International Journal of Intelligent Information and Management Science
ISSN: 2307-0692, Volume 6, Issue 5, October, 2017

Extremal Kirchhoff Index of a Class of Unicyclic Graphs

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Abstract: The Kirchhoff index Kf(G) is the sum of resistance distances between all pairs of vertices of the graph G, where the resistance distance between two vertices of a connected graph G is defined as the effective resistance between them in the corresponding electrical network constructed from G by replacing each edge of G with a unit resistor. In this paper, we shall determine a class of unicyclic graph with the extremal Kirchhoff index.

Keywords: Resistance distance; Kirchhoff index; Unicyclic graph

1. Introduction

For any $v \in V(G)$, $d(v) = d_G(v)$ is the degree of vertex v,the distance between vertices u and v, denoted by $d(u,v) = d_G(u,v)$, is the length of a shortest path between them. Wiener index is the first recorded index, introduced by American
chemist H. Wiener in [1], defined as

$$W(G) = \sum_{\{u,v \subseteq V(G)\}} d(u,v) \tag{1}$$

In 1993, Klein and Randic [2] introduced resistance distance on the basis of electrical network theory. They viewed a graph G as an electrical network N such that each edge of G is assumed to be a unit resistor. The resistance distance between the vertices u and v, are denoted by r(u, v)=rG(u, v), is defined to be the effective resistance between nodes $u, v \in N$. The Kirchhoff index Kf(G) of a graph G is defined as [2, 3]

$$Kf(G) = \sum_{\{u,v \subseteq V(G)\}} r(u,v) \tag{2}$$

If G is a tree, then r(u, v)=d(u, v) for any two vertices u and v. Consequently, the Kirchhoff and Wiener indices of trees coincide.

The Kirchhoff index is an important molecular structure descriptor [4], it has been well studied in both mathematical and chemical literatures. References to recent studies are available from [5-10].

A graph G is called a unicyclic graph if it contains exactly one cycle, the unique cycle Cg=v1v2⋯vgv1 in an unicyclic graphs, simply denoted as

$$G = U(C_g; T_1, T_2, \ldots, T_g) \tag{3}$$

where Ti is the components of G. E(Cg) containing vi, 1≤i≤g, Ti is a tree rooted at vi. Let U(n; g) be the set of unicyclic graphs with n vertices and the unique cycle Cg, U(n) be the set of unicyclic graphs with n vertices. In [11], H. Zhang et al., determined unicyclic graphs with the maximal Kirchhoff index, and H. Deng et al. [12] obtained unicyclic graphs with the second maximal Kirchhoff index. X. Cai in [13] characterized unicyclic graphs with the second maximum Kirchhoff index.

In the paper we'll characterize a class unicyclic graphs Ug(T(l,k,i)), depicted in Figure 1, with extremal Kirchhoff index. The paper is organized as follows, in Section 2 we state some preparatory results, whereas in Section 3 we state our main results.

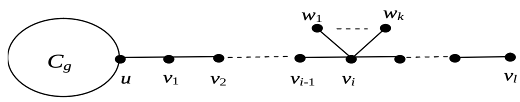

Figure 1. The graph Ug(T(l,k,i))

2. Preliminary Results

For a graph G with $v \in V(G)$, $G - v$ denotes the graph resulting from G by deleting v (and its incident edges). For an edge uv of the graph G (the complement of G, respectively), G-uv(G+uv, respectively) denotes the graph resulting from G by deleting (adding, respectively) uv.

Let $Kf_u(G) = \sum_{v \in V(G)} r(u,v)$, then

$$Kf(G) = \frac{1}{2} \sum_{u \in V(G)} Kf_u(G) \tag{4}$$

Let Ck be the cycle on k≥3 vertices, for any two vertices $v_i, v_j \in V(C_k)$ with i<j, by Ohm's law, we have $r(v_i, v_j) = \dfrac{(j-i)(k+i-j)}{k}$. For any vertex $v \in V(C_k)$, it's suffice to see that $Kf_v(C_k) = \dfrac{k^2-1}{6}$, $Kf(C_k) = \dfrac{k^3-k}{12}$.

ISSN: 2307-0692, Volume 6, Issue 5, October, 2017

Lemma 2.1([2]). Let x be a cut vertex of a connected graph and a and b be vertices occurring in different components which arise upon deletion of x. Then

$$r_G(a,b) = r_G(a,x) + r_G(x,b) \quad (5)$$

Lemma 2.2([11]). Let G1 and G2 be two connected graphs with exactly one common vertex x, and $G = G_1 \cup G_2$. Then

$$Kf(G) = Kf(G_1) + Kf(G_2) + (|V(G_1)|-1)Kf_x(G_2) + (|V(G_2)|-1)Kf_x(G_1) \quad (6)$$

Let T(l,k,i) be the tree depicted in Figure 2.

