Research on Application of Geometric Solid Surface

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Abstract: Based on full analysis of subdivision, the technology utilizes the discrete of four-order geometric flows to successfully construct four-order geometric partial differential equations' finite element method based on quadrilateral surface subdivision. Experimental results show that: surface design which based on geometric flow method and surface subdivision is effective and correct.

Keywords: Surfaces; Mullins; Diffusion Flow

1. Introduction

In surface design, Bloor and others first proposed surface construction method based on partial differential equations. After that the method which based on PDE develops rapidly and it is widely used in computer-aided surface design. Later, it is proposed to use the geometric PDE method to construct surfaces. For example, the mean curvature flow is a second-order geometric PDE, which has good smoothness and noise canceling features, has been successfully applied in the design of the smooth surface, curved stitching and free surface design and other issues. In recent years, surface design method based on fourth-order geometric PDE has became a hot topic in the field, for it can generate G1 continuous stitching surface.

Tessellation technology provides a simple and efficient way to construct arbitrary topology and at the same time has a certain order of smoothness of the surface. Geometric method is a powerful technology for constructing high-quality surfaces. This article organically combines the two together and give full play to the advantages of both, in a unified framework it solves some surface design problems such as surface blending, N side fill holes and others which meet G_1 boundary conditions. This paper has successfully constructed the finite element method for surfaces of four order geometric partial differential equations based on quadrilateral Catmull-Clark subdivision.

Geometric partial differential equations are often used to control surface movement or general manifolds, except the time variable it contains only geometric quantities, it is the geometric nature, which does not rely on a specific parameterization. Using geometric partial differential equations to solve problems of geometric process and design has achieve good results, surfaces which satisfy geometric partial differential equations usually have some global optimum properties, such as small area, tiny squares mean curvature, mean curvature minimal changes, etc [1-4].These properties make the surfaces desirable smooth results. Second order partial differential equations, such as the mean curvature flow can be effectively used to solve the surface smooth or de-noising and other issues. Second order partial differential equations at the boundary of the surface can only reach G_0 smooth, while the fourth-order partial differential equations can solve surface design problems with G_1 boundary conditions. Researches focus on surface diffusion flow are numerous, the equation was introduced in 1957 by Mullins, used to describe the physics of crystal growth interface law of motion [5]. Surface diffusion flow area can be regarded as functional area's $R-1$ gradient flow and corresponding to surface diffusion flow, Willmore functional was proposed in 1923 by Thomsen. These four streams have made the desired results in solving problems such as surface polishing, surface repairing, designing fairing surfaces with G1 continuity and meeting G1 surface boundary conditions. In recent years, geometric analysis methods caused a lot of problems which need to be addressed urgently.

2. Thought of Subdivision

Subdivision method is a discrete modeling approach gradually developed since the late 1974s and was considered geometric modeling industry as one of the key technologies for the next generation of geometric modeling. The basic idea generated from the free surface modeling technology in the early 1970s. As shown in Figure 1, control polygon of cubic *B.zier* curve is $\eta=\left[q_0,q_1,q_2,q_3\right]$.Mapping method or the de Casteljau algorithm can obtain a point $q_3^{'}\left(q_3^{'}=q_0^{'}\right)$ on the curve B□zier. $q_3^{'}$ divides the curve into two child cubic *B.zier* curves which take $\eta=\left[q_0^{'},q_1^{'},q_2^{'},q_3^{'}\right]$ and

$\eta = [q_0, q_1, q_2, q_3]$ as the control polygon .Obviously, the broken line combined by the control polygon η_1 and η_2 is more approximate to *B.zier* curve than η. If repeat the segmentation process on curves of η_1 and η_2, broken line which is more approximate to *B.zier* curve can be got. If the iterative process continues, the resulting line will rapidly converge *B.zier* curve.

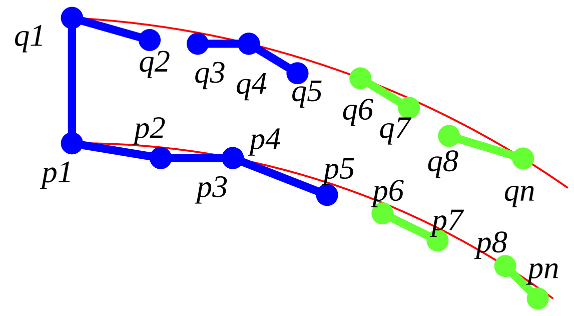

Figure 1. One time subdivision of the cubic B zier curve

Segmentation model discussed in this article is the popularization of the above subdivision thought in the space triangular control grid, first proposed by Loop of Utah University in the United States, the resulted surface is promotion of the box-spline surfaces.

