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Incrementally Updating Method in Dominance- Based Rough Set Approach

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Abstract: Dominance-based rough set approach (DRSA) can handle the attributes with preference orders, and therefore it has been widely applied in multi-criteria decision making problems. In real applications, the collected information is updated from time to time which results in dynamic information systems, especially when the attributes or objects are inserted or deleted. The traditional DRSA needs to update the set approximations whenever the information systems change, which decreases the method efficiency greatly. For classification problems with multiple criteria, this paper presents incremental algorithms to update set approximations when an object is inserted or deleted, which is expected to be more efficient than computing the approximations from the scratch. The related theoretical results are presented with proofs, and illustrative examples are also given to support the effectiveness of the proposed incremental method.

Keywords: Rough Set; Dominance Relation; Set Approximation; Dynamic Information Systems; Incremental Update

1. Introduction

Rough set theory [1] is a mathematical tool developed in recent years to deal with inconsistency and ambiguity information. The traditional rough set theory proposed by Pawlak and based on equivalence relations can only deal with discrete symbolic attributes. In order to solve practical problems with preference-ordered attributes, Greco et al. proposed dominance-based rough sets approach (DRSA) by substitution of the equivalence relation in RST with a dominance relation [4-6], and the condition attributes with preference order are called criteria. Currently, DRSA has been widely applied in multi-criteria decision problems [7,8]. Many collected data are dynamic in practical application, where the involved information systems need to be updated frequently. Some scholars have put forward some methods of incrementally updating approximations and attribute reduction in the framework of rough set [9-14]. When the set of attributes changes, Tianrui Li et al put forward incremental approach for updating set approximations based on characteristic relation rough set [9] to deal with the situation when multiple attributes change at the same time. Hongmei Chen discussed the dynamic maintenance scheme of set approximations when the attribute values in incomplete information increase and decrease, i.e., coarsening and refining the partition granularity [10]. Based on the concept of information entropy, Feng Wang et al designed strategies of attribute increasing and put forward the corresponding calculation method of attribute reduction [11]. In the framework of DRSA, Shaoyong Li introduced the concept of dominance matrix to update the dominant sets /inferior sets and approximations [12]. When the object set changes, i.e., an object is deleted or inserted, Hongmei Chen provided the incremental approach for updating approximations in variable precision rough set [13]; Shaoyong Li discussed the updating computation method for approximations of upward union and downward union under the dominance relation [14]. Compared with classical rough set method, these incremental methods obviously improved the computational efficiency of the approximations and make good preparation for the subsequence. Considering the preference order of attribute values, the methods proposed in [12][14] require both the condition attribute and decision attribute are criteria simultaneously. While in many practical problems, only condition attribute values have preference order, and the decision attribute values are no better or worse. For example, consider Iris in UCI databases which determine the type of flower based on the sepal length, width and the petals length and width. Guoyin Wang [15] and Yan Li [16] et al discussed this situation and proposed the positive domain reduction and rule extraction methods, and this type of problem is called as multi-criteria classification problem. In this paper, we consider multi-criteria classification problem, and discuss rapid methods of updating approximations under the variation of the object set. Distinguished with [14], we only require the condition attributes have preference order. We have obtained simple updating rules by theoretical proofs, which can be used to greatly improve the efficiency of information processing in multi-criteria classification problems.

The remainder of this paper is organized as follows: We present basic notions of DRSA in Section 2; when a single object is inserted into or deleted from the system, the principles of updating approximations and detailed proofs are given in Section 3; on this basis, Section 4 gives the corresponding incremental updating algorithm; a numerical example is given to validate the feasibility of our proposed approach in Section 5; and conclusions are given finally.

2. Basic Concepts

As a prior knowledge, this section describes the involved concepts based on dominance relations of rough set theory, including target information system, dominance relations and dominance classes, upper and lower approximations.

Definition 1. A quadruple S = (U, A, V, f) is an information system, where U is a nonempty finite set of objects, called the universe. A is a nonempty finite set of attributes, $A=C\cup D, C\cap D=\varnothing$, where C and D denote the sets of condition attributes and decision attributes, respectively. $V=\bigcup_{a\in A}V_a$, V_a is the domain of attribute a. f: $U\times A\rightarrow V$ is an information function, which gives values to every object on each attribute, namely, $\forall a\in A, x\in U, f(x,a)\in V_a$.

