Self-adaptive Anisotropy Registration Model in Image Registration Based on Partial Differential Equation

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Abstract: As the optical flow motion estimation algorithm in image registration appears serious image blur, large errors, error messages and other issues, so this paper presents a PDE-based self-adaption anisotropy registration model. The model can adaptively choose the diffusion rate of tangent direction and normal direction according to the local image structure information, to protect the edge of the image in the registration process not to be blurred, at the same time, designs strong data item and improve the robustness of the algorithm. Experimental results show that: this model can make the image clear and the details are not easy to lose, and achieve effective registration.

Keywords: Image Feature; Evolution; Penalty Function; Gradient Modulus

1. Introduction

Image segmentation and image registration are the two main basic tasks in image processing; they are important parts of machine vision, pattern recognition, and other areas. Registration and image segmentation exist from image arising, many researchers have proposed a lot of treatment, but because of the complex content of the image, and user needs vary, each method is often only applicable to a particular image or demand, making these two issues are still research hotspots in the image project [1]. In addition, Yezzi proposed registration and segmentation mixture model based on those linkages between registration and segmentation [2]. Mixture method has also become a hot issue in recent image processing. Partial differential equation, stochastic modeling and wavelet constitute a theoretical basis for image processing, compared to the other two models , the theory of partial differential equations has the following outstanding advantages: a) has a strong scalability, a variety of image processing theory can be well integrated under the theoretical framework of the partial differential equations; b) can be closely connected with multi-scale analysis and provide an efficient algorithm for a variety of image processing problems; c) has a good theoretical support, applied mathematics, differential geometry and other aspects of the theory results provide mature theoretical basis for new method [3-9]. The research based on image segmentation of partial differential equation and image registration methods has lasts for a long time and it is fruitful. Image segmentation method based on active contour model and image registration method based on optical flow field and other physical models are typical. This paper based on theory of partial differential equations, talking about specific issues in image segmentation and image registration, presents an improved method and effective solutions.

As the displacement field which requires solution by registration can be solved through the velocity profile of optical flow, Palos, Hellier introduced the optical flow field model to image registration, and then in the following 20 years, various kinds of new algorithms and improved algorithm continuously appeared and achieved fruitful results. In recent years, with the continuous penetration of partial differential equations, tensor analysis and other mathematical methods in image analysis, optical flow computing technology made great progress in calculation accuracy, reliability]. Currently, there are defects in optical flow computation theory, mainly in the ill-posed problem, the algorithm stableness and optical flow calculation on object boundary. Thus, this paper proposes an anisotropic gradient vector flow according to the problems in classic optical flow field model, this model can adaptively choose the direction of the tangent and normal diffusion rate according to the local image structure information, to protect the image edges not be blurred during registration process, at the same time, designs strong data item, and improved the robustness of the algorithm.

2. Optical Flow Field Model

2.1. Horn-schunck Algorithm Diffusion

$\Delta^2 = \frac{\partial^2}{\partial l^2} + \frac{\partial^2}{\partial j^2}$ is the Laplace operator, $\Delta^2 a$, $\Delta^2 v$ are the Laplace diffusions, I x (I x u+ I y v+ I t), I y (I x u+ I y v+ I t) is a data item. The model at the location of $|\Delta a| \approx 0$, there may be unreliable local optical flow estimation, but regularization $|\Delta b|2 + |\Delta r|2$ can fill this information from the neighborhood area. This is an advantage, but meanwhile there are the following problems: gray consistency assumption for many real image sequences is not suitable. When the illumination condition changes a lot, and the image of the block edge or the target velocity is too high, gray consistency assumption would have a big error; Laplace operator is very strong smooth operator, it is difficult to protect the border, will cause seriously image blur in the evolution process and lost important information. Data entry can only play the role of constraints, can not prevent image blur. So the performance of Horn-Schunck algorithm has a close relationship with Laplace operator.

3. PDE-based Self-adaption Anisotropy Registration Model

Image registration and segmentation are two basic tasks in the project, and has been researched separately as two separate problems for a long time, Bansal, Yezzi and other researchers noted that the registration process and image segmentation process can utilize each other and promote, and proposed coupled model which combined registration and segmentation. The basic idea is: to take full advantage of the two images on the evolution curve shape similarity to achieve mutual promotion registration and segmentation, which is shown in Figure 1. On one hand, the similarity of target curve shapes on two images promote the changes of displacement field; on the other hand, action on the evolution curve in reference map through displacement field to facilitate the evolution of the curve in the figure.

