International Journal of Intelligent Information and Management Science

Volume 4, Issue 3, June 2015

http://www.hknccp.org

Publisher: HongKong New Century Cultural Publishing House

Address: Unit E79, 3/F., Wing Tat Commercial Building, 97 Bonham Strand East, Sheung Wan, HK

International Journal of Intelligent Information and Management Science
ISSN: 2307-0692 Volume 4, Issue 3, June 2015

Contents

H K . N C C P International Journal of Intelligent Information and Management Science
ISSN: 2307-0692 Volume 4, Issue 3, June 2015

Wiener Related Index of Gear Fan and Gear Wheel Related Graph

Yun Gao[1], Tianwei Xu[2], Li Liang[2], Wei Gao[2]
[1]Department of Editorial, Yunnan Normal University, Kunming 650092, CHINA
[2]School of Information Science and Technology, Yunnan Normal University, Kunming 650500, CHINA

Abstract: The modified Wiener index and l -modified hyper-Wiener index as extensions of Wiener index, are an important topological index in Chemistry. It is used for the structure of molecule. There is a very close relation between the physical, chemical characteristics of many compounds and the topological structure of that. The modified Wiener index and l -modified hyper-Wiener index are such topological indices and it has been widely used in Chemistry fields. In this paper, based on previous studies, we determine the modified Wiener index and l -modified hyper-Wiener index of gear fan graph, gear wheel graph and their r-corona graphs. The theoretical conclusions obtained in this paper illustrate the promising prospects of the application for the pharmacy and chemical engineering.

Keywords: Chemical Graph Theory; Organic Molecules; Modified Wiener Index; l -modified Hyper-Wiener Index; r-corona Graph

1. Introduction

Chemical compounds and drugs are often modeled as graphs where each vertex represents an atom of molecule, and covalent bounds between atoms are represented by edges between the corresponding vertices. This graph derived from a chemical compounds is often called its molecular graph, and can be different structures. An indicator defined over this molecular graph, the Wiener index, has been shown to be strongly correlated to various chemical properties of the compounds. The Wiener index of a graph is defined as the sum of distances between all pairs of vertices of the graph. It has been found extensive applications in chemistry. Several years later, mathematician began to pay attention to the Wiener index and study it from the mathematical point of view. In such background, since each structural feature of organic molecule can be expressed as a graph, chemical graph theory as a branch of combinatorial chemistry is introduced to research the structure of molecule from graph theory standpoint. Recently, several articles contributed to reporting certain distance-based indices of special molecular graph (See Yan et al., [1-2], Gao et al., [3-4], Gao and Shi [5], Gao and Wang [6], Xi and Gao [7-8], Xi et al., [9], Gao et al., [10] for more detail). The notation and terminology used but undefined in this paper can be found in [11].

The graphs considered in this paper are simple and connected. The vertex and edge sets of G are denoted by V(G) and E(G), respectively. The Wiener index is defined as the sum of distances between all unordered pair of vertices of a graph G, i.e.,

$$W(G)=\sum_{\{u,v\}\subseteq V(G)} d(u,v),$$

where $d(u,v)$ is the distance between u and v in G.

Several papers contributed to determine the Wiener index of special graphs. Chen [12]gained the exact expression for general pepoid graph. Xing and Cai [13] characterized the tree with third-minimum wiener index and introduce the method of obtaining the order of the Wiener indices among all the trees with given order and diameter, respectively. A tricyclic graph is a connected graph with n vertices and n+2 edges. Wan and Ren [14] studied the Wiener index of tricyclic graph t_n^3 which have at most a common vertex between any two circuits, and the smallest, the second-smallest Wiener indices of the tricyclic graphs t_n^3 are given.

The Modified Wiener index of graph G is defined by

$$W_l(G)=\sum_{\{u,v\}\subseteq V(G)} d(u,v)^l$$

The l -modified hyper-Wiener index is denoted as

$$WW_l(G)=\frac{1}{2}\Big(\sum_{\{u,v\}\subseteq V(G)} d(u,v)^{2l}+\sum_{\{u,v\}\subseteq V(G)} d(u,v)^l\Big)$$

Pan [15] deduced the formula of Wiener number and Hyper-Wiener number of two types of polyomino systems. More results on Wiener related index can refer to [16-23].

The graph Fn={v}Pn is called a fan graph and the graph Wn={v}Cn is called a wheel graph, where Pn is a path with n vertices and Cn is a cycle with n vertices. Graph Ir(G) is called r- crown graph of G which splicing r hang edges for every vertex in G. The vertex set of hang edges that splicing of vertex v is called r-hang vertices, note v*.By adding one vertex in every two adjacent vertices of the fan path Pn of fan graph Fn, the resulting graph is a subdivision graph called gear fan graph, denote as $\bar{F}_n$. By adding one vertex in every two adjacent vertices of the wheel cycle Cn of wheel graph Wn, Theresulting graph is a subdivision graph, called gear wheel graph, denoted as $\bar{W}_n$.

In the past ten years, the calculation of distance-based and degree-based indices for special structure of chemical molecular had raised many attention among researchers. Although there have been several advances in distance based indices of molecular graphs, the study of Wiener related indices for special chemical structures are still largely limited. In addition, as widespread and critical chemical structures, fan graph, wheel graph, gear fan graph, gear wheel graph and their r-corona graphs are widely used in medical science and pharmaceutical field. For these reasons, many kinds of indices for special structures of fan graph, wheel graph, gear fan graph, gear wheel graphand their r-corona graphs have been studied in recent years.

In this paper, we present the modified Wiener index and l -modified hyper-Wiener index of gear fan and gear wheel related graph.

