Arithmetic Research of Image Details Protection Based PG

Mingjun WANG, Yanyan ZHENG
Institute of Technology, Lishui University, Lishui, 323000, CHINA

Abstract: General restoring algorithms based on PDE could lose local feature and degenerate reading and understanding on image. In this paper, we recommend a technique to better preserve local feature by PDE and Gabor (PG) in image restoring processes. It borrows ideas from Gabor Wavelet Transformation (GWT) and is achieved by splitting the image regions which comprise ordinary regions and preserved regions. In order to preserve the regions of textures or certain details, we take advantage of GWT to search and write down these regions. Then, we may employ the algorithm PDE-based to dispose them respectively. Our results show these regions, which are preserved, retain the characteristics of original image.

Keywords: Part Protection; Image Restoration; Gabor; PDE Variation

1. Introduction

Traditional restoration algorithm based on PDE (Partial Differential Equation) image can well protect the image edge and other important information, but it is not very good to protect the local characteristics of the image, such as texture details, and these local characteristics could be removed in the process of image restoration, as shown in figure 1 on the right. Therefore, the restoration algorithm based on PDE image is likely to lose partial important characteristics of the image, which can lower the image readability and understandability. The purpose of this study is to find a better algorithm to protect the local detail features in the process of image restoration. This algorithm is based on the PDE algorithm, and use GWT (Gabor Wavelet Transformation) to find the image feature. By automatic analysis of the image regional composition, the algorithm may adaptive smooth the image and effectively protect more local features of the image. In order to protect the local image details, the algorithm will use the GWT to recognize the image details, and then weight local features and better protect the important information. The process of this algorithm is relatively simple, and adapt to handle different types images.

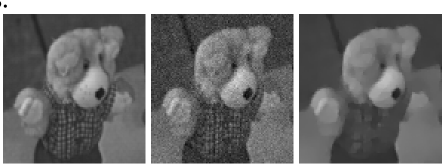

Figure 1. Left is original image, middle is dirty image, right is restoration image

Gabor filter based on Gabor wavelet transform can recognize the image characteristics of the fine feature. According to the need, the Gabor filter is consist of multi-filters. By adjusting the parameters of filter, the filter may joint spatial and frequency domain to identify the image details. In the past twenty years, Gabor theory has been applied in the field of image analysis, such as image texture classification[5]. The key of Gabor filter is to identify a set of images which contain the image information as much as possible. If Gabor filters are too much, then they may lead to the redundant of image information expression and cause a large amount of calculation. But the Gabor filters is too little, the useful information of image may loss. Therefore, it is difficult to determine the relationship between the number of Gabor filters and the extraction of useful information. How to choose these parameters of the Gabor filter, such as the center frequency, measurement and direction? the latter is more important than the others.

2. Algorithm Description

Firstly, this paper is to study how to use the GWT to represent the local feature of image and recognize the mechanism of the image feature. The second is to study the automatically mechanism of the restored image based on PDE algorithm, focus on avoiding the algorithm error and image partition mechanism. The third is to study the organic combination between the algorithm of PDE and GWT image restoration (PG algorithm). That is how to automatically identify the details of the image and use the PDE algorithm of adaptive parameter to take measures, so as to avoid the important details of the images are filtered out. Finally, verify whether the algorithm is consistent with the expected results by the experiments.

2.1. Local Features and Detail Recognition

The standard Fourier transform is a powerful tool for digital signal processing, but it can only reflect the signal in the overall character of the whole real axis, while, it does not reflect the characteristics of signal in the local time range. For images, we focus on the local characteristics, such as image edge. The Gabor function possesses the ability to recognize the local features, so it is widely used in signal processing, especially for low vision such as texture segmentation, optical flow estimation, data compression and edge detection etc.

The Gabor filter is a set of filters. According to the corresponding parameters, each filter captures the local image structure. These parameters include the center frequency, spectral bandwidth and the direction angle. Each Gabor filter in the form is the following formula[6, 7].

$$f(x, y, q_k, b, s) = \frac{1}{2ps^2} e^{-\frac{x^2+y^2}{2s^2}} e^{(x\cos q_k + y\sin q_k)bi} \quad (1)$$

Where x and y is the Coordinate of the image pixel point, s expresses the spectral bandwidth, b expresses the center frequency, q_k is the direction angle, its definition is as follow:

$$q_k = \frac{p}{n}(k-1) \qquad k = 1, 2, \cdots, n. \qquad (2)$$

k indicates the number of filter Gabor filter.

