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Updating Approximations of VPRS Model based on Dominance Relations

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Abstract: In real world problems, the collected data vary from time to time, and therefore, the approximations of a concept by a variable precision rough set model (VPRS) should be correspondingly updated. This paper focuses on developing incremental method to update set approximations of VPRS based on dominance relations. Under dynamic environments where an object is inserted or deleted, we present the updating principles and then develop the incremental method for updating approximation sets. The related theoretical results are presented with proofs, and illustrative examples are also given to support the effectiveness of the proposed method.

Keywords: Variable Precision Rough Set; Dominance Relation; Set Approximation; Dynamic Information Systems; Incremental Update

1. Introduction

Rough set theory [1] is a mathematical tool developed in recent years to deal with inconsistency and ambiguity information. Traditional rough set theory (TRS) is described by equivalence relations in universe, and then forms equivalence class and lower/upper approximation sets of concepts in the given universe. One limitation of TRS is that the classification must be completely correct or certain. It only considers complete "including" or "non-including," but not "including" in a certain degree. In real-life applications, since the imprecise and missing data are common, it is difficult for a class (i.e., a concept) to be completely included in an equivalence class. The Variable Precision Rough Set model (VPRS) [4] proposed by Ziarko gives a classification strategy in which the error rate is less than a given threshold $b\left(0 \le b < 0.5\right)$. Thus, the VPRS models are fault-tolerant in a certain degree. On the other hand, in order to solve practical problems with preference-ordered attributes, Greco et al. extended TRS by introducing the concept of dominance relation and proposed dominance-based rough sets approach (DRSA) [4-6]. Here, the condition attributes with preference order are called criteria. Currently, DRSA has been widely applied in multi-criteria decision problems[7,8].

In practical applications, many collected data are dynamic and therefore the involved information systems need to be updated frequently. The re-computation of approximations sets is very time consuming in TRS. In the literature, some methods of incrementally updating approximations and attribute reductions have been put forward in the framework of rough set[9-14]. Compared with classical rough set method, these incremental methods obviously improved the computational efficiency. To our best knowledge, incremental updating approximations of VPRS based on dominance relations in multi-criteria problems has not yet been discussed so far. In this paper, we discuss rapid methods of updating approximations under the variation of the object set. We have obtained some updating rules by theoretical proofs, which can be used to greatly improve the efficiency of information processing in multi-criteria classification problems.

The remainder of this paper is organized as follows: We present basic notions of DRSA in Section 2;and the principles of updating approximations with detailed proofs are presented in Section 3. Section 4 gives a numerical example to show the feasibility of our proposed approach, and conclusions are given finally.

2. Basic concepts

As a prior knowledge, this section describes the involved concepts based on dominance relations of rough set theory.

Definition 1. A quadruple $S = (U, A, V, f)$ is an information system, where U is a nonempty finite set of objects, called the universe. A is a nonempty finite set of attributes, $A = C \cup D, C \cap D = \varnothing$, where C and D denote the sets of condition attributes and decision attributes, respectively. $V = \bigcup_{a \in A} V_a$, V_a is the domain of attribute a. f: $U \times A \to V$ is an information function, which gives values to every object on each attribute, namely, $\forall a \in A, x \in U, f\left(x,a\right) \in V_a$.

Definition 2. Let $S = (U, A, V, f)$ is an information system, for $B \subseteq A$, we denote

$$R_B^{\le} = \{(x_i, x_j) \in U \times U : f_l(x_i) \le f_l(x_j), \forall a_l \in B\}$$

$R_B^{\le}$ is the dominance relations of information system. Based on definition 2,

$$D_P^{+}(x_i) = \{x_j \in U : (x_i, x_j) \in R_P^{\le}\} =$$
$$\{x_j \in U : f_l(x_i) \le f_l(x_j), \forall a_l \in P, P \subseteq A\}$$

is the dominance class of x_i.

Assume there is only one decision attribute *d*. Based on the above definitions, the universe *U* is divided by the decision attribute *d* into a family of equivalence classes with preference-ordered, called decision classes. Let **Cl** = {Cl_n, *n Î* T} be a set of decision classes, T = {1, …,*m*}. If r, s *Î* T such that r > s, the objects from Cl_r are preferred to the objects from Cl_s.

Definition 3. Let *m* be the number of attribute values. Define the upward union of classes as

$$Cl_n^{\ge} = \bigcup_{n' \ge n} Cl_{n'} \left(\forall n, n' \in T, \ T = \{1, \ldots, m\}\right),$$

where $x \in Cl_n^{\ge}$ means "x belongs to at least class Cl_n".

Definition 4. Downward union of classes is defined as $Cl_n^{\le}$,

$$Cl_n^{\le} = \bigcup_{n' \le n} Cl_{n'} \left(\forall n, n' \in T, \ T = \{1, \ldots, m\}\right)$$

where $x \in Cl_n^{\le}$ means "x belongs to at most class Cl_n".