2. Modified Wiener Index

Theorem 1.

$$W_l(I_r(F_n)) =$$
$$(nr^2+2n+r-1)+(4nr+\frac{n^2}{2}-\frac{3n}{2}+2r-1)2^l+$$
$$((2n-1)r^2+n^2-3n+2)3^l+\frac{(n-1)(n-2)}{2}4^l.$$

Proof.Let Pn=v1v2…vnand the rhanging vertices of vibe v_i^1, v_i^2,…, v_i^r (1≤i≤n). Let vbe a vertex in Fnbeside Pn, and the rhanging vertices of vbe v^1, v^2, …, v^r.

By the definition of modified Wiener index, we have

$$W_l(I_r(F_n)) = \sum_{i=1}^{r} d(v,v^i)^l + \sum_{i=1}^{n} d(v,v_i)^l + \sum_{i=1}^{n}\sum_{j=1}^{r} d(v,v_i^j)^l$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{r} d(v_i,v^j)^l$$
$$+ \sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{r} d(v_i^j,v^k)^l + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} d(v_i,v_j)^l +$$
$$\sum_{i=1}^{n}\sum_{j=1}^{r} d(v_i,v_i^j)^l + \sum_{i=1}^{n}\sum_{j\in\{1,2,\ldots,n\}-i}\sum_{k=1}^{r} d(v_i,v_j^k)^l$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{r-1}\sum_{k=j+1}^{r} d(v_i^j,v_i^k)^l + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{k=1}^{r}\sum_{t=1}^{r} d(v_i^k,v_j^t)^l$$

=r+n+ $nr2^l + nr2^l + nr^2 3^l$ +

$$(n-1)+\frac{(n-1)(n-2)}{2}2^l$$ +nr+

$$r((2n-2)2^l+(n-1)(n-2)3^l)$$

+nr(r-1)+ $r^2((n-1)3^l+\frac{(n-1)(n-2)}{2}4^l)$

$$= \begin{matrix}(nr^2+2n+r-1)+(4nr+\frac{n^2}{2}-\frac{3n}{2}+2r-1)2^l+\\((2n-1)r^2+n^2-3n+2)3^l+\frac{(n-1)(n-2)}{2}4^l\end{matrix}.$$

In this way, we get the decision.

Corollary1. $W_l(F_n)$ =

$$(2n-1)+(\frac{n^2}{2}-\frac{3n}{2}-1)2^l+(n^2-3n+2)3^l+\frac{(n-1)(n-2)}{2}4^l.$$

Theorem 2.

$$W_l(I_r(W_n)) =$$
$$(nr^2+r+n^2-n)+4nr2^l+(2nr^2+n^2r-3nr)3^l$$
$$+\frac{n(n-3)r^2}{2}4^l.$$

Proof.Let Cn=v1v2…vn and v_i^1, v_i^2 ,…, v_i^r be the r hanging vertices of vi (1 i n). Let v be a vertex in Wnbeside Cn, and v^1, v^2, …, v^r be the r hanging vertices of v.

By the definition of modified Wiener index, we have

$$W_l(I_r(W_n)) = \sum_{i=1}^{r} d(v,v^i)^l + \sum_{i=1}^{n} d(v,v_i)^l + \sum_{i=1}^{n}\sum_{j=1}^{r} d(v,v_i^j)^l$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{r} d(v_i,v^j)^l$$
$$+ \sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{r} d(v_i^j,v^k)^l + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} d(v_i,v_j)^l +$$
$$\sum_{i=1}^{n}\sum_{j=1}^{r} d(v_i,v_i^j)^l + \sum_{i=1}^{n}\sum_{j\in\{1,2,\ldots,n\}-i}\sum_{k=1}^{r} d(v_i,v_j^k)^l$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{r-1}\sum_{k=j+1}^{r} d(v_i^j,v_i^k)^l + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{k=1}^{r}\sum_{t=1}^{r} d(v_i^k,v_j^t)^l$$

=r+n+ $nr2^l$ + $nr2^l$ + $nr^2 3^l$ +n(n-2)+nr+

$r(2n2^l+n(n-3)3^l)$ +nr(r-1)+ $r^2(n3^l+\frac{n(n-3)}{2}4^l)$

$$= \begin{matrix}(nr^2+r+n^2-n)+4nr2^l+(2nr^2+n^2r-3nr)3^l\\+\frac{n(n-3)r^2}{2}4^l\end{matrix}.$$

Hence, we derivethe desire conclusion.

Corollary2. $W_l(W_n)=n^2-n$.

Theorem3.

$$W_l(I_r(\bar{F}_n)) = (n+nr^2+5nr-4r) +$$
$$(\frac{(r^2-r)(n-1)}{2}-5+3n+n^2+2nr)2^l +$$

$(3nr^2+3r^2n-3nr-4r^2)3^l$ +

$((\frac{n^2}{2}+\frac{3}{2}n-3)r^2+(\frac{3}{2}n^2-\frac{11}{2}n+5))4^l$ +

$((n^2-3n+2)r^2+(n^2-5n+6)r)5^l$.

Proof. Let Pn=v1v2…vn and $v_{i,i+1}$ be the addingvertex between vi and vi+1. Let v_i^1, v_i^2,…, v_i^r be the r hanging vertices of vi (1 i n). Let $v_{i,i+1}^1$, $v_{i,i+1}^2$,…, $v_{i,i+1}^r$ be the r hanging vertices of $v_{i,i+1}$ (1 i n-1). Let v be a vertex in Fn beside Pn, and the r hanging vertice-sof vbe v^1, v^2, …, v^r.