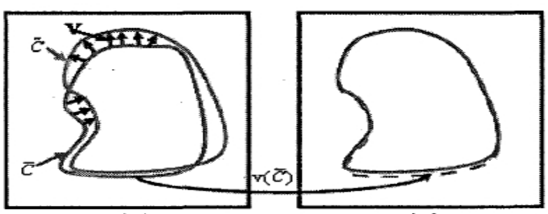


Figure 1. Registration Divides Heterosexual Registration Model

Differential optical flow field basic model is constituted by data item and regular item, the improvements of this article include: structure anisotropic regularization term in order to maintain the discontinuity of the image, protect the information on the edge of the image, and use of non-quadratic penalty function to enhance the strength of the model.

Wherein, α represents the size of the amount of diffusion along tangential direction; β represents the size of the amount of diffusion along normal direction. Obviously, the diffusion process is determined by the coefficient, so the structure of the regular item which has specific spread function, attributed to determine the problem about the size of α, β. In order to keep the edges of the image during the evolution process, the image evolution should maintain the diffusion of tangential direction, and inhibit the diffusion of normal direction; besides, during the registration process, consider protecting the corners not being diffused. Since the target sharp corner has a large gradient and curvature, and the character of target edge is large gradient, small curvature; while the noise location has large curvature, small gradient, gradient in smooth area is small, and the curvature is small, so α, β definition can take values as follows:

$$\begin{cases} \partial = 1 - \left[1 - \exp\left(-\left(\dfrac{\Delta j}{l_1}\right)^2\right)\right]\left[1 - \exp\left(\dfrac{l}{l_2}\right)^2\right] \\ b = \exp\left(-\left(\dfrac{\Delta j}{l_1}\right)\right) \end{cases} \tag{1}$$

In it, Δj is the gradient modulus, and κ is the curvature, l_1, l_2 are diffusion threshold. At present, many algorithms select diffusion threshold depend on experience, which is a limitation of the algorithm. In year 1998, Black pointed out that the design of the anisotropic diffusion problem itself may be regarded as estimate smoothing region in a noise pollution input image by analyzing the relationship between anisotropy strength statistics, and the design problem about "edge off" spread function is equivalent to strength estimation error criterion design issues, and proposed diffusion threshold can be obtained through automatically estimate by strength statistics. It means diffusion threshold is equal to the median absolute deviation (MAD) of gradient.

$$l = \frac{1}{0.6755} median\left[\left|\Delta j - median\left(|\Delta j|\right)\right|\right] \tag{2}$$

Among them, setting the constant is because that the value of normally distributed MAD is 0.6755, the mean is 0 and variance is 1. The value for l_1, l_2 in this article use the median absolute deviation function of Black, it is:

$$\begin{cases} l_1 = \dfrac{1}{0.6755} median\left[\left|\Delta j - median\left(|\Delta j|\right)\right|\right] \\ l_2 = \dfrac{1}{0.6755} median\left[\left|l - median\left(|l|\right)\right|\right] \end{cases} \tag{3}$$

If Δj is much larger than l_1, and l is much larger than l_2, then α tends to 0, otherwise α tends to 1; if Δj is much larger than l_1, then b tends to 0, otherwise b tends to 1. In this way, the diffusion rate of ∂, b along with the tangential and normal direction can be adaptively changed based on the image of its information structure (as shown in Table 1)

Table 1. Relationship Between Image Feature and the Value of b , ∂

Feature	Gradient	Curvature	∂	b
Sharp	Ralatively large	Ralatively smalle	$\partial \to 1$	$b \to 0$
Edge	Ralatively large	Ralatively large	$\partial \to 0$	$b \to 0$
Noise	Ralatively smalle	Ralatively large	$\partial \to 1$	$b \to 1$
Smooth region	Ralatively smalle	Ralatively smalle	$\partial \to 1$	$b \to 1$

