# Analysis of Chinese Stock Market based on Stochastic Models

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**Abstract:** We divide the Chinese stock market into two periods, Jan 2001-Dec 2005 and Jan 2006-Aug 2013 according to the real market features. Two most important Stock Indexes of China (Shanghai Composite Index and Shenzhen Composite Index) are selected to reflect the stock market. Generalized tempered stable (GTS)Levy processes are applied on the indexes during each period separately. C-GMM is used to estimate the parameters in GTS model. From the parameters in two periods, we can analyze the feature and difference of the two different market periods. We find that the stock market during Jan 2001-Dec 2005 has less jumps than that during Jan 2006-Aug 2013. During Jan 2001-Dec 2005, upward jumps are more than downward jumps, while the case is opposite during Jan 2006-Aug 2013. The big jumps in 2001-2005 are less often than those in 2006-2013.

**Keywords:** Chinese Stock Market; GTS Lèvy Model; C-GMM Estimation; Characteristic Function

## 1. Introduction

Exponential Lèvy processes have long been used in mathematical finance. Osborne [11] proposed the exponential $e^{B_t}$ of Brownian motion as a stock price model. The process $e^{B_t}$ called exponential or geometric Brownian motion was also used by Samuelson [12] in a more systematic manner.

In recent years, Madan etc. [9,10], Barndorff-Nielsen [1,2,3] have proposed several non-stable Lèvy processes such as VG(Variance Gamma) and NIG(Normal Inverse Gauss) process respectively to model financial market. VG and NIG processes are both subordinations(time changed Brownian motion) of Brownian motion in which the subordinators are gamma process and inverse Gaussian process respectively. A wider class that contains VG and NIG is CGMY process, which is described in detail by Koponen [8] and Carr, Geman, Madan, and Yor [6]. Levy Flight is also applied by some researchers.

Definition 1.1 ([7]). A generalized tempered stable process(GTS process) is a Lèvy process with no Gaussian component and a Lèvy density of the form

$$n(x)=\frac{c_-}{|x|^{1+a_-}}e^{-l_-|x|}1_{x<0}+\frac{c_+}{|x|^{1+a_+}}e^{-l_+x}1_{x>0}, \qquad (1)$$

with $a_+<2$ and $a_-<2$.

Let $X_t$ be a GTS Lèvy process with drift $gt$, the CF (characteristic function) of $X_1$ (denoted by $\Psi(s)$) is given in Cont (2004). The CF is

$$\Psi(s)=exp\{isg+\Gamma(-a_+)l_+^{a_+}c_+\left[(1-\frac{is}{l_+})^{a_+}-1+\frac{isa_+}{l_+}\right]+\Gamma(-a_-)l_-^{a_-}c_-\left[(1+\frac{is}{l_-})^{a_-}-1+\frac{isa_-}{l_-}\right]\}.$$

The parameters $l_\pm$ determine the tail behavior of the Lèvy measure, they tell us how far the process may jump, and from the viewpoint of a risk manager this corresponds to the amount of money that we can lose (or gain) during a short period of time. The parameters $c_\pm$ determine the overall and relative frequency of upward and downward jumps; of course, the total frequency of jumps is infinite, but if we are interested only in jumps larger than a given value, then these two parameters tell us how often we should expect such events.

In section 2, we review the China stock market during 1990-2011, the C-GMM method for estimating parameters, and then calculate the necessary quantity in our problem in order to apply the C-GMM method. In section 3, we use matlab to calculate the parameters in the model and then we draw some conclusions based on the parameter tables.

## 2. China Stock Market and Estimating Methods of Parameters

### 2.1. Review of China Market

In mainland China, the stock market which appeared before 1949 reappeared in the 1980's and has experienced a lot of growth ever since.

China capital market has been stagnant from 2001 to 2005. In this period, Shanghai and Shenzhen Composite



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Indexes range from about 1500 to about 2500 and from about 3000 to about 6000 respectively. But in 2006 and 2007, China stock market experienced surprisingly rapid and large rise, Shanghai and Shenzhen Composite Indexes reaching more than 6000 and more than 19600 respectively.

From 2008, China stock market has been undergoing dramatic rise and fall of big size frequently until now. So we separately study the relatively stagnant period 2001 to 2005 and the fluctuating period 2006 to 2013. For each period, we use GTS process to model the log-index movement and apply the method in next section to estimate parameters, and compare the parameters in two periods to see the market movement difference between two periods.