Definition 2. Let S = (U, A, V, f) is an information system, for $B\subseteq A$, we denote

$$R_B^{\leq}=\{(x_i,x_j)\in U\times U: f_l(x_i)\leq f_l(x_j), \forall a_l\in B\}$$

$R_B^{\leq}$ is the dominance relations of information system. Based on definition 2,

$$[x_i]_B^{\leq}=\{x_j\in U:(x_i,x_j)\in R_B^{\leq}\}=\{x_j\in U: f_l(x_i)\leq f_l(x_j), \forall a_l\in B\}$$

is the dominance class of x_i.

Definition 3. With each subset $X\subseteq U$, we associate two subsets:

$$\underline{R_B^{\leq}}(X)=\{x_i\in U:[x_i]_B^{\leq}\subseteq X\}$$

$$\overline{R_B^{\leq}}(X)=\{x_i\in U:[x_i]_B^{\leq}\cap X\neq\varnothing\}$$

Which are respectively called the lower and upper approximations of X with respect to dominance relation. The dominance relations discussed in this paper only reflect on condition attribute, and the decision attribute is still based on equivalence relation.

3. The Updating Principles

Let $S=(U,A,V,f)$ be an information system, $U=\{x_1,x_2,...,x_n\}, A=C\cup D$. In the multi-criteria classification problems, the values of condition attribute are with preference order, and the decision attributes are only class labels. Let R be the dominance relation defined on C, then U/R forms the overlap for U rather than partition. The decision attribute is no preference relation, then introduce equivalence relation on D, U/D forms a partition of the universe.

Considering the universe U is dynamic, and the attribute set of A is invariant. We will discuss the rapid method of updating upper and lower approximations for a given object set $X\subseteq U/D$ when delete or insert an object x. The object set X' represents X after changing.

3.1. Updating Approximations of $X\subseteq U/D$ when Deleting an Object $x\in U$

1) *Update the lower approximations of* X

Case 1: if $x\notin X$, *i.e.*, $X'=X$, then the lower approximations do not require to re-calculate, i.e., $\underline{R}_A^{\leq}(X')=\underline{R}_A^{\leq}(X)$.

Proof: suppose the overlapping which original dominance classes made on U is $E=\{[x_1]_R^{\leq},[x_2]_R^{\leq},...,[x_n]_R^{\leq}\}$, then the dominance classes after updating is

$E'=\{[x_1]_R^{\leq}-\{x\},[x_2]_R^{\leq}-\{x\},...,[x_n]_R^{\leq}-\{x\}\}$.

Because $x\notin X$, $x\in[x]_R^{\leq}$, then $[x]_R^{\leq}\not\subset X$

If $\exists$ $x_j\neq x$ and $[x_j]_R^{\leq}\subseteq X$, for $x\notin X$, obviously $x\notin[x_j]_R^{\leq}$, so the dominance categories x_j after updating is $[x_j]'^{\leq}_R\subseteq X$, i.e., $[x_j]'^{\leq}_R=[x_j]_R^{\leq}\subseteq X$.

By the generality of *X* we can conclude that the dominance classes contained in *X* are still contained in *X'* after updating. For this situation, the lower approximations do not require re-calculation, i.e., $\underline{R}_A^{\leq}(X')=\underline{R}_A^{\leq}(X)$.

Case 2: if $x\in X$, *i.e.*, $X'=X-\{x\}$, the formula of updating lower approximations is defined as follows:

$$\underline{R}_A^{\leq}(X')=\begin{cases}(\underline{R}_A^{\leq}(X)-\{x\}) & [x]_R^{\leq}\subseteq X\\ \underline{R}_A^{\leq}(X) & [x]_R^{\leq}\not\subset X\end{cases}$$

Proof: Omitted because it is simple to prove.

2) *Update the upper approximations of* X

Case 3: if $x\notin X$, *i.e.*, $X'=X$, the upper approximations after updating can be written as the following formulation:

$$\overline{R}_A^{\leq}(X')=\begin{cases}(\overline{R}_A^{\leq}(X)-\{x\}) & x\in\overline{R}_A^{\leq}(X)\\ \overline{R}_A^{\leq}(X) & x\notin\overline{R}_A^{\leq}(X)\end{cases}$$

Proof*:* it is samilar to the proof of case 1.

Case 4: $x\in X$, *i.e.*, $X'=X-\{x\}$, for $\forall x_j\in\overline{R}_R^{\leq}(X), x_j\neq x$, we discuss the following two conditions:

(a) $x \in [x_j]_R^{\leq} \cap X$ and $[x_j]_R^{\leq} \cap X = \{x\}$, then update the upper approximations of X' as $\overline{R}_A^{\leq}(X') = \overline{R}_R^{\leq}(X) - \{x\} - \{x_{j\in(a)}\}$, where $j \in (a)$ represents the subscript j which is satisfied the condition $[x_j]_R^{\leq} \cap X = \{x\}$ of (a).