As analyzed above, we construct a self-adaptive anisotropic regularization term with discontinuous maintenance

$$\begin{cases} T(\Delta j, \Delta u) = \partial u_{jj} + b v_{ll} + b v_{zz} \\ b = \exp\left(-\dfrac{\Delta j}{l_1}\right)^2 \end{cases} \tag{4}$$

4. Experimental Simulation and Analysis

4.1. Registration Evaluation

The following experiment is for single-mode and multi-modal brain MR images, all experimental parameters take " $b = 0.03$, $h = 3.5$, " $r = 0.3$, the time step $w_1 = 0.3$, the basis of iteration termination judgment takes the maximum amount of displacement which is no more than 0.01. In each figure, contour curve is red. In addition, we compared the present model and representative Wang model. Quantitative analysis of segmentation result commonly uses Js. and De to measure; registration results use the average gray difference, MSE and mutual information for evaluation.

In this paper, we introduce edge-aligned degrees based on these three common evaluation parameters, in order to better detect the registration results about the edges of image.

Mean Intensity Subtraction

$$Mean = \frac{\sum_{j=1}^{m}\sum_{j-1}^{m}\left(w_1(j,l)\right) - w_2(j,l)}{m \times n} \tag{5}$$

In it, $w_1(j,l)$ represents the image after registration; $w_2(j,l)$ represents the reference image; $m \times n$ is the pixel number. If the mean intensity subtraction is smaller, the registration would be better.

Mean Square Error

$$Var = \sqrt{\frac{\sum_{j=1}^{m}\sum_{j-1}^{m}\left(w_1(j,l)\right) - w_2(j,l) - Mean)^2}{m \times n}} \tag{6}$$

In it, $w_1(j,l)$ represents the image after registration; $w_2(j,l)$ represents the reference image; $m \times n$ is the number of pixel; Mean is the mean intensity subtraction.

When mean square error is smaller, which means the registration is better.

Peak Signal to Noise Ratio

$$PSNR = 103g\frac{256^2}{Var} \tag{7}$$

In it, Var is the mean square error, if the Peak signal to noise ratio is larger, the better.

Alignment Metric of Edge

The concept of alignment metric is from the understanding of contents of two aligned images from human eyes; from microscopic point of view, this means that each gray level of an image at pixel location corresponding to another image gray level is stable most. The process of solving alignment metric is

$$CW(w_2, w_2) = \frac{\bar{s}_{1,2}^{\,2}}{s_1^2} + \frac{\bar{s}_{1,2}^{\,2}}{s_2^2} \tag{8}$$

In formula (23) to (26), $w_1(j,l)$ represents the image after registration, $w_2(j,l)$ represents the reference image; $E(1,2)(n)$ and $E(2,1)(n)$ are gray value in two images that one image whose pixel gray level is n corresponding to another image; $h_1(n), h_2(n)$ represents that the number of pixel gray value is n; $p_1(n), p_2(n)$ represent the pixel rate of which the pixel value is n in two images. CW in the range $[0,2]$, use Kuang Yabin's infrared and visible image registration algorithm directly request reciprocal for CI, then get the alignment metric (AM), that is

$$AM = \frac{1}{CI} \tag{9}$$

However, the range of alignment metric is too wide, so alignment metric in this article is

$$AM' = e^{-CI} \tag{10}$$

Because when registration between two images, the value of alignment metric AM ' is the greatest.

4.2. Optical Flow Experimental of Rubik Chart

Rubik Chart is one of the most representative test charts of the optical flow field; first, choose this chart to validate the registration result of the model in this paper. Use Rubik Chart placed at an angle 2 (a) to be chart waiting registration. Rubik Chart rotates in certain angle 2 (b) as a benchmark figure, three models were used 100 times iterations. The results can be seen from Hom model 2 (c), Hom model results in blurred images quickly , as well as the edges of the image is difficult to distinguish; Registration result in Figure 2 (d) of Weickert model shows that the diffusion model defined by Weickert has enhanced ability to maintain the edge of the model, it is better than Hom method, but on the top of Rubik whose working speed is fast (fierce change of the performance of the image), image characteristics can not be effectively maintained and edges severely blurred; registration results of this model (Figure 2 (e)) shows that this model can maintain image features and applies to all forms of image motion, and has better adaptability on fast motion, and the registration accuracy is significantly better than the other two models. In table 4.1 and Table 2, the mean and variance of registration results in our model is minimum, which further explains the registration result of our model is the closest to reference image.