Therefore, the Gabor filter is a tunable bandpass filter. By tuning parameter of Gabor filter, we can obtain the corresponding different filter.

According to the characteristics of the Gabor filter, we represent the local features by convolving the local features with image data. If $I(x, y)$ expresses the gray value of the coordinate (x, y), then the definition of $I(x, y)$ and k^{th} Gabor filter f_k is as follow.

$$I_k(x, y) = I(x, y) * f_k(x, y), \qquad k = 1, 2, \cdots, n \qquad (3)$$

Where * is convolution operator, $I_k(x, y)$ is the k-th convolution in accordance with the parameters q_k, b, s of the k^{th} Gabor, n is the total number of the Gabor filters. So, the Gabor representation of image $I(x, y)$ may define $G = \{I_k(x, y) : k = 1, 2, \cdots, n\}$.

The Gabor features representation of these images can be combined to form a feature vector of the image, therefore, through this vector we can find image feature.

In order to simplify the algorithm, we set a fixed suitable value about the center frequency and bandwidth of the Gabor filter. Then, we adjust the direction angle of the Gabor filter, in order to find out the details of images. According to the reference [10], we set the values of the center frequency and bandwidth of the Gabor filter, they were 1.25 and 4.5 respectively. The direction angle is set to 18 faces from 0 to $\frac{17p}{18}$, the interval $\frac{p}{18}$. Therefore, in the Gabor filter set, the number of Gabor filters are 18.

2.2. Automatic Image Restoration Mechanism

The most basic model of Digital image restoration is as follows.

$$I_0 = BI + h, \qquad (4)$$

Where $I(x, y) : \Omega \subset R^2 \to R$ depict the mapping of original image, It represents the gray value mapping of the image coordinate point (x, y). I_0 is the dirty image. In Eq. (4), h indicates additive white Gauss noise, and B indicates the linear operator. Supposing a given image I_0 may reconstruct I by Eq. (4) . However, the solution of this problem is ill posed, We can construct an approximate image I .

In order to overcome the defects of the model (4) and ensure the well posedness of solutions of the minimization problem, the model should be introduced into a regularization term. In 1977, Tikhonov and Arsenin proposed the minimize problem.

$$E(I) = \int_\Omega |I_0 - BI|^2 \, dxdy + l \int_\Omega |\nabla I|^2 \, dxdy \qquad (5)$$

Where the first term of the right side in the above equation is the fidelity term of image data, the second term is smooth term. In other words, To find the most suitable gradient minimum I (which would eliminate the "noise"). The parameter l is a positive weight constant.

The minimization problem of Eq. (5) is a Euler- Lagrange equation, containing a Laplacian operator in the equation. Because of the characteristics of the Laplacian operator has the strong isotropic, so it can not effectively protect the edge of the image using it in image processing, and it makes image fuzzy. In Eq. (5), the gradient L^2 norm can enable us to easily remove the "noise". However, the gradient of the image edge is larger, so smooth effect of the edge is more apparent, and simultaneously remove the image edge features. In order to protect the edge of the image, we should lower the gradient norm from L^2 to L^1. Rudin, Osher and Fatemi proposed the equation (5) [11] . In order to study more precisely the effect of smoothing, we consider the energy equation[12] of the following.

$$E(I) = \frac{1}{2} \int_\Omega |I_0 - BI|^2 \, dxdy + l \int_\Omega f(|\nabla I|) dxdy \qquad (6)$$

Here, we assume that the operator B in the formula is the identity operator and the boundary conditions are Neumann boundary conditions. In order to directly use the change method to process the image, assuming that the function in the formula is a strict convex function, and is a non decreasing function from R^+ to R^+. Assume the existence of two constants ($c>0$ and $b>0$), and the function meet the following conditions.

$$cs-b\le f(s)\le cs+b \quad \forall s\ge 0 \qquad (7)$$

In this study, we use the function $f(s)=2\sqrt{1+s^2}-2$.

Next, we calculate the parameters l of equation (6). The minimization problem of Eq. (6), i.e. Euler-Lagrange equation is as follows.

$$I-I_0=l\,div(\frac{f'(|\nabla I|)}{|\nabla I|}\nabla I) \qquad (8)$$

According to Eq. (4) and Eq. (8), we obtain the below.

$$\frac{1}{l}=\frac{1}{|\Omega|h^2}\int_\Omega div(\frac{f'(|\nabla I|)}{|\nabla I|}\nabla I)(I_0-I)dxdy \qquad (9)$$

Parameters l may be regarded as a metric parameter. Applying the pollution of image I_0, we can construct a cluster of image $\{I(x,y,l)\}_{l>0}$. These images are a series of smooth image changing with the variation of the parameters l. The research in this way has been done by many scholars, and they is also playing an increasingly important role in the image analysis.