### 2.2. CGMM Scheme

Let $\{X_t\}$ be a $p$-dimension stochastic process indexed by parameter $q \in \Theta \subset R^q$. Singleton(2001) propose an estimating method under CCF(conditional characteristic function). The CCF of $X_{t+1}$ given $\{X_t\}$ is

$$\Psi_q(s \mid X_t) = E^q(e^{isX_{t+1}} \mid X_t) \qquad (2)$$

From the property of conditional expect, we have

$$E^q[(e^{isX_{t+1}} - \Psi_q(s \mid X_t))A(X_t)] = 0 \qquad \forall s \in R^p, \qquad (3)$$

where $A(X_t)$ is an arbitrary instrument. The equation (3) is called moment condition. The expression $(e^{isX_{t+1}} - \Psi_q(s \mid X_t))A(X_t)$ is called moment function. Besides being a function of $X_t$, $A$ may be a function of an index $r$ equal to or different from $s$. Two types of moment functions are used often: Single Index (SI) and Double Index (DI) [14].

Carrasco and Florens [4] use DI instrument:

$A(r, X_t) = e^{irX_t}$.

Denote $(r,s)' \in R^{2p}$ by $t$, then the corresponding moment function is

$$h_t(t;q) = (e^{isX_{t+1}} - \Psi_q(s \mid X_t))e^{irX_t}. \qquad (4)$$

In the next subsection, we will calculate the moment function in our problem The C-GMM estimator is based on the arbitrary set of moment conditions:

$$E^{q_0} h_t(t;q_0) = 0, \quad \forall t. \qquad (5)$$

Let $\hat{h}_T(t;q_0) = \sum_{t=1}^{T} h_t(t;q_0)/T$. It is an estimator of $E^q h_t(t;q_0)$, the problem is reduced to find the estimator $\hat{q}$ which best satisfies the equation $\hat{h}_T(t;\hat{q}) = 0$ for all $t$.

Adapting operator theory and probability theory, they obtain the following essential and nice results under some regularity conditions in their paper.

Theorem 2.1 A good estimate is the solution of the following problem

$$\min_q \underline{w}(q)[a_T I_T + C^2]^{-1}\underline{v}(q), \qquad (6)$$

where $C$ is a $T \times T$-matrix with $(t,l)$ element $c_{tl}/(T-q)$, $t,l = 1,\ldots,T$, $I_T$ is the $T \times T$ identity matrix, $\underline{v} = [v_1, \mathbf{L}, v_T]'$ and $\underline{w} = [w_1, \mathbf{L}, w_T]'$ with

$v_t(q) = \int \overline{h}_t(t;\hat{q}_T^1)\hat{h}_T(t;q)p(t)dt$,

$w_t(q) = \left\langle h_t(t;\hat{q}_T^1), \hat{h}_T(t;q) \right\rangle$,

$c_{tl} = \int \overline{h}_t(t;\hat{q}_T^1)h_l(t;\hat{q}_T^1)p(t)dt$.

Let $\{S_t\}_{t \ge 0}$ denote the stock index process. The log-index $X_t = \ln(S_t)$ is assumed to be a Lèvy process as usual. We write $\Psi_q(s)$ for unconditional characteristic function $\Psi(s)$ in section 1, where $q$ denotes the parameter vector $(a_+, a_-, l_+, l_-, c_+, c_-, g)'$. Due to the independent increments of $X_t$ and the properties of conditional expect, we have

Proposition 2.2 The moment function (4) for $X_t$ is

$$h_t(t;q) = (e^{isX_{t+1}} - e^{isX_t}\Psi_q(s))e^{irX_t}. \qquad (7)$$

with $h_t(t;q)$ in the above proposition, the formulas in Theorem 2.1, programming in Matlab, we can obtain parameters in GTS model for China market in the next section.

## 3. Empirical Study and Analysis

From the analysis in section 1, we divide China stock market into two periods: 1st Jan 2001-31st Dec 2005 and 1st Jan 2006-31st Aug 2013. The data we used are Shanghai Composite Index and Shenzhen Composite Index(from Yahoo Finance).

We use C-GMM based on CF(characteristic function), coding Matlab 7.6 programme to obtain the parameter estimates. The results are in Table 1 and Table 2, where the numbers in parentheses are the corresponding standard errors.

**Table 1. Parameter estimates for Shanghai Composite Index**

| Period | $a_+$ | $a_-$ | $l_+$ | $l_-$ | $c_+$ | $c_-$ | $g$ |
|---|---|---|---|---|---|---|---|
| 2006.1-2013.8 | 0.2341 (0.0170) | 0.3546 (0.0215) | 42.1073 (5.7843) | 32.2510 (2.2007) | 0.1205 (0.0218) | 0.2519 (0.0310) | 0.0012 (0.0018) |



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|---|---|---|---|---|---|---|---|
| 2001.1-2005.12 | 0.6284 (0.1003) | 0.4356 (0.0528) | 62.5170 (8.0120) | 45.7208 (3.1043) | 2.0824 (0.2100) | 1.1983 (0.0681) | 0.0063 (0.0250) |