(b) if $\exists x_k \neq x \in [x_j]_R^{\leq} \cap X$, we still have $x_j \in \overline{R}_A^{\leq}(X')$, then $\overline{R}_A^{\leq}(X') = \overline{R}_A^{\leq}(X) - \{x\}$

3.2. Updating Set Approximations for Given X when Inserting an Object x^+

Since inserting an object is more complex than deleting an object with the variation of dominance class and approximations, we describe the main conclusions through the discussions of different cases.

1) *Update the lower approximations* $\underline{R}_A^{\leq}(X')$

Case 1: $x^+ \notin X', i.e., X' = X$

Firstly, we calculate the dominance class $[x^+]_R^{\leq}$ of x^+ as follows: (a) if x^+ is better than x_j, i.e., $x_j \in [x^+]_R^{\geq} \Leftrightarrow x^+ \in [x_j]_R^{\leq}$, then $[x_j]'^{\leq}_R = [x_j]_R^{\leq} \cup \{x^+\}$. While $x^+ \notin X$, therefore $[x_j]'^{\leq}_R \not\subset X$.

(b) $x_j \notin [x^+]_R^{\geq} \Leftrightarrow x^+ \notin [x_j]_R^{\leq}$, then $[x_j]'^{\leq}_R = [x_j]_R^{\leq}$.

Second, we compute the updating approximations based on original lower approximation. For $\forall x_j \in U'$, if $x_j = x^+$, then $[x_j]_R^{\leq} = [x^+]_R^{\leq} \not\subset X$, there is no need to update $\underline{R}_A^{\leq}(X')$; if $[x_j]_R^{\leq} \subseteq X$ and satisfy with situation (a), then $[x_j]'^{\leq}_R = [x_j]_R^{\leq} \cup \{x^+\} \not\subset X$, we have $\underline{R}_A^{\leq}(X') = \underline{R}_A^{\leq}(X) - \{x_j\}$; if $\exists$ $[x_j]_R^{\leq} \subseteq X$ and satisfy with situation (b), then $[x_j]'^{\leq}_R \subseteq X'$, there is no need to update $\underline{R}_A^{\leq}(X')$.

Therefore we obtain
$$\underline{R}_A^{\leq}(X') = \underline{R}_A^{\leq}(X) - \{x_{j\in(a)}\}$$
$$where \{x_{j\in(a)}\} = \bigcup_{j\in(a)} \{x_j\}$$

Case 2: $x^+ \in X', i.e., X' = X \cup \{x^+\}$

(a) $x_j \in [x^+]_R^{\geq} \Leftrightarrow x^+ \in [x_j]_R^{\leq}$, $\therefore [x_j]'^{\leq}_R = [x_j]_R^{\leq} \cup \{x^+\}$

(b) $x_j \notin [x^+]_R^{\geq} \Leftrightarrow x^+ \notin [x_j]_R^{\leq}$, $\therefore [x_j]'^{\leq}_R = [x_j]_R^{\leq}$

The inclusion relation of $[x^+]_R^{\leq}$ and X' needs to be checked firstly.

If $[x^+]_R^{\leq} \subseteq X'$, then $\underline{R}_A^{\leq}(X') = \underline{R}_A^{\leq}(X) \cup \{x^+\}$;

If $[x^+]_R^{\leq} \not\subset X'$, for $\forall$ $x_j \in U'$, if $[x_j]_R^{\leq} \subseteq X$ and satisfy with situation (a) or (b), then $\underline{R}_A^{\leq}(X') = \underline{R}_A^{\leq}(X)$.

2) *Update the upper approximations* $\overline{R}_A^{\leq}(X')$

Case 1: $x^+ \notin X', i.e., X' = X$, for $\forall$ x_j, consider the following two situation:

(a) $x_j \in \overline{R}_A^{\leq}(X)$, i.e., $[x_j]_R^{\leq} \cap X \neq \varnothing$, then $[x_j]'^{\leq}_R \supseteq [x_j]_R^{\leq} \Rightarrow [x_j]'^{\leq}_R \cap X \neq \varnothing$, obviously $x_j \in \overline{R}_A^{\leq}(X')$

(b) $x_j \notin \overline{R}_A^{\leq}(X)$, i.e., $[x_j]_R^{\leq} \cap X = \varnothing$. Since $x^+ \notin X'$ and $[x_j]'^{\leq}_R = [x_j]_R^{\leq} \cup \{x^+\}$ *or* $[x_j]_R^{\leq}$ we have $[x_j]'^{\leq}_R \cap X = \varnothing$ therefore $x_j \notin \overline{R}_A^{\leq}(X')$.