By virtue ofthe definition of modified Wiener index, we get

$$W_l(I_r(F_n)) = \sum_{i=1}^{r} d(v,v^i)^l + \sum_{i=1}^{n} d(v,v_i)^l + \sum_{i=1}^{n}\sum_{j=1}^{r} d(v,v_i^j)^l + \sum_{i=1}^{n}\sum_{j=1}^{r} d(v_i,v^j)^l$$
$$+ \sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{r} d(v_i^j,v^k)^l + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} d(v_i,v_j)^l + \sum_{i=1}^{n}\sum_{j=1}^{r} d(v_i,v_i^j)^l + \sum_{i=1}^{n}\sum_{j\in\{1,2,L,n\}-i}\sum_{k=1}^{r} d(v_i,v_j^k)^l$$
$$+ \sum_{i=1}^{n}\sum_{j=1}^{r-1}\sum_{k=j+1}^{r} d(v_i^j,v_i^k)^l + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{k=1}^{r}\sum_{t=1}^{r} d(v_i^k,v_j^t)^l + \sum_{i=1}^{n-1} d(v,v_{i,i+1})^l + \sum_{i=1}^{n-1}\sum_{j=1}^{r} d(v,v_{i,i+1}^j)^l$$
$$+ \sum_{i=1}^{r}\sum_{j=1}^{n-1} d(v^i,v_{j,j+1})^l + \sum_{i=1}^{r}\sum_{j=1}^{n-1}\sum_{k=1}^{r} d(v^i,v_{j,j+1}^k)^l + \sum_{i=1}^{n}\sum_{j=1}^{n-1} d(v_i,v_{j,j+1})^l + \sum_{i=1}^{n}\sum_{j=1}^{n-1}\sum_{k=1}^{r} d(v_i,v_{j,j+1}^k)^l$$
$$+ \sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{n-1} d(v_i^j,v_{k,k+1})^l + \sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{n-1}\sum_{t=1}^{r} d(v_i^j,v_{k,k+1}^r)^l + \sum_{i=1}^{n-2}\sum_{j=i+1}^{n-1} d(v_{i,i+1},v_{j,j+1})^l$$
$$+ \sum_{i=1}^{n-1}\sum_{j=1}^{r} d(v_{i,i+1},v_{i,i+1}^j)^l + \sum_{i=1}^{n-1}\sum_{j\in\{1,2,L,n-1\}-i}\sum_{k=1}^{r} d(v_{i,i+1},v_{j,j+1}^k)^l$$
$$+ \sum_{i=1}^{n-1}\sum_{j=1}^{r-1}\sum_{k=j+1}^{r} d(v_{i,i+1}^j,v_{i,i+1}^k)^l + \sum_{i=1}^{n-2}\sum_{j=i+1}^{n-1}\sum_{k=1}^{r}\sum_{t=1}^{r} d(v_{i,i+1}^k,v_{j,j+1}^t)^l$$

=r+n+ $nr2^l$ + $nr2^l$ + $nr^2 3^l$ + $n(n-1)2^l$ +nr+ $n(n-1)r3^l$ +nr(r-1)+ $\frac{n(n-1)}{2}r^2 4^l$ + $(n-1)2^l$ + $r(n-1)3^l$ + $r(n-1)3^l$ + $r^2(n-1)4^l$ + $(2n-2)2^l+(n^2-3n+2)4^l$ + $(2n-2+(n^2-3n+2)3^l)r$ + $(2n-2+(n^2-3n+2)3^l)r$ + $r^2((2n-2)3^l+(n^2-3n+2)5^l)$ + $(n-2)2^l+\frac{(n-2)(n-3)}{2}4^l$ +r(n-1)+ $r((2n-4)3^l+(n^2-5n+6)5^l)$ + $\frac{r(r-1)(n-1)}{2}2^l$ + $((n-2)4^l+\frac{(n-2)(n-3)}{2}6^l)r^2$

= $(n+nr^2+5nr-4r)$ + $(\frac{(r^2-r)(n-1)}{2}-5+3n+n^2+2nr)2^l$ + $(3nr^2+3r^2n-3nr-4r^2)3^l$ + $((\frac{n^2}{2}+\frac{3}{2}n-3)r^2+(\frac{3}{2}n^2-\frac{11}{2}n+5))4^l$ + $((n^2-3n+2)r^2+(n^2-5n+6)r)5^l$.

Thus, the result is hold.

Corollary 3. $W_l(F_n) = n + (-5+3n+n^2)2^l + (\frac{3}{2}n^2-\frac{11}{2}n+5)4^l$.

Theorem 4. $W_l(I_r(W_n)) = (3n+r+2nr)$ + $(3nr+\frac{3}{2}n+\frac{n^2}{2}+nr^2)2^l$ + $(3nr+4nr^2+n^2+nr^2-2n)3^l$ +

$((\frac{n^2}{2}+\frac{3n}{2})r^2+(2n^2-4n)r+\frac{n^2}{2}-\frac{3n}{2})4^l$ + $((n^2-2n)r^2+n(n-3)r)5^l+\frac{n(n-3)r^2}{2}6^l$.

Proof. Let Cn=v1v2…vn and v be a vertex in Wn beside Cn. $v_{i,i+1}$ be the adding vertex between vi and vi+1. Let v^1, v^2, …, v^r .be the r hanging vertices of v and v_i^1, v_i^2 ,…, v_i^r \be the r hanging vertices of vi (1 i n). Let $v_{n,n+1}=v_{1,n}$ and $v_{i,i+1}^1$, $v_{i,i+1}^2$,…, $v_{i,i+1}^r$ be the r hanging vertices of $v_{i,i+1}$ (1 i n). In view of the definition ofHyper-Wienerindex, we deduce