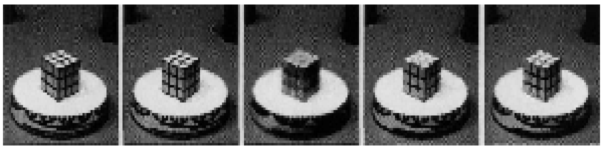


Figure 2. The Diffusion Model Defined by Weickert has Enhanced Ability to Maintain the Edge

4.3. Brain MR Image Registration

Brain MR Chart is the most representative figure of optical flow field experiment, this experiment choose two brain MR images to be an image waiting registration and the reference image, in the case of constant illumination, choose Horn model and Lucas-Kanade model for comparative experiment. Horn model and model of this paper do iteration for 30 times respectively, window size of Lucas-Kanade model is 6, 6 layers iteration deformation between two images, and optical flow vectors shown in Figure 1, and then get the registration image which is shown in Figure 3. Figure 6.1 (g) shows the enlarged partial ventricle area in each image, you can see the registration and segmentation results more clearly of each model. The parameter value in table 6.1 further illustrates the effectiveness of our model to deal with complex images.

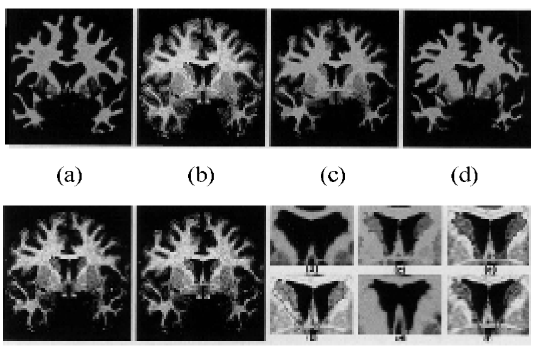


Figure 3. Registration of Brain MR Image

From the experimental results Figure 2 (c), we can see Horn model lead to image blurred quickly, and image edges and corners severely blurred and difficult to distinguish. Figure 2 (d) shows the registration results of Lucas-Kanade model, the upper edge of the image appears error message, verify that the model select window registration make image edge information loss, but the registration for the target image is superior than Horn model. The registration results of model presented in this paper (Figure 2 (e)) shows that this model is able to maintain image features, good attention to detail, and the closest to the reference image.

4.3.1. Different Pictures of Multi-modal Human Brain

The experiment realizes the registration of brain MR images of T1, T2 two modals and the segmentation of upper ventricles organization. Treat the clearer organization T1 picture as basic map (Figure 4(a)), and treat the fuzzy picture T2 to be segmented, the initial reference curve uses a known figure ventricle contour curve, as shown in 4 (b). Our model and the Wang model can overcome brightness inconsistencies, which can achieve the valid registration from T1 to TZ.

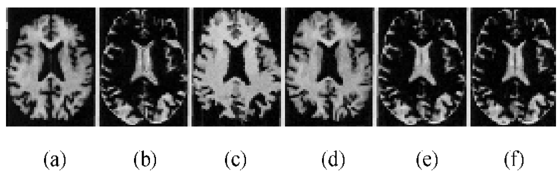


Figure 4. Experiment Result of Multi-modal Brain Figure MR

The data in Table 2 adequately describes the registration result of our model is significantly better than the Horn model and the Lucas-Kanade model, and is closest to the reference image, the peak signal to noise ratio is also the largest, indicating that this model has better strength. In addition, because Horn model is fuzzy seriously, especially the edge of error is too large, resulting alignment metric is very small; while Lucas-Kanade model appears false information on edge position which cause the alignment metric is small as well; our model has the characteristic of self-adaptive anisotropic, so it protect edges on the registration process avoid being lost.
results evaluation of brain MR image registration

Table 2. The registration result of our model

model	*PSNR*	*Mean*	*Var*
Horn model	34.02	-0.12	31.24
Lucas-Kanade model	32.34	1.62	38.65
Proposed model	38.87	-009	9.23