Consequently if $x^+ \in \overline{R}_A^{\leq}(X')$, then $\overline{R}_A^{\leq}(X') = \overline{R}_A^{\leq}(X) \cup \{x^+\}$, otherwise remain unchanged.

Case 2: all the dominance classes after updating have $[x_j]'^{\leq}_R = [x_j]_R^{\leq} \cup \{x^+\}$ *or* $[x_j]_R^{\leq}$

(a) the same as (a) in case 1

(b) if $\{x_j\} \notin \overline{R}_A^{\leq}(X)$, namely $[x_j]_R^{\leq} \cap X = \Phi$. If $[x_j]'^{\leq}_R = [x_j]_R^{\leq} \cup \{x^+\}$, then $[x_j]'^{\leq}_R \cap X' = \{x^+\}$, i.e., $x_j \in \overline{R}_A^{\leq}(X')$

Therefore, for $\forall$ $x_j \in U'$, if $x_j = x^+$, then $[x_j]_R^{\leq} = [x^+]_R^{\leq}$. $x^+ \in X'$, $\therefore$ $[x^+]_R^{\leq} \cap X' \neq \varnothing$, then $\overline{R}_A^{\leq}(X') = \overline{R}_A^{\leq}(X) \cup \{x^+\}$; if x_j is satisfied with (a), then $\overline{R}_A^{\leq}(X') = \overline{R}_A^{\leq}(X)$; if x_j is satisfied with (b), i.e. $[x_j]'^{\leq}_R = [x_j]_R^{\leq} \cup \{x^+\}$, then $\overline{R}_A^{\leq}(X') = \overline{R}_A^{\leq}(X) \cup \{x_j\}$.

The final result of case 2 is $\overline{R}_A^{\leq}(X') = \overline{R}_A^{\leq}(X) \cup \{x^+\} \cup_{j\in(b)} \{x_j\}$.

4. Numerical Examples

Table 1. Information System after Deleting an Object

U	a_1	a_2	a_3	d
x_1	1	2	1	3
x_2	3	2	2	2
x_3	1	1	2	1
x_4	2	1	3	2
x_5	3	3	2	3
x_6	3	2	3	1

Table 2. Information System after Insertion of x_7

U	a_1	a_2	a_3	d
x_1	1	2	1	3
x_2	3	2	2	2
x_3	1	1	2	1
x_4	2	1	3	2
x_5	3	3	2	3
x_6	3	2	3	1

x_7	2	1	2	1

According to the definition, we obtain the dominance relation based on condition attributes:

$$R_A^{\leq}=\{(x_1,x_1),(x_1,x_2),(x_1,x_5),(x_1,x_6),(x_2,x_2),$$
$$(x_2,x_5),(x_2,x_6),(x_3,x_2),(x_3,x_3),(x_3,x_4),(x_3,x_5),$$
$$(x_3,x_6),(x_4,x_4),(x_4,x_6),(x_5,x_5),(x_6,x_6)\}$$

The corresponding condition dominance classes are as follows: $[x_1]_A^{\leq}=\{x_1,x_2,x_5,x_6\}$,

$$[x_2]_A^{\leq}=\{x_2,x_5,x_6\},\ [x_3]_A^{\leq}=\{x_2,x_3,x_4,x_5,x_6\},$$
$$[x_4]_A^{\leq}=\{x_4,x_6\},\ [x_5]_A^{\leq}=\{x_5\},\ [x_6]_A^{\leq}=\{x_6\}$$

Let $X=\{x_1,x_5\}$, i.e., an equivalence class based on decision attribute *d*, then $\underline{R}_A^{\leq}(X)=[x_5]_A^{\leq}=\{x_5\}$, and

$$\overline{R}_A^{\leq}(X)=\{x_1,x_2,x_3,x_5\}$$

4.1. Deleting an Object

After deleting x_6, $X'=X=\{x_1,x_5\}$, which satisfies the condition of case 1. The traditional rough set method and the proposed incremental method are used to update the upper and lower approximations as follows.

a) Re-calculation *by traditional method:* By comparing the merits of the relationship between two objects, we obtain the dominance class set after updating $E'=\{\{x_1,x_2,x_5\},\{x_2,x_5\},\{x_2,x_3,x_4,x_5\},\{x_4\},\{x_5\}\}$, the amount of calculation is (*n*-1) (*n*-2)**m*/2, here *n*=6, i.e., the computation complexity is O(mn^2).