A good image restoration model can capture many important information in the image, image restoration algorithm based on the PDE may better process the image "noise" pollution, and better protect the image edge information. But the restoration algorithm based on PDE image exists some defects, which will also clear away the important details of image features at the same time of eliminating the "noise" pollution, Such as texture information, some small details, and even some large scale characteristic.

2.3. PG Image Restoration Algorithm

According to [13], we divide image regions into four parts. "Cartoon" part I_C is relatively smooth regions, including image edge; "Noise" part is the area of the contaminated image; "Local features" part is the local features of the image, such as texture, meaningful small details etc.. So, $I_0=I+I_n$, and $I=I_C+I_{NC}$.

In order to protect the local image details, we first must recognize the detail characteristics using the Gabor filters. The algorithm is as follow.

$$\hat{I}_k=\int_w I_k(\tilde{x},\tilde{y})w_{x,y}(\tilde{x},\tilde{y})d\tilde{x}d\tilde{y} \qquad (10)$$

The above equation is the average on $w\subset\Omega$, where $w_{x,y}(\tilde{x},\tilde{y})=w(|\tilde{x}-x|,|\tilde{y}-y|)$ is the smooth window of radial symmetry, and $\int_w w_{x,y}(\tilde{x},\tilde{y})d\tilde{x}d\tilde{y}=1$.

When the local details of the image and the Gabor filter convolve, the direction of Gabor filter and image orientation are closer, the greater the value of the convolution. Therefore, by judging the maximum value $\hat{I}_k$ of Gabor filter in the collection, we may find the most suitable number k of the Gabor filters.

$$\hat{I}_{\max k}(x,y)=\max(\hat{I}_k(x,y)) \qquad (11)$$

Simultaneously, we define the weight:

$$I_{V_{NC}}(x,y)=\begin{cases}e, & if\ I_{\max k}(x,y)>z\\ 0, & if\ I_{\max k}(x,y)\le z\end{cases} \qquad (12)$$

Where $e>0$ is positive real numbers, and $z=(\max_\Omega(I_0*f_k)-\min_\Omega(I_0*f_k))/2$.

Next, we define the local energy:

$$M_z(x.y)=\frac{1}{|\Omega|}\int_\Omega(I_z(\tilde{x},\tilde{y})+I_{V_{NC}}(\tilde{x},\tilde{y})-m[I_z])^2 w_{x,y}(\tilde{x},\tilde{y})d\tilde{x}d\tilde{y} \qquad (13)$$

From [13], if the "noise" in Eq. (5) is additive white Gauss noise, then we can derive the following conclusion:

$$\min_I\int_\Omega f(|\nabla I_V|)dxdy \quad subject\ to\ M_{\hat{R}}(x,y)=S(x,y), \qquad (14)$$

where $I_{\hat{R}}=(I-I_0-C)$, $I_V=I+I_{V_{NC}}$, $S(x,y)\ge 0$ is assumed to be given a priori value, c is a constant. Equation $I_V=I+I_{V_{NC}}$ depicts the increasing weight $I_{V_{NC}}$ of the local image features. In order to effectively enhance the image local feature against the smooth function of PDE image restoration algorithm, and protect the image local feature effect.

We use the Lagrange product to solving optimization problems:

$$E(I)=\frac{1}{2}\int_\Omega M_z(x,y)dxdy+l\int_\Omega f(|\nabla I_V|)dxdy. \qquad (15)$$

The Euler-Lagrange variational equation is as follows:

$$I-I_0-C-\bar{l}(x,y)div(\frac{f'(|\nabla I_V|)}{|\nabla I_V|}\nabla I_V)=0 \qquad (16)$$

In order to calculate the value l, we multiply $I-I_0-C$ in two side of equation (16) and integrate it. Changing the integral order of l term, we can obtain:

$$\int_\Omega(S(x,y)-l(x,y)(I-I_0-C)div(f'\frac{\nabla I_V}{|\nabla I_V|}))dxdy=0 \qquad (17)$$