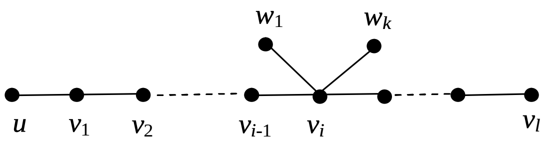

Figure 2. The tree T(l,k,i)

As we'll need the Kirchhoff index of both the star Sn and the path Pn on several occasions, we list the results in advance:

$$Kf(S_n) = (n-1)^2,$$
$$Kf(P_n) = \binom{n+1}{3} = \frac{1}{6}n(n^2-1). \quad (7)$$

Lemma 2.3. Let $T(l,k,i)$ be the tree depicted in Figure 2, then

$$Kf(T(l,k,i)) = \frac{(l+1)(l+2)(l+3k)}{6} + k(i^2 - il) + k(k-1). \quad (8)$$

Proof. Let $V_1 = \{u, v_1, v_2, \cdots, v_l\}$, $V_2 = V(T(l,k,i)) - V_1$. By the definition of Kirchhoff index, one has

$$\begin{aligned} Kf(T(l,k,i)) &= \sum_{\{u,v\}\subseteq V(G)} r(u,v) \\ &= \sum_{\{u,v\}\subseteq V_1} r(u,v) + \sum_{\{u,v\}\subseteq V_2} r(u,v) + \sum_{u\in V_1, v\in V_2} r(u,v) \\ &= \binom{l+2}{3} + 2\binom{k}{2} + \\ & k\left[\frac{1}{2}(i+1)(i+2) + \frac{1}{2}(l+1-i)(l+2-i) - 1\right] \end{aligned} \quad (9)$$

3. Main Results

Firstly, by Lemma 2.2, one has,

Theorem 3.1. Let Ug(l, k, i) be the graph depicted in Figure 1, then

$$\begin{aligned} Kf(U_g(l,k,i)) &= \frac{k^3-k}{12} + \frac{1}{6}(l+1)(l+2)(l+3k) + \\ & k(k-1+i^2-il) + (g-1)[\frac{1}{2}l(l+1) + k(i+1)] + \frac{(k^2-1)(k+l)}{6} \end{aligned} \quad (10)$$

Proof. By the formula (2.3), one has,

$$\begin{aligned} & Kf(U_g(l,k,i)) = Kf(C_g) + Kf(T(l,k,i)) + \\ & (g-1)Kf_u(T(l,k,i)) + (l+k)Kf_u(C_g) \\ & = \frac{g^3-g}{12} + \frac{1}{6}(l+1)(l+2)(l+3k) + \\ & k(k-1+i^2-il) + (g-1)[\frac{1}{2}l(l+1) + k(i+1)] + \frac{(k+l)(k^2-1)}{6} \end{aligned} \quad (11)$$

This completes the proof.

Nextly, we'll investigate the extremal Kirchhoff index of Ug(l, k, i).

Theorem 3.2. Let $G \in U_g(l,k,i), 1 \le i \le l-1$, for the fixed k and l, where k≥3, one has,

(i) If $g \ge l+1$, then $Kf(U_g(l,k,1)) \le Kf(G) \le Kf(U_g(l,k,l-1))$.

(ii) If $l-1 \le g \le l+1$, then $Kf(U_g(l,k,1)) \le Kf(G) \le Kf(U_g(l,k,l-1))$.

(iii) If $g \le l-1$, then $Kf(G) \le Kf(U_g(l,k,l-1))$.

Proof. By Theorem 3.1, one has,

$$\begin{aligned} & Kf(U_g(l,k,i)) = k[i^2 - i(l+1-k)] + \frac{g^3-g}{12} + \\ & \frac{1}{6}(l+1)(l+2)(l+3k) + k(k-1) + \\ & \frac{1}{2}l(l+1)(g-1) + \frac{1}{6}(l+k)(k^2-1) + k(g-1) \end{aligned} \quad (12)$$

Let $f(i) = i^2 - i(l+1-k) + a$, for $2 \le i \le l-1$, then Theorem 3.2 holds.

It's easy to see that,

Corollary 3.1. Let $G \in U_g(l,k,i)$, for the fixed k and l, where g≥3, one has,

$$\begin{aligned} & Kf(G) \le \frac{g^3}{12} + \frac{(l+k)g^2}{6} + \frac{g(6l^2+12kl+6l-1)}{12} + \\ & \frac{l^3}{6} + \frac{kl^2}{2} - \frac{kl}{2} - \frac{l}{3} + k^2 + \frac{5k}{6} \end{aligned} \quad (13)$$

with the equality holds if and only if $G \cong U_g(l,k,l-1)$.

In the following, we determine the extremal Kirchhoff index of $U_g(l,k,l-1)$ for k.

Theorem 3.3. Let $G \in U_g(l,k,l-1)$ and for the fixed $g \ge 3$,

$$\begin{aligned} & Kf(U_g(l,k,l-1)) \le Kf(U_g(l+1,k-1,l-1)) \\ & \le \cdots \le Kf(U_g(l+k,0,l-1)) \end{aligned} \quad (14)$$

Proof. By Corollary 3.1, one has,

$$\begin{aligned} & Kf(U_g(l,k,l-1)) - Kf(U_g(l+1,k-1,l-1)) \\ & = k(2-l-g) < 0 \end{aligned} \quad (15)$$

Then, Theorem 3.3 holds.