$$W_l(I_r(W_n)) = \sum_{i=1}^{r} d(v,v^i)^l + \sum_{i=1}^{n} d(v,v_i)^l + \sum_{i=1}^{n}\sum_{j=1}^{r} d(v,v_i^j)^l + \sum_{i=1}^{n}\sum_{j=1}^{r} d(v_i,v^j)^l + \sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{r} d(v_i^j,v^k)^l + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} d(v_i,v_j)^l + \sum_{i=1}^{n}\sum_{j=1}^{r} d(v_i,v_i^j)^l + \sum_{i=1}^{n}\sum_{j\in\{1,2,\ldots,n\}-i}\sum_{k=1}^{r} d(v_i,v_j^k)^l + \sum_{i=1}^{n}\sum_{j=1}^{r-1}\sum_{k=j+1}^{r} d(v_i^j,v_i^k)^l + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{k=1}^{r}\sum_{t=1}^{r} d(v_i^k,v_j^t)^l + \sum_{i=1}^{n} d(v,v_{i,i+1})^l + \sum_{i=1}^{n}\sum_{j=1}^{r} d(v,v_{i,i+1}^j)^l + \sum_{i=1}^{r}\sum_{j=1}^{n} d(v^i,v_{j,j+1})^l + \sum_{i=1}^{r}\sum_{j=1}^{n}\sum_{k=1}^{r} d(v^i,v_{j,j+1}^k)^l + \sum_{i=1}^{n}\sum_{j=1}^{n} d(v_i,v_{j,j+1})^l + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{r} d(v_i,v_{j,j+1}^k)^l + \sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{n} d(v_i^j,v_{k,k+1})^l + \sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{n}\sum_{t=1}^{r} d(v_i^j,v_{k,k+1}^r)^l + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} d(v_{i,i+1},v_{j,j+1})^l + \sum_{i=1}^{n}\sum_{j=1}^{r} d(v_{i,i+1},v_{i,i+1}^j)^l + \sum_{i=1}^{n}\sum_{j\in\{1,2,\ldots,n\}-i}\sum_{k=1}^{r} d(v_{i,i+1},v_{j,j+1}^k)^l + \sum_{i=1}^{n}\sum_{j=1}^{r-1}\sum_{k=j+1}^{r} d(v_{i,i+1}^j,v_{i,i+1}^k)^l + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{k=1}^{r}\sum_{t=1}^{r} d(v_{i,i+1}^k,v_{j,j+1}^t)^l$$

$$=r+n+ nr2^l + nr2^l + nr^2 3^l + \frac{n(n-1)}{2}2^l + nr + n(n-1)r3^l + \frac{nr(r-1)}{2}2^l + \frac{n(n-1)r^2}{2}4^l + n2^l + nr3^l + nr3^l + nr^2 4^l + 2n + (n^2-2n)3^l + (2n2^l+(n^2-2n)4^l)r + (2n2^l+(n^2-2n)4^l)r + (2n3^l+(n^2-2n)5^l)r^2 + n2^l + \frac{n(n-3)}{2}4^l + rn + (2n3^l+n(n-3)5^l)r + \frac{nr(r-1)}{2}2^l + (n4^l+\frac{n(n-3)}{2}6^l)r^2$$

$$= (3n+r+2nr) + (3nr+\frac{3}{2}n+\frac{n^2}{2}+nr^2)2^l + (3nr+4nr^2+n^2+nr^2-2n)3^l + ((\frac{n^2}{2}+\frac{3n}{2})r^2+(2n^2-4n)r+\frac{n^2}{2}-\frac{3n}{2})4^l + ((n^2-2n)r^2+n(n-3)r)5^l+\frac{n(n-3)r^2}{2}6^l.$$

As conclusion, we obtainthe finalconclusion.

Corollary4. $W_l(W_n) = 3n + (\frac{3}{2}n+\frac{n^2}{2})2^l + (n^2-2n)3^l + (\frac{n^2}{2}-\frac{3n}{2})4^l$.

3. l -Modified Hyper-Wiener Index

Theorem 5. $WW_l(I_r(F_n)) =$

$$(nr^2+2n+r-1)+(4nr+\frac{n^2}{2}-\frac{3n}{2}+2r-1)\frac{2^l+2^{2l}}{2}$$

$$+((2n-1)r^2+n^2-3n+2)\frac{3^l+3^{2l}}{2}$$
$$+\frac{(n-1)(n-2)}{2}\frac{4^l+4^{2l}}{2}.$$

Proof.By the definition of l -modified hyper-Wiener index, we have

$$WW_l(I_r(F_n))=\frac{1}{2}\{\{\sum_{i=1}^{r}d(v,v^i)^l+\sum_{i=1}^{n}d(v,v_i)^l+\sum_{i=1}^{n}\sum_{j=1}^{r}d(v,v_i^j)^l+\sum_{i=1}^{n}\sum_{j=1}^{r}d(v_i,v^j)^l$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{r}d(v_i^j,v^k)^l+\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}d(v_i,v_j)^l+\sum_{i=1}^{n}\sum_{j=1}^{r}d(v_i,v_i^j)^l+\sum_{i=1}^{n}\sum_{j\in\{1,2,\ldots,n\}-i}\sum_{k=1}^{r}d(v_i,v_j^k)^l$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{r-1}\sum_{k=j+1}^{r}d(v_i^j,v_i^k)^l+\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{k=1}^{r}\sum_{t=1}^{r}d(v_i^k,v_j^t)^l\}+\{\sum_{i=1}^{r}d(v,v^i)^{2l}+\sum_{i=1}^{n}d(v,v_i)^{2l}$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{r}d(v,v_i^j)^{2l}+\sum_{i=1}^{n}\sum_{j=1}^{r}d(v_i,v^j)^{2l}+\sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{r}d(v_i^j,v^k)^{2l}+\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}d(v_i,v_j)^{2l}$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{r}d(v_i,v_i^j)^{2l}+\sum_{i=1}^{n}\sum_{j\in\{1,2,\ldots,n\}-i}\sum_{k=1}^{r}d(v_i,v_j^k)^{2l}+\sum_{i=1}^{n}\sum_{j=1}^{r-1}\sum_{k=j+1}^{r}d(v_i^j,v_i^k)^{2l}$$
$$+\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{k=1}^{r}\sum_{t=1}^{r}d(v_i^k,v_j^t)^{2l}\}\}$$
$$=(nr^2+2n+r-1)+(4nr+\frac{n^2}{2}-\frac{3n}{2}+2r-1)\frac{2^l+2^{2l}}{2}+((2n-1)r^2+n^2-3n+2)\frac{3^l+3^{2l}}{2}+\frac{(n-1)(n-2)}{2}\frac{4^l+4^{2l}}{2}.$$

In this way, we get the decision.