Using Eq. (17), we can get sufficient conditions:

$$l(x,y)=\frac{S(x,y)}{(I-I_0-C)div(f'\frac{\nabla I_V}{|\nabla I_V|})} \quad (18)$$

From [13], we can assume

$$S(x,y)=h^2\frac{1}{1+M_{NC}(x,y)/h^2}.$$

Finally, we calculate the constant C,

$$C=\int_\Omega(\int_\Omega(I(\tilde{x},\tilde{y})-I_0(\tilde{x},\tilde{y})+I_{V_{NC}}(\tilde{x},\tilde{y})-m[I])w_{x,y}(\tilde{x},\tilde{y})d\tilde{x}d\tilde{y})dxdy$$
$$=\int_\Omega(\overline{I}-\overline{I}_0+\overline{I}_{V_{NC}}-\overline{m})dxdy, \quad (19)$$

Here, we define local average of any $Y(x,y)$.

$$\overline{Y}=\int_\Omega Y(\tilde{x},\tilde{y})w_{x,y}(\tilde{x},\tilde{y})d\tilde{x}d\tilde{y}.$$

In general, the smooth charateristic of function f can act on the "noise", and perform on the local features of the image. While, in the procedure of processing the local features of the image more energy will be lost. In the algorithm, just complement the missing energy, so as to effectively protect the local features of the image, that is the important thought of algorithm.

4. Experiment and Analysis

4.1. Experiment

This paper verify the PG algorithm by choosing four types of image. In Figure 2, these images are from the image library of image processing research institutions. In this paper, we use the explicit Euler scheme to realize the iterative process. Choose Gauss function of standard deviation $s=5$ to slide the window. In the process of restoration image we use the function $f(s)=2\sqrt{1+s^2}-2$.

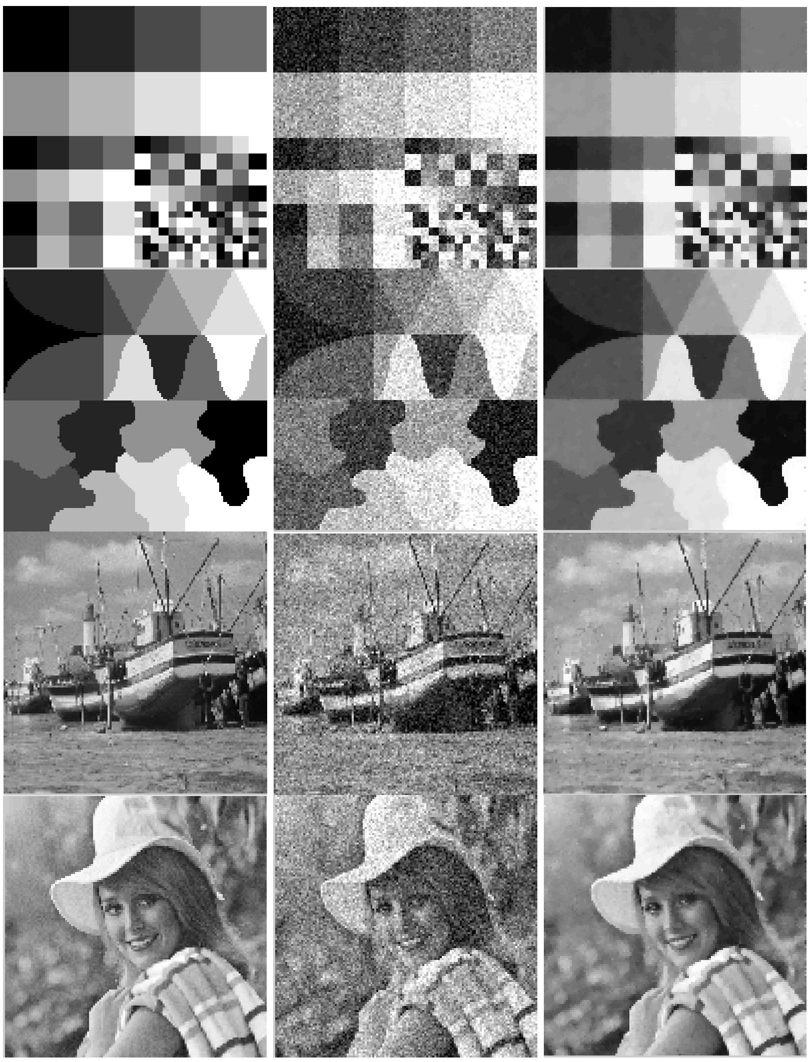

Figure 2. PG image restoration algorithm

In Fig.2, the first column is the original image, the second column is the Gauss white noise image of the variance 0.4, the third column is the removal noise image based on PG algorithm.

