$  and  $t_{mm}$  respectively represent the second derivative along with the normal direction and tangential direction

$$t_{ee} = (t_j^2 t_{ll} + t_l^2 t_{jj} - 2t_j t_l t_{jl}) / |\Delta t|^2 \quad (2)$$

$$t_{mm} = (t_j^2 t_{jj} + t_l^2 t_{ll} - 2t_j t_l t_{jl}) / |\Delta t|^2 \quad (3)$$

In fact, the weighted sum Laplace operator which can be decomposed into tangential and normal direction is the most classical diffusion operator. Therefore the regularization term of differential model is rewriting as

$$R(\Delta j, \Delta l) = \partial \alpha_{ee} + \partial b_{mm} + \beta v_{mm} \quad (4)$$

Wherein,  $\alpha$  represents the size of the amount of diffusion along tangential direction;  $\beta$  represents the size of the amount of diffusion along normal direction. Obviously, the diffusion process is determined by the coefficient, so the structure of the regular item which has specific spread function, attributed to determine the problem about the size of  $\alpha$ ,  $\beta$ . In order to keep the edges of the image during the evolution process, the image evolution should maintain the diffusion of tangential direction, and inhibit the diffusion of normal direction; besides, during the registration process, consider protecting the corners not being diffused. Since the target sharp corner has a large gradient and curvature, and the character of target edge is large gradient, small curvature; while the noise location has large curvature, small gradient, gradient in smooth area is small, and the curvature is small, so  $\alpha$ ,  $\beta$  definition can take values as follows:

$$\begin{cases} \partial = 1 - \left[ 1 - \exp\left(-\left(\frac{\Delta j}{l_1}\right)^2\right) \right] \\ \beta = \exp\left(-\left(\frac{\Delta j}{l_1}\right)\right) \end{cases} \left[ 1 - \exp\left(\frac{l}{l_2}\right)^2 \right] \quad (5)$$

first provide initial velocity field  $(u_0, v_0)$ , here set the initial velocity field as 0, then do iterative solution, in each iteration, the solution for velocity field are calculated through the velocity field obtained from previous iteration calculation, the specific expression is

$$\begin{cases} v^{j+1} = p \frac{(\partial v_{mm} + \beta a_{ee})}{l^2} \\ u^{j+1} = p \frac{(\partial u_{mm} + \beta u_{ee})}{l^2} \end{cases} \quad (6)$$

Here,  $k$  is the iteration number,  $w \in [0, 2]$  is the relaxation coefficient,  $h$  is the step size. In formula (20),  $\partial$ ,  $\beta$  use formula (16) to get the solution;  $a_{ee}$ ,  $a_{mm}$ ,  $v_{mm}$ ,  $v_{ee}$  are the most advanced differential operator, they can use formula (10) and the formula (11) to achieve, for numerical stability, LUCIDO L proposed to use adaptive template to calculate  $a_{ee}$  which shows in Figure 1.

$$\partial a_{ee}(l, j) = -5\lambda_0 u(l, j) + \lambda_1 (u(l, j-1)) \quad (7)$$

For first derivative  $u_j$  and  $u_l$ , this paper adopt the template which is similar to Sobel gradient operator to calculate, there is a certain kind of noise immunity in this template, which is able to estimate the value of image gradient better

$$u_j = \begin{cases} \frac{3-\sqrt{3}}{9} & \frac{\sqrt{3}-1}{3} & \frac{3-\sqrt{3}}{9} \\ \frac{3-\sqrt{3}}{9} & \frac{\sqrt{3}-1}{3} & \frac{3-\sqrt{3}}{9} \end{cases} u_l = \begin{cases} \frac{3-\sqrt{3}}{9} & 0 & \frac{2-\sqrt{3}}{9} \\ \frac{\sqrt{3}-1}{3} & 0 & \frac{2-\sqrt{3}}{9} \\ \frac{3-\sqrt{3}}{9} & 0 & \frac{2-\sqrt{3}}{9} \end{cases} \quad (8)$$

As  $v_{ee}$  is the same as  $u_{ee}$ , both of them need solution of speed on tangential direction, so solving method is the same; on the other hand,  $u_{ee}$  and  $v_{ee}$  have similarities in the expression form, the numerical calculating method for  $u_{ee}$  only need to change  $u_j$  and  $u_l$  in the template for calculating UTT to  $u_j$  and  $u_l$  respectively; solving  $v_{ee}$  is similar with  $u_{ee}$ .