Corollary5. $WW_l(F_n)=$
$$(2n-1)+(\frac{n^2}{2}-\frac{3n}{2}-1)\frac{2^l+2^{2l}}{2}+(n^2-3n+2)\frac{3^l+3^{2l}}{2}+\frac{(n-1)(n-2)}{2}\frac{4^l+4^{2l}}{2}.$$

Theorem 6. $WW_l(I_r(W_n))=$
$$(nr^2+r+n^2-n)+4nr\frac{2^l+2^{2l}}{2}+(2nr^2+n^2r-3nr)\frac{3^l+3^{2l}}{2}+\frac{n(n-3)r^2}{2}\frac{4^l+4^{2l}}{2}.$$

Proof.By the definition of l -modified hyper-Wiener index, we have

$$WW_l(I_r(W_n))=\frac{1}{2}\{\{\sum_{i=1}^{r}d(v,v^i)^l+\sum_{i=1}^{n}d(v,v_i)^l+\sum_{i=1}^{n}\sum_{j=1}^{r}d(v,v_i^j)^l+\sum_{i=1}^{n}\sum_{j=1}^{r}d(v_i,v^j)^l$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{r}d(v_i^j,v^k)^l+\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}d(v_i,v_j)^l+\sum_{i=1}^{n}\sum_{j=1}^{r}d(v_i,v_i^j)^l+\sum_{i=1}^{n}\sum_{j\in\{1,2,\ldots,n\}-i}\sum_{k=1}^{r}d(v_i,v_j^k)^l$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{r-1}\sum_{k=j+1}^{r}d(v_i^j,v_i^k)^l+\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{k=1}^{r}\sum_{t=1}^{r}d(v_i^k,v_j^t)^l\}+\{\sum_{i=1}^{r}d(v,v^i)^{2l}+\sum_{i=1}^{n}d(v,v_i)^{2l}$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{r}d(v,v_i^j)^{2l}+\sum_{i=1}^{n}\sum_{j=1}^{r}d(v_i,v^j)^{2l}+\sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{r}d(v_i^j,v^k)^{2l}+\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}d(v_i,v_j)^{2l}$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{r}d(v_i,v_i^j)^{2l}+\sum_{i=1}^{n}\sum_{j\in\{1,2,\ldots,n\}-i}\sum_{k=1}^{r}d(v_i,v_j^k)^{2l}+$$

$$\sum_{i=1}^{n}\sum_{j=1}^{r-1}\sum_{k=j+1}^{r}d(v_i^j,v_i^k)^{2l}$$

$$+\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{k=1}^{r}\sum_{t=1}^{r}d(v_i^k,v_j^t)^{2l}\ \}\}$$

$$=$$

$$(nr^2+r+n^2-n)+4nr\frac{2^l+2^{2l}}{2}$$

$$+(2nr^2+n^2r-3nr)\frac{3^l+3^{2l}}{2}+\frac{n(n-3)r^2}{2}\frac{4^l+4^{2l}}{2}$$

..

Hence, we derivethe desire conclusion.

Corollary6. $WW_l(W_n)=n^2-n$.

Theorem7. $WW_l(I_r(F_n))=(n+nr^2+5nr-4r)+$

$$(\frac{(r^2-r)(n-1)}{2}-5+3n+n^2+2nr)\frac{2^l+2^{2l}}{2}+$$

$$(3nr^2+3r^2n-3nr-4r^2)\frac{3^l+3^{2l}}{2}+$$

$$((\frac{n^2}{2}+\frac{3}{2}n-3)r^2+(\frac{3}{2}n^2-\frac{11}{2}n+5))\frac{4^l+4^{2l}}{2}+$$

$$((n^2-3n+2)r^2+(n^2-5n+6)r)\frac{5^l+5^l}{2}.$$

Proof. By virtue ofthe definition of l -modified hyper-Wiener index, we get

$$WW_l(I_r(F_n))=\frac{1}{2}\{\{\sum_{i=1}^{r}d(v,v^i)^l+\sum_{i=1}^{n}d(v,v_i)^l+$$

$$\sum_{i=1}^{n}\sum_{j=1}^{r}d(v,v_i^j)^l+\sum_{i=1}^{n}\sum_{j=1}^{r}d(v_i,v^j)^l$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{r}d(v_i^j,v^k)^l+\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}d(v_i,v_j)^l+$$

$$\sum_{i=1}^{n}\sum_{j=1}^{r}d(v_i,v_i^j)^l+\sum_{i=1}^{n}\sum_{j\in\{1,2,\ldots,n\}-i}\sum_{k=1}^{r}d(v_i,v_j^k)^l$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{r-1}\sum_{k=j+1}^{r}d(v_i^j,v_i^k)^l+\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{k=1}^{r}\sum_{t=1}^{r}d(v_i^k,v_j^t)^l+$$

$$\sum_{i=1}^{n-1}d(v,v_{i,i+1})^l+\sum_{i=1}^{n-1}\sum_{j=1}^{r}d(v,v_{i,i+1}^j)^l$$

