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Divergent Thinking in the Process of Solving Advanced Mathematics Problem

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Abstract: This article primarily discusses the significance of the students' divergent thinking in teaching of mathematics, and further explores how to train students' divergent thinking abilities in the process of solving problems, that is to say to train the fluency of thought through the adoption of divergent questions,; to train broad thinking through seeking several answers to a problem; to train the flexibility of thinking through patchwork techniques; to train the creativity of thought through delaying the assessment.

Keywords: Teaching of Advanced Mathematics; Divergent Thinking; Convergence of Thinking; Reverse Thinking

1. Introduction

Starting from the available information, the so-called divergent thinking in creating and problem-solving process of thinking identifies problems, asks questions and looks for clues and ways to solve them, through observation, analysis, induction, association, analogy, and other methods. Divergent thinking, also known as the proliferation of ideas, radiation thinking or difference-seeking thinking, is multi-dimensional and open. This way of thinking, not subject to the limitations of modern knowledge, free from the shackles of traditional knowledge, is directly linked to the core of the creative thinking. Take the famous GOLDBACH (Goldbach, 1690-1764) conjecture as an example: "any even number greater than 4 can be expressed as two odd prime numbers". This conjecture was offered by Goldbach through repeated observation and testing, 250 years has passed so far, but this conjecture has not received the final settlement yet been hailed as the "crown jewel". Many conjectures in the mathematical field are the products of divergent thinking.

Divergent thinking is based on feeling or intuition-based, non-logical form of thinking, which is a less rigorous way of thinking. It has a strong personality and is less common than others. So, the convergence of thinking or logical thinking is often preferred in the current mathematics teaching, whereas the divergent thinking is often neglected. The comfortable, boring and passive pursuit of understanding, memorizing consolidating and reproducing of the teachers in the teaching ready-made conclusions limits the students' thinking space and confines their creative thinking and innovation capacity. Therefore, students should be trained to high-quality innovative talents, while teachers must strive to create scenarios to help students to think independently, to provide incentives for students so that students have the opportunity to personally explore the unknown, to solve the problem of the unknown, to capture the knowledge and the ability to grow. In short, the teaching must vigorously strengthen training in divergent thinking, which is necessary to train a generation of creative talents. Undoubtedly, the strengthening of problem-solving training and practice is an effective means to achieve the above purpose.

2. Divergent Questions to Train the Fluency of Thinking

Since thinking starts from the problem, divergent questions can motivate students to think positively. The goal of this question is not to pursue a single answer, but the creative idea. What's more, it's of a more direct and more realistic significance to develop students' creative thinking. In conjunction with the actual teaching in the classroom teaching, to stimulate students' passion of active participation, positive thinking and lively discussion, a situation should be set up with fresh unique and challenging mathematical problems. Special attention should be paid to guide students to break the routine, to open up new approaches from different aspects, to think divergently from different channels, to read, understand, analyze and summarize problems and problem-solving skills, and to improve the abilities of analyzing and solving problems. This kind of training really does much good to students' divergence of thinking capacity. For example, after teaching the theorem and the proposition, teachers should be aware of asking the students to examine whether the converse proposition is established. If it's not established, they should guide the students to give counter-examples. One additional example is that in learning the concept of derivative we mention the derivative is a very important mathematical model. Although it's introduced by the instantaneous velocity, its significance goes far beyond the scope of the mechanics and penetrates the various fields of science and technology. The entire division defined concept (physical) in the uniform case is the vast majority of derivative in the case of non-uniform. Here some simple examples can be given: speed, acceleration, current intensity, linear velocity, angular velocity and so on. Then a question can be asked: "Can you cite other examples?" At this time, all the students are willing to offer such answers as "growth rate and mortality", "the rate of decay of radioactive substances", "war material and the loss rate of the combat effectiveness", "the rate of temperature change of the cooling process". Students come up with many different examples showing their active thinking. At this time the teacher should lose no time to summarize that mathematics, collectively referred to as the rate of change of function, is all derivative bonded.