### 3. Experimental

The following experiment is for single-mode and multi-modal brain MR images, all experimental parameters take "  $\beta = 0.03$ ,  $\eta = 3.5$ , "  $r = 0.3$ , the time step  $w_1 = 0.3$ , the basis of iteration termination judgment takes the maximum amount of displacement which is no

more than 0.01. In each figure, contour curve is red. In addition, we compared the present model and representative Wang model. Quantitative analysis of segmentation result commonly uses Js. and Deto measure; registration results use the average gray difference, MSE and mutual information for evaluation.

In this paper, we introduce edge-aligned degrees based on these three common evaluation parameters, in order to better detect the registration results about the edges of image.

A) Mean Intensity Subtraction

$$Mean = \frac{\sum_{j=1}^m \sum_{j=1}^m (w_1(j, l) - w_2(j, l))}{m \times n} \quad (9)$$

In it,  $w_1(j, l)$  represents the image after registration;  $w_2(j, l)$  represents the reference image;  $m \times n$  is the pixel number. If the mean intensity subtraction is smaller, the registration would be better.

2) Mean Square Error

$$Var = \sqrt{\frac{\sum_{j=1}^m \sum_{j=1}^m (w_1(j, l) - w_2(j, l) - Mean)^2}{m \times n}} \quad (10)$$

In it,  $w_1(j, l)$  represents the image after registration;  $w_2(j, l)$  represents the reference image;  $m \times n$  is the number of pixel; Mean is the mean intensity subtraction. When mean square error is smaller, which means the registration is better.

3) Peak Signal to Noise Ratio

$$PSNR = 103g \frac{256^2}{Var} \quad (11)$$

In it,  $Var$  is the mean square error, if the Peak signal to noise ratio is larger, the better.

3) Alignment Metric of Edge

The concept of alignment metric is from the understanding of contents of two aligned images from human eyes; from microscopic point of view, this means that each gray level of an image at pixel location corresponding to another image gray level is stable most. The process of solving alignment metric is

$$CW(w_2, w_2) = \frac{\sigma_{1,2}^2}{\sigma_1^2} + \frac{\sigma_{1,2}^2}{\sigma_2^2} \quad (12)$$

In formula (23) to (26),  $w_1(j, l)$  represents the image after registration,  $w_2(j, l)$  represents the reference image;  $E(1, 2)(n)$  and  $E(2, 1)(n)$  are gray value in two images that one image whose pixel gray level is  $n$  corresponding to another image;  $h_1(n), h_2(n)$  represents that the number of pixel gray value is  $n$ ;  $p_1(n), p_2(n)$  represent the pixel rate of which the pixel value is  $n$  in two images.

$CW$  in the range  $[0, 2]$ , use KuangYabin's infrared and visible image registration algorithm directly request reciprocal for  $CI$ , then get the alignment metric ( $AM$ ), that is

$AM = \frac{1}{CI}$  However, the range of alignment metric is too

wide, so alignment metric in this article is  $AM' = e^{-CI}$  Because when registration between two images, the value of alignment metric  $AM'$  is the greatest.

#### 4. Conclusion

This paper is based on the error of the existing optical flow image processing of moving in the pull, the image is not clear, more fuzzy defects, the proposed adaptive anisotropic image standard model research based on partial differential model, the simulation results show that the adaptive image using partial differential equation, the resolution of the image is improved obviously, read the image information is enough fast enough, effectively protect the diffusion direction and the tangent direction

of inhibition, successfully protecting the image edge and edge angle.

#### References

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