For non-triangular meshes (like quadrilateral mesh), triangulation is required before subdivision. On a triangle conduct one Loop subdivision can generate four subtriangles (as Figure 2), the vertexes of the triangle can be divided into sub-parent triangular top bit (vertex-point) and edge points (edge-point). The former is subdivided from the parent triangle vertexes before changing, as $[q_0^{'}, q_1^{'}, q_2^{'}]$ in Figure 2; the latter is new vertex generated from triangle edge of the parent by the segmentation process, such as q_1, q_2, q_3. Vertex-point and edge points are generated by the parent triangle vertexes and directly adjacent vertexes in the grid by using vertex template and edge template by affine combinations.

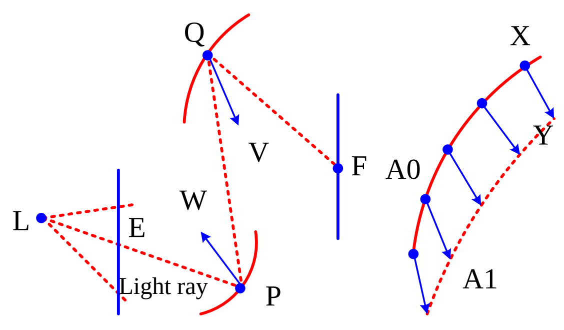

Figure 2. Triangle 1- 4 schematic subdivision

3. Simulation and Analysis A Effects of SDF, ASDF and WF

All four order equations have surfaces polished or denoising function, as can be seen from Figure 3, three short-term four order stream's evolution effect is similar, but the long-term evolution effect is significantly different.

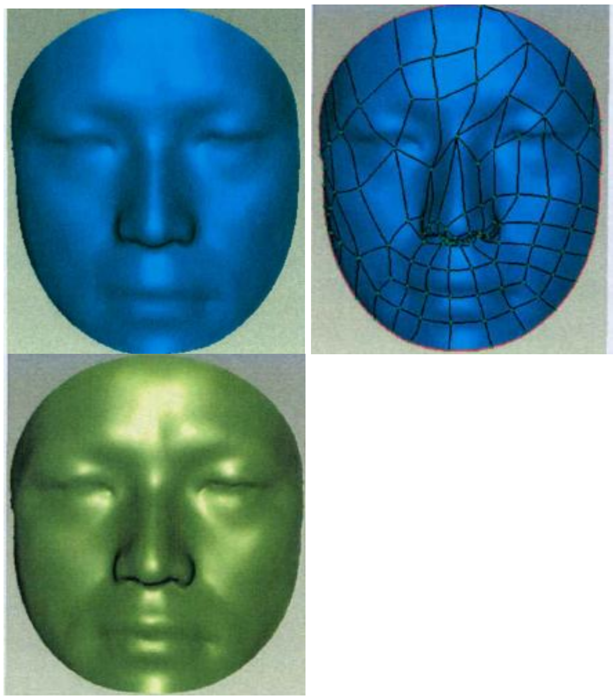

Figure 3. Effect comparison of SDF, QSDF and WF

In Figure 3, from left to right the first is an input surface mesh, the second and third one are respectively the results after twice and four times evolutions through QSDF. The fourth and the fifth one are the results are respectively the results after four times and seven times evolutions through QSDF and time step length is taken as 0.00005. As can be seen, since SDF has a volume-preserving nature, it can remain initial surface shape better than QSDF and WF, so SDF is more suitable for solving the problem of smooth surface. In addition, QSDF can rapidly shrink surface, while WF expands the surface. Therefore, according to the actual need to select the appropriate geometric flow, the three streams' different evolutionary effects can also be reflected in the splicing problems.

4. Conclusion

All the above methods are represented with the discrete grid method of surface. As the B-spline, NURBS emerge and develop, the spline surface representation is gradually introduced into the surface design. Earlier, Bloor and others applied B-spline as the solution of biharmonicequation.Later, Terzopoulos etc., put forward a dynamic NURBS method, using a second-order differential equation to determine the evolution of DNURBS [1-3]. But the equation is not intrinsic geometry and cannot get NURBS' G1 smooth stitching.

References

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