$$+\sum_{i=1}^{r}\sum_{j=1}^{n-1}d(v^i,v_{j,j+1})^l+\sum_{i=1}^{r}\sum_{j=1}^{n-1}\sum_{k=1}^{r}d(v^i,v_{j,j+1}^k)^l+$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n-1}d(v_i,v_{j,j+1})^l+\sum_{i=1}^{n}\sum_{j=1}^{n-1}\sum_{k=1}^{r}d(v_i,v_{j,j+1}^k)^l$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{n-1}d(v_i^j,v_{k,k+1})^l+$$

$$\sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{n-1}\sum_{t=1}^{r}d(v_i^j,v_{k,k+1}^r)^l+\sum_{i=1}^{n-2}\sum_{j=i+1}^{n-1}d(v_{i,i+1},v_{j,j+1})^l$$

$$+\sum_{i=1}^{n-1}\sum_{j=1}^{r}d(v_{i,i+1},v_{i,i+1}^j)^l+$$

$$\sum_{i=1}^{n-1}\sum_{j\in\{1,2,\ldots,n-1\}-i}\sum_{k=1}^{r}d(v_{i,i+1},v_{j,j+1}^k)^l+$$

$$\sum_{i=1}^{n-1}\sum_{j=1}^{r-1}\sum_{k=j+1}^{r}d(v_{i,i+1}^j,v_{i,i+1}^k)^l$$

$$+\sum_{i=1}^{n-2}\sum_{j=i+1}^{n-1}\sum_{k=1}^{r}\sum_{t=1}^{r}d(v_{i,i+1}^k,v_{j,j+1}^t)^l\}+\{\sum_{i=1}^{r}d(v,v^i)^{2l}+$$

$$\sum_{i=1}^{n}d(v,v_i)^{2l}+\sum_{i=1}^{n}\sum_{j=1}^{r}d(v,v_i^j)^{2l}$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{r}d(v_i,v^j)^{2l}+\sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{r}d(v_i^j,v^k)^{2l}+$$

$$\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}d(v_i,v_j)^{2l}+\sum_{i=1}^{n}\sum_{j=1}^{r}d(v_i,v_i^j)^{2l}$$

$$+\sum_{i=1}^{n}\sum_{j\in\{1,2,\ldots,n\}-i}\sum_{k=1}^{r}d(v_i,v_j^k)^{2l}+$$

$$\sum_{i=1}^{n}\sum_{j=1}^{r-1}\sum_{k=j+1}^{r}d(v_i^j,v_i^k)^{2l}+\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{k=1}^{r}\sum_{t=1}^{r}d(v_i^k,v_j^t)^{2l}$$

$$+\sum_{i=1}^{n-1}d(v,v_{i,i+1})^{2l}+\sum_{i=1}^{n-1}\sum_{j=1}^{r}d(v,v_{i,i+1}^j)^{2l}+$$

$$\sum_{i=1}^{r}\sum_{j=1}^{n-1}d(v^i,v_{j,j+1})^{2l}+\sum_{i=1}^{r}\sum_{j=1}^{n-1}\sum_{k=1}^{r}d(v^i,v_{j,j+1}^k)^{2l}$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{n-1}d(v_i,v_{j,j+1})^{2l}+\sum_{i=1}^{n}\sum_{j=1}^{n-1}\sum_{k=1}^{r}d(v_i,v_{j,j+1}^k)^{2l}+$$

$$\sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{n-1}d(v_i^j,v_{k,k+1})^{2l}$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{n-1}\sum_{t=1}^{r}d(v_i^j,v_{k,k+1}^r)^{2l}+$$

$$\sum_{i=1}^{n-2}\sum_{j=i+1}^{n-1}d(v_{i,i+1},v_{j,j+1})^{2l}+\sum_{i=1}^{n-1}\sum_{j=1}^{r}d(v_{i,i+1},v_{i,i+1}^j)^{2l}$$

$$+\sum_{i=1}^{n-1}\sum_{j\in\{1,2,\ldots,n-1\}-i}\sum_{k=1}^{r}d(v_{i,i+1},v_{j,j+1}^k)^{2l}+$$

$$\sum_{i=1}^{n-1}\sum_{j=1}^{r-1}\sum_{k=j+1}^{r}d(v_{i,i+1}^j,v_{i,i+1}^k)^{2l}+$$

$$\sum_{i=1}^{n-2}\sum_{j=i+1}^{n-1}\sum_{k=1}^{r}\sum_{t=1}^{r}d(v_{i,i+1}^k,v_{j,j+1}^t)^{2l}\}\}$$

$$=(n+nr^2+5nr-4r)+$$

$$(\frac{(r^2-r)(n-1)}{2}-5+3n+n^2+2nr)\frac{2^l+2^{2l}}{2}+$$

$$(3nr^2+3r^2n-3nr-4r^2)\frac{3^l+3^{2l}}{2}+$$

$$((\frac{n^2}{2}+\frac{3}{2}n-3)r^2+(\frac{3}{2}n^2-\frac{11}{2}n+5))\frac{4^l+4^{2l}}{2}+$$

$$((n^2-3n+2)r^2+(n^2-5n+6)r)\frac{5^l+5^l}{2}.$$

Thus, the result is hold.