Table 3 shows for brain MR images, this algorithm with other algorithms comparison of computing time. As can be seen by comparing, the computation time of this proposed algorithm is longer than the Horn model, than Lucas-Kanade model runs a short time. Mainly due to: 1) This article is a Horn model improvement in the data items and regular items on the design than the Horn model complexity; 2) Lu-cas-Kanade model is a hierarchical sub-window iteration, the window is smaller, the more stratified and more complex the model which is the Lucas-Kanade one of the drawbacks. As can be seen from Table 3, this model greatly improved the accuracy, computing time and did not slow down too much, indicating that the algorithm is effective.

computing time comparing among our algorithm with other algorithms

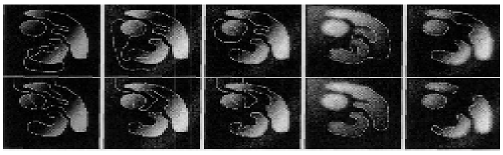
Table 3. Brain MR images

model	Image Size	Iterations	Computation time/s
Horn model	260×220	32	1.40031
Lucas-Kanade model	260×220	Size of window is 5.6 layer iteration	3.80625
Proposed model	260×220	32	1.56051

Since the differential optical flow field model constructs regularization term starting from a smoothness constraint, so the evolution of image will inevitably result in blurring, the registration process is a process of iterations, therefore, the image with increasing number of iterations becomes smooth is an inevitable trend, results also reflect this phenomenon, Hom method is simple, fast, but can cause severe image blur; use edge enhanced model of WIckert on image registration, although able to keep an edge better than Hom model, but because the model is proposed for static noise images, so that the model can not achieve accurate registration on the part which change violently. The model in this paper uses anisotropic diffusion as a flow-driven regularization term, according to the local image structures to control the evolution of the image, the character of the image can be maintained well; data item uses non-quadratic penalty function which effects on the brightness constant assumption, and make the model suitable for brightness changes caused by various motion, and it is more robust.

4.4. Registration of Human Face Images

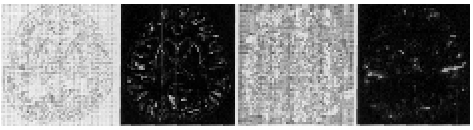
To validate our model is suitable for image with illumination changes, this experiment choose two face images for registration. Observe from Figure 5 , in Figure 6 illumination changed significantly, all three models were used 20 iterations, the obtained vector of optical flow field is shown in Figure 6. Experimental results show that both the whole or partial view, Horn model seriously obscure the image, so that we can not recognize (see Figure 5 (c)), once again validate the limitation of the Horn model; the registration result of Lucas-Kanade model obviously loss the moving edges; our model has better retention ability to image detail, and the illumination image can be changed to achieve more satisfactory results, the data in Table 4 further illustrate the registration result of this is the closest to the reference chart.



(a) (b) (c) (d) (e)

Figure 5. Registration of Face Image

Figure 5 Result: The division result comparison of four methods on artificial image, line 1: the original image and the initialization curve; Line 2: segmentation of Contrast constraint LBF model; line 3: division result of the original LBF model: Line 4: segmentation result of PS model



(a) (b) (c) (d)

Figure 6. Optical Flow Vector Field of Face Image

evaluation for registration results of face image

Table 4. Further illustrate the registration result

model	*PSNR*	*Mean*	*Var*
Horn model	34.03	-4.17	35.25
Lucas-Kanade model	32.36	10.62	38.68
Proposed model	38.89	1.24	9.25

Table 5 shows the comparison of the running time from our algorithm and other algorithms in the experiment of face image. The above results show this improvement is valid.

computation time compared with our algorithm and other algorithms

Table 5. The comparison of the running time

model	Image Size	Iterations	Computation time/s
Horn model	200*200	30	1.50031
Lucas-Kanade model	200*200	Size of window is 5.6 layer iteration	7.80624
Proposed model	200*200	30	2.86041

5. Conclusion

Because PDE-based image registration model does not need to extract image feature points, and for the characteristic of automatic registration process, it is getting more and more attention. But the existing optical flow motion estimation method causes serious blurring and big error when the image displacement is large, even error message appears in the registration process. This paper presents a PDE-based self-adaptive anisotropic registration model, which makes the image self-adaptively protect tangential diffusion according to the local image structure in the registration process, and inhibit the diffusion of normal direction, in order to better protect the edge and sharp corners of the image.

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