Divergent thinking ability of such students has been training. At the same time they will be greatly attracted by the charm of the derivative.

3. Several Answers to One Problem to Train Broad Thinking

Several answers To a given problem is an effective way to cultivate students' creative thinking. The reason it helps to cultivate divergent thinking lies in the following: first, it is conducive to the basic theory learnt, helpful to the organic link between the mathematical each part and contributed to the mastery of the knowledge; second, it requires students to think more, not limited to a single point of view, free from the shackles of an idea. In order to seek the solution of the problem, it requires to find a variety of solutions, and pursue many possibilities. Teachers should guide students, who are not confined to the given questions, to train and strengthen their broad thinking ability, which can play a subtle role in the students' ability to innovate. At the same time, teachers should ask students to make comparison and choices among a variety of ideas and finally to find a clear, concise problem-solving approach. In this case, students tend to be inventive to find new ways to solve the problem.

For example, Here's such a problem in the study of double integrals:

Seeking $I=\iint_D (x+y)ds$, D is Circle area $x^2+y^2\le 2(x+y)$.

there are a number of different solutions to the question. Teachers can guide students to analyze several ideas of different solutions:

[Solution 1] Polar coordinates, D's boundary equation can be reduced to $r=2(\cos q+\sin q)$

Closed area D Can be expressed as

$$0\le r\le 2(\cos q+\sin q),\quad -\frac{p}{4}\le q\le\frac{3p}{4},$$

So $I=\int_{-\frac{p}{4}}^{\frac{3p}{4}} dq\int_0^{2(\cos q+\sin q)}(r\cos q+r\sin q)rdr$

$$=\frac{8}{3}\int_{-\frac{p}{4}}^{\frac{3p}{4}}(\cos q+\sin q)^4 dq=\frac{8}{3}\int_{-\frac{p}{4}}^{\frac{3p}{4}}(\frac{3}{2}+2\sin 2q-\frac{1}{2}\cos 4q)dq=4p.$$

[Solution 2] using a transform $x=1+r\cos q, y=1+r\sin q$, In this transformation, D ' sboundary equation can be reduced to $r^2=2$

Closed area D Can be expressed as

$$0\le r\le\sqrt{2},\quad 0\le q\le 2p,$$

so $I=\int_0^{2p} dq\int_0^{\sqrt{2}}(2+r\cos q+r\sin q)rdr$

$$=\int_0^{2p}[2+\frac{8}{3}(\cos q+\sin q)]dq=4p.$$

Solution 1 and 2 belong to the general method. Teachers should guide the students to examine more closely the question, analyze the information implied and weigh their importance on solving the question.

First of all, students should realize that integral domain is a point with $C(1,1)$ as the Centre and $r=\sqrt{2}$ as the RADIUS field. Second, $I=\iint_D (x+y)ds=\iint_D xds+\iint_D yds$. So what do students think? It seems as an integral in seeking uniform slices in Center of gravity formula of static moment! If plane D in xoy occupies the closed area on a flat sheet, , when surface density is $r=1$ in a thin uniform, slice of Barycentric coordinates $\bar{x}=\frac{1}{A}\iint_D xds$, $\bar{y}=\frac{1}{A}\iint_D yds$, which $A=\iint_D ds$ is for the area of a region D. So $\iint_D xds=\bar{x}\cdot A$, $\iint_D yds=\bar{y}\cdot A$. When the area of a plane figures and Barycentric coordinates, a simple integral $\iint_D xds$ or $\iint_D yds$ can be calculated. After examining the above, a new method of recognition and ssociation into this series of thought has been achieved.

[Solution 3] integral domain D can be reduced to: $(x-1)^2+(y-1)^2\le 2$.