Corollary 7. $WW_l(\tilde{F}_n)=n+(-5+3n+n^2)\frac{2^l+2^{2l}}{2}$

$$+(\frac{3}{2}n^2-\frac{11}{2}n+5)\frac{4^l+4^{2l}}{2}.$$

Theorem 8. $WW_l(I_r(\tilde{W}_n))=(3n+r+2nr)+$

$$(3nr+\frac{3}{2}n+\frac{n^2}{2}+nr^2)\frac{2^l+2^{2l}}{2}+$$

$$(3nr+4nr^2+n^2+nr^2-2n)\frac{3^l+3^{2l}}{2}+$$

$$((\frac{n^2}{2}+\frac{3n}{2})r^2+(2n^2-4n)r+\frac{n^2}{2}-\frac{3n}{2})\frac{4^l+4^{2l}}{2}+$$

$$((n^2-2n)r^2+n(n-3)r)\frac{5^l+5^{2l}}{2}+$$

$$\frac{n(n-3)r^2}{2}\frac{6^l+6^{2l}}{3}.$$

Proof. In view of the definition of l -modified hyper-Wiener index, we deduce

$$WW_l(I_r(\tilde{W}_n))=\frac{1}{2}\{\{\sum_{i=1}^{r}d(v,v^i)^l+\sum_{i=1}^{n}d(v,v_i)^l+$$

$$\sum_{i=1}^{n}\sum_{j=1}^{r}d(v,v_i^j)^l+\sum_{i=1}^{n}\sum_{j=1}^{r}d(v_i,v^j)^l$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{r}d(v_i^j,v^k)^l+\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}d(v_i,v_j)^l+$$

$$\sum_{i=1}^{n}\sum_{j=1}^{r}d(v_i,v_i^j)^l+\sum_{i=1}^{n}\sum_{j\in\{1,2,\ldots,n\}-i}\sum_{k=1}^{r}d(v_i,v_j^k)^l$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{r-1}\sum_{k=j+1}^{r}d(v_i^j,v_i^k)^l+\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{k=1}^{r}\sum_{t=1}^{r}d(v_i^k,v_j^t)^l+$$

$$\sum_{i=1}^{n}d(v,v_{i,i+1})^l+\sum_{i=1}^{n}\sum_{j=1}^{r}d(v,v_{i,i+1}^j)^l$$

$$+\sum_{i=1}^{r}\sum_{j=1}^{n}d(v^i,v_{j,j+1})^l+\sum_{i=1}^{r}\sum_{j=1}^{n}\sum_{k=1}^{r}d(v^i,v_{j,j+1}^k)^l+$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}d(v_i,v_{j,j+1})^l+\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{r}d(v_i,v_{j,j+1}^k)^l$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{n}d(v_i^j,v_{k,k+1})^l+$$

$$\sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{n}\sum_{t=1}^{r}d(v_i^j,v_{k,k+1}^r)^l+\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}d(v_{i,i+1},v_{j,j+1})^l$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{r}d(v_{i,i+1},v_{i,i+1}^j)^l+$$

$$\sum_{i=1}^{n}\sum_{j\in\{1,2,\ldots,n\}-i}\sum_{k=1}^{r}d(v_{i,i+1},v_{j,j+1}^k)^l+$$

$$\sum_{i=1}^{n}\sum_{j=1}^{r-1}\sum_{k=j+1}^{r}d(v_{i,i+1}^j,v_{i,i+1}^k)^l$$

$$+\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{k=1}^{r}\sum_{t=1}^{r}d(v_{i,i+1}^k,v_{j,j+1}^t)^l\}+\{\sum_{i=1}^{r}d(v,v^i)^{2l}+$$

$$\sum_{i=1}^{n}d(v,v_i)^{2l}+\sum_{i=1}^{n}\sum_{j=1}^{r}d(v,v_i^j)^{2l}$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{r}d(v_i,v^j)^{2l}+\sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{r}d(v_i^j,v^k)^{2l}+$$

$$\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}d(v_i,v_j)^{2l}+\sum_{i=1}^{n}\sum_{j=1}^{r}d(v_i,v_i^j)^{2l}$$

$$+\sum_{i=1}^{n}\sum_{j\in\{1,2,\mathbf{L},n\}-i}\sum_{k=1}^{r}d(v_i,v_j^k)^{2l}+$$

$$\sum_{i=1}^{n}\sum_{j=1}^{r-1}\sum_{k=j+1}^{r}d(v_i^j,v_i^k)^{2l}+\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{k=1}^{r}\sum_{t=1}^{r}d(v_i^k,v_j^t)^{2l}$$

$$+\sum_{i=1}^{n}d(v,v_{i,i+1})^{2l}+\sum_{i=1}^{n}\sum_{j=1}^{r}d(v,v_{i,i+1}^{j})^{2l}+$$

$$\sum_{i=1}^{r}\sum_{j=1}^{n}d(v^i,v_{j,j+1})^{2l}+\sum_{i=1}^{r}\sum_{j=1}^{n}\sum_{k=1}^{r}d(v^i,v_{j,j+1}^{k})^{2l}$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{n}d(v_i,v_{j,j+1})^{2l}+\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{r}d(v_i,v_{j,j+1}^{k})^{2l}+$$

$$\sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{n}d(v_i^j,v_{k,k+1})^{2l}$$

$$+\sum_{i=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{n}\sum_{t=1}^{r}d(v_i^j,v_{k,k+1}^{r})^{2l}+$$

$$\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}d(v_{i,i+1},v_{j,j+1})^{2l}+\sum_{i=1}^{n}\sum_{j=1}^{r}d(v_{i,i+1},v_{i,i+1}^{j})^{2l}$$

$$+\sum_{i=1}^{n}\sum_{j\in\{1,2,\mathbf{L},n\}-i}\sum_{k=1}^{r}d(v_{i,i+1},v_{j,j+1}^{k})^{2l}+$$

$$\sum_{i=1}^{n}\sum_{j=1}^{r-1}\sum_{k=j+1}^{r}d(v_{i,i+1}^{j},v_{i,i+1}^{k})^{2l}+$$

$$\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{k=1}^{r}\sum_{t=1}^{r}d(v_{i,i+1}^{k},v_{j,j+1}^{t})^{2l}\ \}\}$$

$$=(3n+r+2nr)+(3nr+\frac{3}{2}n+\frac{n^2}{2}+nr^2)\frac{2^l+2^{2l}}{2}$$

$$+(3nr+4nr^2+n^2+nr^2-2n)\frac{3^l+3^{2l}}{2}+$$

$$((\frac{n^2}{2}+\frac{3n}{2})r^2+(2n^2-4n)r+\frac{n^2}{2}-\frac{3n}{2})\frac{4^l+4^{2l}}{2}+$$

$$((n^2-2n)r^2+n(n-3)r)\frac{5^l+5^{2l}}{2}+$$

$$\frac{n(n-3)r^2}{2}\frac{6^l+6^{2l}}{3}.$$

As conclusion, we obtainthe finalconclusion.