**Table 2. Parameter estimates for Shenzhen Composite Index**

| Period | $a_+$ | $a_-$ | $l_+$ | $l_-$ | $c_+$ | $c_-$ | $g$ |
|---|---|---|---|---|---|---|---|
| 2006.1-2013.8 | 0.2671 (0.0315) | 0.4012 (0.0807) | 65.2750 (8.2009) | 57.2108 (4.7901) | 0.7489 (0.2062 ) | 2.1005 (1.2108) | 0.0204 (0.033) |
| 2001.1-2005.12 | 0.8250 (0.1120) | 0.4916 (0.0413) | 81.0970 (7.9020) | 67.2290 (6.3215) | 4.3154 (0.8870) | 2.7408 (0.5201) | 1.8942 (1.1895) |

From the results in the Tables, we see the following facts: (1) Using the estimating parameters to calculating the cumulants [7] such as the mean value, variance, kurtosis, skewness, we find they are almost consistent with the empirical results. This indicates the GTS Lèvy process can capture these important characteristics very well.

(2) The parameters $\frac{c_1}{c_1+c_2}$ and $\frac{c_1}{c_1+c_2}$ reflect the relative frequencies of positive and negative jumps respectively. For Shanghai Composite Index, we see the upward jumps frequency is higher than downward jumps frequency in 2001-2005, while the case is the opposite way in 2006-2013. The results of Shenzhen Composite Index are similar.

(3) For the two Indexes, $l_+$ is greater than $l_-$, which shows the decay rate of right tail is faster than that of left tail. That leads to the left-skewness of the distribution. In addition, $l_+$ ($l_-$) in 2001-2005 is bigger than $l_+$ ($l_-$) in 2006-2013, which means the big jumps in 2001-2005 are less often than those in 2006-2013. This corresponds to the market's situation.

(4) The parameters $a_+$ and $a_-$ capture the features of small jumps. For the two indexes, $a_+ < a_-$ in 2006-2013, while $a_+ < a_-$ in 2001-2005. This shows small upward jumps are less than small downward jumps in 2006-2013, while the period 2001-2005 is the opposite way. From the trend figure of stock market, we can intuitively see this fact. In addition, $a_+$ ($a_-$) in 2001-2005 is larger than $a_+$ ($a_-$) in 2006-2013, showing small jumps in 2001-2005 are more frequent than those in 2006-2013, which is opposite to big jumps' feature in (3). Discussion Our study of the most important stock indexes of China, Shanghai Composite Index and Shenzhen Composite Index, shows jump is the intrinsic feature of financial asset movement, positive and negative jumps happening often with different frequency. The parameters $a_\pm$ for the two indexes during two periods are all in the interval (0, 1) which means the GTS Lèvy process for stock markets has infinitely jumps (also called infinitely activity by researchers), but has finite variation. From $l_\pm$, we see the indexes movement has scarce big jumps, yet with a lot of small jumps between big jumps.

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## References

[1] O.E., Barndoff-Nielson. Normal inverse Gaussian distribution and the modelling of stock returns. Technical report, Research Report NO. 300, Department of Theoretical Statistics, Aarhus University. 1995.

[2] O.E., Barndoff-Nielson. Normal inverse Gaussian distribution and stochastic volatility modelling. Scandinavian Journal of Statistics, vol 24, pp.1-13, 1997.

[3] O.E., Barndoff-Nielson. Processes of normal inverse Gaussian type. Finance and Stochastics , vol 2, pp. 41-68, 1998.

[4] M., Carrasco, M. Chernov, J. Florens, Efficient estimation of general dynamic models with a continuum of moment conditions. Journal of Econometrics , vol.140, pp.529–573, 2007 .

[5] M., Carrasco, J. Florens,. Generalization of GMM to a continuum of moment conditions. Econometric Theory, vol 16, pp.797–834, 2000.

[6] P. Carr, H. Geman, D. Madan and M. Yor, The fine structure of asset returns: an empirical investigation. Journal of Business, vol. 75, 2002.

[7] R. Cont, and P. Tankov, Financial modelling with jump processes. Chapman and Hall/CRC., 2004.

[8] I. Koponent, Analytic approach to the problem of convergence of truncated Lèvy flights towards the Gaussian stochastic process. Physical Review E, vol 52, pp. 1197-1199, 1995.

[9] D.B. Madan, and E. Seneta, Chebyshev polynomial approximations and characteristic function estimation. Journal of the Royal Statistical Society Series B, vol. 49, no. 2, pp.163-169, 1987

[10] D.B. Madan, and E. Seneta, The V.G. model for share market returns. Journal of Business, vol. 63, pp. 511-524, 1990.

[11] M.F. Osborne, Brownian motion in the stock market. Operations Research, vol.7, pp.145-173, 1959.

[12] P. Samuelson, Rational theory of warrant pricing. Industrial: Management Review, vol. 6, pp.13-32, 1965.

[13] K. Singleton, Estimation of affine pricing models using the empirical characteristic function. Journal of Econometrics, vol. 102, pp. 111–141, 2001.



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