It's suffice to see that,

Theorem 3.4. Let $G \in U_g(l,k,l-1)$, and for the fixed g, one has,

$$Kf(U_g(l,k,l-1)) \le \frac{g^3}{12}+\frac{g^2 l}{6}+ \frac{g(6l^2+6l-1)}{12}+\frac{l^3}{6}-\frac{l}{3}. \quad (16)$$

The equality holds if and only if $G \cong U_g(l,0,l-1)$.

Theorem 3.4 coincide with the Theorem 3.4 in reference [11].

In the next, it's natural to investigate the extremal $Kf(U_g(l,0,l-1))$ according to g.

Theorem 3.5[11]. Let $G \in U_g(l,0,l-1)$, and for g≥3,

$$Kf(U_g(l,0,l-1)) \le \frac{n^3-11n+18}{6}. \quad (17)$$

The equality holds if and only if $G \cong U_3(n-3,0,n-4)$.

In the following, we'll determine the graph with the second maximum Kirchhoff index among $G \cong U_g(l,k,i)$.

Theorem 3.6. Let $G \in U_g(l,0,l-1) \setminus U_3(n-3,0,n-4)$, one has

$$Kf(G) \le \frac{n^3-17n+36}{6}, \quad (18)$$

The equality holds if and only if $G \cong U_3(n-4,1,n-5)$.

Proof. From the previous inference, we can draw the conclusion,

$$\begin{aligned} Kf(G) &= \max\{Kf(U_3(n-4,1,n-5)), Kf(U_4(n-4,0,n-5))\} \\ &= \max\left\{\frac{n^3}{6}-\frac{17n}{3}+17, \frac{n^3-17n+36}{6}\right\} \quad (19) \\ &= \frac{n^3-17n+36}{6}. \end{aligned}$$

4. Acknowledgment

Projects supported by NSFC(Grant no. 11401192), Scientific Research Fund of Hunan Provincial Education Department (Grant no. 15C0248), Natural Science Foundation of Hunan Province(Grant no. 2015JJ3031).

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Innovative Method for Vocational College English Teaching under the Background of MOOC

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Abstract: With the reform of quality education, the all-round development of students' abilities in various fields can be promoted. China's higher vocational universities (HVU) have carried on the corresponding teaching method reform, in practical study process the student's practice ability has been improved and developed. In the process of English teaching in HVU, as a new teaching mode, MOOCs break traditional school teacher-teaching mode, and has brought great challenges to the reform of higher vocational education (HVE). In English MOOCs, teachers should have higher teaching ability, make rational use of teaching resources, strengthen the initiative of students in the process of learning, and improve the quality of English teaching in HVE.

Keywords: MOOC; Higher vocational education (HVE); English teaching; Classroom evaluation; Innovative practice

1. Introduction

The reform of English teaching in higher vocational universities (HVU) under MOOC background requires of higher vocational teachers to strengthen the cultivation and development of teaching ability of MOOCs in the process of reform, promote the students' interest in learning, and strengthen the combination with traditional teaching modes in taking reasonable and scientific teaching measures. Under the guidance of ideology of HVE reform, establish a new teaching mode, strengthen the construction of teaching system for MOOCs, improve teachers' teaching ability, so that in the classroom, teachers are good at using MOOC teaching method, and establishing MOOC teaching platform in reasonable teaching evaluation system. In HVE teaching, we should create a reasonable form of classroom teaching, increase the attention of students' learning methods, keep pace with the times, and establish a new teaching concept, making the teaching mode more epochal.

2. Application Background of MOOC Teaching

With the rapid development of social science and technology, the scientific development of quality education has been promoted, which brings many challenges to the innovation and reform model of education, especially for education and technology, the awareness of innovation in schools increases. In the process of teaching, we should strengthen the use of information technology, and bring a better way for the sustainable development of education, and promote the improvement of the ability of school education. In recent years, MOOC is an online teaching mode, which is based on the open teaching platform and scientific teaching mode. Led by the keynote teachers, students are taught by video lectures online, and video teaching is managed by a specific management personnel responsible. The number of students is not limited by the time and space, and large-scale student participation is supported. The form of class teaching is mainly through video teaching, after-school practice, spatial interaction, mail examination and other forms of network teaching. The rise of MOOC teaching mode has injected fresh blood into the reform and development of education, and has established a teaching platform through the network environment with the support of information technology. The use of network platform for online learning model serious impacts on the traditional classroom teaching mode, breaking the limitation of traditional learning and classroom teaching, and bringing a powerful reform force to the innovation of teaching mode. In the traditional teaching management process of HVE, information technology easily arouses students' interest in learning, promotes students to form a new driving force of learning, and stimulates their interest in learning. Under the guidance of the new teaching concept, the students' enthusiasm to learning can be effectively promoted. In the process of English teaching reform the focus of teaching form of many vocational schools is to learn passively and accept knowledge, and teachers play a major role in the teaching process. Obviously, this teaching mode greatly limits the initiative of students' learning, reduces the fre-