Corollary8. $WW_l(W_n)=3n+(\frac{3}{2}n+\frac{n^2}{2})\frac{2^l+2^{2l}}{2}+$

$$(n^2-2n)\frac{3^l+3^{2l}}{2}+(\frac{n^2}{2}-\frac{3n}{2})\frac{4^l+4^{2l}}{2}.$$

4. Conclusion

Combinatorial chemistry is a new powerful technology in molecular recognition and drug design. It is a wet-laboratory methodology purposed to massively parallel screening of chemical compounds for the founding of compounds that have certain biological activities. The power of trick draws from the interaction between computational modeling and experimental design.

Fan graph, wheel graph, gear fan graph, gear wheel graph and their r-corona graphs are common structural features of organic molecules. The contributions of our paper are determining the l -modified Wiener index and l -modified hyper-Wiener index of these special structural features of organic molecules.

5. Acknowledgements

We would like to thank the anonymous referees for providing us with constructive comments and suggestions.

References

[1] L. Yan, Y. Li, W. Gao,J. S. Li, On the extremal hyper-wiener index of graphs, Journal of Chemical and Pharmaceutical Research, 2014, 6(3): 477-481.

[2] L. Yan, W. Gao, J. S. Li, General harmonic index and general sum connectivity index of polyomino chains and nanotubes, Journal ofComputational and Theoretical Nanoscience, In press.

[3] W. Gao, L. Liang, Y. Gao, Some results on wiener related index and shultz index of molecular graphs, Energy Education Science and Technology: Part A, 2014, 32(6): 8961-8970.

[4] W. Gao, L. Liang, Y. Gao, Total eccentricity, adjacent eccentric distance sum and Gutman index of certain special molecular graphs, The BioTechnology: An Indian Journal, 2014, 10(9): 3837-3845.

[5] W. Gao, L. Shi, Wiener index of gear fan graph and gear wheel graph, Asian Journal of Chemistry, 2014, 26(11): 3397-3400.

[6] W. Gao, W. F. Wang, Second atom-bond connectivity index of special chemical molecular structures, Journal of Chemistry, Volume 2014, Article ID 906254, 8 pages, http://dx.doi.org/10.1155/2014/906254.

[7] W. F. Xi, W. Gao, Geometric-arithmetic index and Zagreb indices of certain special molecular graphs, Journal of Advances in Chemistry, 2014, 10(2): 2254-2261.

[8] W. F. Xi, W. Gao, l -Modified extremal hyper-Wiener index of molecular graphs, Journal of Applied Computer Science & Mathematics, 2014, 18 (8): 43-46.

[9] W. F. Xi, W. Gao, Y. Li, Three indices calculation of certain crown molecular graphs, Journal of Advances in Mathematics, 2014, 9(6): 2696-2304.

[10] Y. Gao, W. Gao, L. Liang, Revised Szeged index and revised edge Szeged index of certain special molecular graphs, International Journal of Applied Physics and Mathematics, 2014, 4(6): 417-425.

[11] J. A. Bondy, U. S. R. Mutry, Graph Theory, Spring, Berlin, 2008.

[12] D. Chen, The value of the Wiener index of the peptoids, J. Wuhan Inst. Chem. Tech., 2005, 27(2): 82-84.

[13] B. Xing, G. Cai, The Wiener index of trees with prescribed diameter, Operations Research Transactions, 2011, 15(4): 36-44.

[14] H. Wan, H. Ren, The Wiener index of a class of tricyclic graphs, Journal of Mathematical Study, 2012, 45(2): 207-212.

[15] Y. Pan, Wiener number and hyper-wiener number of two types of polyomino systems, Journal of Mathematical Study, 2013, 46(3): 260-269.

[16] D. Stevanovic, Maximizing Wiener index of graphs with fixed maximum degree, Match Commun. Math. Comput. Chem., 2008, 60:71-83.

[17] Y. Hong, H. Liu, X. Wu, On the Wiener index of unicyclic graphs, Hacettepe Journal of Mathematics and Statistics, 2011, 40(1): 63-68.

[18] X. Zhang, Q. Xiang, The Wiener index of trees with given degree sequences, Match Commun. Math. Comput. Chem., 2008, 60: 623-644.

[19] G. Cash, Three methods for calculation of the Hyper-wiener index of molecular graphs, J. Chem. Inf. Comput. Sci., 2002, 42, 571-576.

[20] E. Castro, Ivan Gutman, Damian Marino, Pablo Peruzzo, Upgrading the wiener index, J. Serb. Chem. Soc., 2002, 67(10):647-651.

[21] B. Wu, Wiener index of line graphs, Match Commun. Math. Comput. Chem., 2010, 64: 699-706.

[22] A. Vijayabarathi, G.S.G.N. Anjaneyulu, Wiener index of a graph and chemical applications, International Journal of ChemTech Research, 2013, 5(4): 1847-1853.

[23] P. Dankelmann, I. Gutman, S. Mukwembi, H.C. Swart, The edge-Wiener index of a graph, Discrete Mathematics, 2009, 309: 3452-3457.