Then consider the inclusion relation between dominance class and X', the computation complexity is O(*n*). Finally we get $\underline{R}_A^{\leq}(X')$ = $[x_5]_A^{\leq}=\{x_5\}$ and $\overline{R}_A^{\leq}(X')=\overline{R}_A^{\leq}(X)=\{x_1,x_2,x_3,x_5\}$

b) Incremental updating *method proposed in this paper:* Update the lower approximations:

Using the corresponding formula directly, we can obtain $\underline{R}_A^{\leq}(X')=\underline{R}_A^{\leq}(X)=[x_5]_A^{\leq}=\{x_5\}$. There is no need to calculate the dominance classes of each object and the inclusion relation with X' after updating, then save a lot.

Update the upper approximations: we only need to consider whether the dominance classes of deleted object include in original upper approximations or not. The result is consistent with the re-calculation as $\overline{R}_A^{\leq}(X')=\overline{R}_A^{\leq}(X)=\{x_1,x_2,x_3,x_5\}$.

4.2 Inserting an Object

After Inserting $x_7=\{2,1,2,1\}$, $X'=X=\{x_1,x_5\}$, which satisfies the condition of case 1.

a) Re-calculation by *traditional method*: The traditional methods still need to update every dominance classes. $E'=\{\{x_1,x_2,x_6\},\{x_2,x_5,x_6\},\{x_2,x_3,x_4,x_5,x_6,x_7\},\{x_4,x_6\},\{x_5\},\{x_6\},\{x_2,x_4,x_5,x_6,x_7\}\}$ then determine whether they are contained in X' or have intersections with X', finally the lower and upper approximations are computed as

$$\underline{R}_A^{\leq}(X')=\underline{R}_A^{\leq}(X)=\{x_5\},\ \overline{R}_A^{\leq}(X')=\{x_1,x_2,x_3,x_5,x_7\}$$

The computation complexity is still O(mn^2).

b) Incremental *updating method*.

Incremental updating lower approximations: update the dominance classes and inferior classes of $x_7=\{2,1,2,1\}$ firstly, we have $[x_7]_R^{\leq}=\{x_2,x_4,x_5,x_6,x_7\},[x_7]_R^{\mathbf{f}}=\{x_3\}$, the amount of calculation is *m*(*n*-1); then update the dominance class of objects in $[x_7]_R^{\mathbf{f}}$, and add x_7 based on the original, i.e., $[x_3]_R'^{\leq}=[x_3]_R^{\leq}\cup\{x_7\}$. Since other dominance classes are unchanged, the amount of calculation of this step is $|[x_7]_R^{\mathbf{f}}|$. The third step tests whether these dominance classes include in X before updating. Finally, we have $\underline{R}_A^{\leq}(X')=\underline{R}_A^{\leq}(X)=[x_5]_A^{\leq}=\{x_5\}$. The computation complexity is O(*mn*).

Incremental updating upper approximations: the same as obove to update the dominance classes and inferior classes of inserted object, and update the dominance classes of elements in inferior classes as well. The dominance classes are either increased or unchanged after inserting objects, and if they have intersections with $X'=X$ remains unchanged. The third tests if there is intersection between the dominance class $[x_7]_R^{\leq}=\{x_2,x_4,x_5,x_6,x_7\}$ of inserted object and X. In this case the intersection is nonempty, we directly have $\overline{R}_A^{\leq}(X')=\overline{R}_A^{\leq}(X)\cup\{x_7\}=\{x_1,x_2,x_3,x_5,x_7\}$, and the computation complexity is O(n2).

Conclusion

In dynamic environment, information is constantly updated, and how to effectively deal with this kind of information system is an important topic. Here the dynamic environment generally include the change of attribute sets and object sets, reflecting on the increasing or decreasing of attribute and object. In this paper, we proposed an incremental approach for updating the approximations of DRSA under the variation of the object set. The difference with existing studies is that the proposed method only requires that the condition attributes have preference order, namely it is suitable to the multi-criteria classification problems. We gave detailed theoretical results with proofs and a numerical example to support our incremental method. We can conclude that the proposed method for dynamically updating the approximations of DRSA is feasible and can effectively reduce the computational time when the set of objects changes. One of our future work is to conduct some experiments with real datasets and combine the method with variable precision rough set.

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