4.2. Comparison with Other Algorithms

Signal noise ratio (SNR) formula is SNR=S/N, where N is the noise, S is the signal. Visual effects (Visual) refer to the human visual sense to the image, If the image details is fine enough, visual effect is good. This paper gives a comparison of four algorithms, such as Median, Wiener, PDE, PG. Their SNR were calculated.

Table 1. Algorithm Comparison

Algorithm	**Median**	**Wiener**	**PDE**	**PG**
SNR	16.7023	17.0872	20.1267	20.9286
Visual	Ordinary	Ordinary	Good	Very Good

From Fig. 1, the PG algorithm is better than the other three algorithms both SNR and Visual, Therefore, our proposed algorithm is feasible, and the effect is good.

Conclusion

In this paper, we propose a better image restoration algorithm protecting the local image features. This algorithm references the Gabor wavelet transform (GWT) theory, dividing the image region into common region and local characteristics region protected area. In order to protect the texture or other details, we use GWT to find details of the characteristics. Then, according to the different regions automatic to adjust the parameter, PDE image restoration algorithm is used to deal with them. Experimental results show that, comparing with other algorithm the proposed algorithm can preserve more detail features. And the algorithm process is relatively simple, it might be used in image processing and computer vision.

From the experiment, the proposed algorithm can filter various image noise on different scenarios, the SNR has been improved to a certain extent. From the visual effect, the proposed algorithm can better protect the local features, such as the edge of objects in the image, and some important image information. The image processing algorithm from the visual effect is more natural and more close to the source image. Therefore, the proposed algorithm based on PG better protects the image details, and improves the image readable and understandable.

References

[7] N. Paragios, Y. Chen, and O. Faugeras, editors.The Handbook of Mathematical Models in Computer Vision.Springer, 2005.

[8] R. Kimmel. Numerical Geometry of Images: Theory, Algorithms, and applications. Springer, November 2004.

[9] G. Sapiro. Geometric Partial Differential Equations and Image Analysis.Cambridge University Press, 2001.

[10] Y. You, W. Xu, A. Tannenbaum, M. Kaveh.Behavioral analysis of anisotropic diffusion in image processing.IEEE Trans. Image Process, Vol. 5, No. 11, 1996.

[11] A. C. Bovik, M. Clark, and W. S. Geisler.Multichannel texture analysis using localized spatial filters. IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 12, No.1, pp. 55-73, January 1990.

[12] J. G. Daugman. Two-dimensional spectral analysis of cortical receptive fieldprofiles. Vision Res. 20:847-856, 1980.

[13] J. Daugman. Uncertainty relation for resolution in space, spatial frequency andorientation optimized by two-dimensional visual cortical filters. Journal of theOptical Society of America A. 2: 1160-1169, 1985.

[14] T.S. Lee. Image representation using 2D Gabor wavelets.IEEE Transaction onPattern Analysis and Machine Intelligence. 18(10): 959-971, 1996.

[15] D. H. Liu, K. M. Lam, L.Shen. Optimal sampling of Gabor features for facerecognition. Pattern Recognition Letters. 25: 267-276, 2004.

[16] A. D. Dileep, and C. Chandra Sekhar. Selection of Non-Uniformly Spaced Orientations for Gabor Filters Using Multiple Kernel Learning. Proceedings of the IEEE International Workshop on Machine Learning for Signal Processing (MLSP 2010), Kittila, Finland, September 2010, pp. 415-420.

[17] L. Rudin, S. Osher, and E. Fatemi. Nonlinear total variation based noise removal algorithms. Physica D, 60:259-268, 1992.

[18] G. Aubert and L. Vese.A variational method in image recovery. SIAM Journal of Numerical Analysis, 34(5): 1948-1979, 1997.

[19] Gilboa G. Sochen N. and Zeevi Y. Texture Preserving VariationalDenoising Usingan Adaptive Fidelity Term [A]. ProcVLSM 2003[C].Nice, France:IEEE, 2003. 137-144.

[20] Anil K. Jain. Fundamentals of Digital Image Processing. Prentice-Hall International Editions, 1989.

Subscriptions and Individual Articles:

User	**Hard copy:**
Institutional:	800 (HKD/year)
Individual:	500 (HKD/year)
Individual Article:	20 (HKD)