D is the density is 1 sheet, its center of gravity. C(1,1) .Because the slice area $A=pr^2=2p$,

Deduced from the gravity,

$$\iint_D xds=\bar{x}\cdot A=1\times 2p=2p,$$

$$\iint_D yds=\bar{y}\cdot A=1\times 2p=2p,$$

So $I=\iint_D (x+y)ds=4p$.

Comparing these three methods we can easily find the third solution clear, concise and is a typical reverse thinking. So, getting one answer does not mean the completion of the problem solving. We should also consider the possibility of other ways to solve? When the answers to the questions are long and complexed, we naturally think whether there are more concise way. Even if we are successful to find a satisfactory solution, we should not feel proud. Instead we should seriously consider other solutions and then compare a variety of solutions too. Which is the easiest? Which is the best? To a given problem can not only develop students' divergent thinking ability, can greatly stimulate students' learning enthusiasm and keen interest of the advanced mathematics.

4. Piecing Together the Skills to Train Flexibility of Thinking

piecing together a clear intention of deformation techniques belongs to divergent thinking. We often encounter such difficulties that the problem solving process is too complicated. With conventional methods, students are always separated by an insurmountable gap between the known and unknown. This makes people give up routine to seek a new path, so they re-examine the problem, and have a careful study of the problem in a variety of conditions. They often find more subtle information. And how to use the idea of this information makes us we think of patchwork skills, which can simplify the complexed problem-solving process. But Patchwork does not asks for general principles and only is effective on specific problems. It is an excellent opportunity for training the divergent thinking ability. Facing the problem, we must first recognize the type of problem to grasp the nature of the problem, seize the goal land find possibilities of patchwork. And finally seeking the specific method, we can train flexibility of t students' thinking.

For example, finding indefinite integral $\int \frac{1}{x^{11}+2x}dx$.

This is a relatively simple integrals of rational functions, but regular "partial fraction" method is very complexed. The examination of the integrand, $x^{11}+2x = x(x^{10}+2)$ probably would have caused the thought of the numerator and the denominator of integrand x^9, So the following method is got:

Solution: $\int \frac{1}{x^{11}+2x}dx = \int \frac{x^9}{x^{10}(x^{10}+2)}dx$

$= \frac{1}{20}\int (\frac{1}{x^{10}} - \frac{1}{x^{10}+2})dx^{10} = \frac{1}{20}(\ln \frac{x^{10}}{x^{10}+2}) + C$

This is concocted by changing the structure of the integrand, and exports new different points . successful problem solving is often due to a flash in people thought. n inspiration jumps into the mind so that the problem can be successfully settled.

5 Delaying Evaluation to Tain Oiginality of Tinking

Delaying evaluation can speak freely to the students to create an atmosphere of mutual inspiration, so that students can create more ideas in a limited period of time, which helps to develop students' divergent thinking ability. There must be a process of thinking for students to find the answers, especially the new and unique ones. This process slowly expands just as the machine starts. In the process of the students' thinking, premature evaluations from others, especially from teachers, often become obstacles. Because of this, we , in the classroom, should have a great deal of patience to give the students sufficient time to let them express their views. In this case, without any hesitation, the students will have a "sense of security" or "sense of freedom" to tackle the problem with positive thinking and language.

Sometimes in advanced mathematics teaching appropriate knowledge can be also found to create an atmosphere easy for research to enable students to take collective discussion on the conclusions of the research questions given to speculation and estimation. This is a very creative process and belongs to divergent thinking. Such a discussion without teachers' intervention helps students to speak out their minds, brainstorm and give rise to the generation of creative thinking. During collective discussion, students' thinking is in a positive state so collective discussion is useful. The students really understand the knowledge of advanced mathematics.

In short, divergent thinking can make students active and innovative with quick thinking. A large number of approaches become available, in particular, can raise some fresh insights to miraculously solve some completely unexpected problems. Therefore, as a college teacher, no matter which subject he teaches, should be aware of developing students' divergent thinking.

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