Definition 5. The b -lower approximation of the upward union is defined as

$$\underline{P}_b(Cl_n^{\ge}) = \left\{x \in U : \frac{|D_P^{+}(x) \cap Cl_n^{\ge}|}{|Cl_n^{\ge}|} \ge b\right\}$$

In DRSA, b -upper approximation of the upward union

$$\overline{P}_b(Cl_n^{\ge}) = \left\{x \in U : \frac{|D_P^{+}(x) \cap Cl_n^{\ge}|}{|Cl_n^{\ge}|} \ge 1 - b\right\}$$

For convenient, here $c(x, Cl_n^{\ge}) = \frac{|D_P^{+}(x) \cap Cl_n^{\ge}|}{|Cl_n^{\ge}|}$ denotes the degree of object x belongs to upward union.

Definition 6. (β-lower/upper approximations of the downward union**)**

In DRSA, b -lower approximation of the downward union

$$\underline{P}_b(Cl_n^{\le}) = \left\{x \in U : \frac{|D_P^{-}(x) \cap Cl_n^{\le}|}{|Cl_n^{\le}|} \ge b\right\}$$

In DRSA, b -upper approximation of the downward union

$$\overline{P}_b(Cl_n^{\le}) = \left\{x \in U : \frac{|D_P^{-}(x) \cap Cl_n^{\le}|}{|Cl_n^{\le}|} \ge 1 - b\right\}$$

For convenient, denote $c(x, Cl_n^{\le}) = \frac{|D_P^{-}(x) \cap Cl_n^{\le}|}{|Cl_n^{\le}|}$ as the degree of object x belongs to downward union.

3. Updating Principles of The Incremental Method

In this section, we present the theoretical results of the incremental method which can be used to develop incremental algorithms. For convenience, we use U, $D_P^{+}(x)$, $Cl_n^{\ge}$, $Cl_n^{\le}$ denote the universe, dominance class, the upward and downward unions of the original information systems; and $U', D_P^{+}(x), Cl_n^{'\ge}, Cl_n^{'\le}$ denote the updated ones, respectively. Considering the dynamic environments where an object is inserted or deleted. Use x^{+}, x^{-} denote the inserted and deleted object respectively.

LEMMA 1[14]. Let $x^{+} \in Cl_{n'}$ be an added object, $n, n' \in T$, and then we have

$$Cl_n'^{\ge} = \begin{cases} Cl_n^{\ge}, n > n' \\ Cl_n^{\ge} \cup \{x^{+}\}, n \le n' \end{cases}$$

$$Cl_n'^{\le} = \begin{cases} Cl_n^{\le}, n < n' \\ Cl_n^{\le} \cup \{x^{+}\}, n \ge n' \end{cases}$$

LEMMA 2[14]. Let $x^{-} \in Cl_{n'}$ be an removed object, $n, n' \in T$, and then we have

$$Cl_n'^{\ge} = \begin{cases} Cl_n^{\ge}, n > n' \\ Cl_n^{\ge} - \{x^{-}\}, n \le n' \end{cases}$$

$$Cl_n'^{\le} = \begin{cases} Cl_n^{\le}, n < n' \\ Cl_n^{\le} - \{x^{-}\}, n \ge n' \end{cases}$$

3.1. Updating the lower/upper approximations of upward union

1. Insert an object x^{+}, $U' = U \cup \{x^{+}\}$,

Proposition 1 If $n > n'$, $Cl_n'^{\ge} = Cl_n^{\ge}$, then

$$\underline{P}_b(Cl_n'^{\ge}) = \underline{P}_b(Cl_n^{\ge}) \cup \{x^{+} | c(x^{+}, Cl_n'^{\ge} \ge b)\}$$

Proof: If $n > n'$, then $Cl_n'^{\ge} = Cl_n^{\ge}$

$\forall x \in U$,

$$c\left(x, Cl'^{\geq}_n\right)=\frac{\left|D'^{+}_P(x)\cap Cl'^{\geq}_n\right|}{\left|Cl^{\geq}_n\right|}=\frac{\left|D^{+}_P(x)\cap Cl^{\geq}_n\right|}{\left|Cl^{\geq}_n\right|}=c\left(x, Cl^{\geq}_n\right)$$

So $\underline{P}_b\left(Cl'^{\geq}_n\right)=\underline{P}_b\left(Cl^{\geq}_n\right)$.

If $x = x^+$,

$$c\left(x^+, Cl'^{\geq}_n\right)=\frac{\left|D^{+}_P(x^+)\cap Cl'^{\geq}_n\right|}{\left|Cl'^{\geq}_n\right|}=\frac{\left|D^{+}_P(x^+)\cap Cl^{\geq}_n\right|}{\left|Cl^{\geq}_n\right|}\geq b.$$

Then $x^+ \in \underline{P}_b\left(Cl'^{\geq}_n\right)$,

and $\underline{P}_b\left(Cl'^{\geq}_n\right)=\underline{P}_b\left(Cl^{\geq}_n\right)\cup\left\{x^+\right\}$.

Proposition 2 If $n \leq n'$, $Cl'^{\geq}_n = Cl^{\geq}_n \cup \left\{x^+\right\}$, then

$$\underline{P}_b\left(Cl'^{\geq}_n\right)=\underline{P}_b\left(Cl^{\geq}_n\right)\cup\left\{x^+ \mid c\left(x^+, Cl'^{\geq}_n\right)\geq b\right\}$$
$$\cup\left\{x \mid x^+ \in D'^{+}_P(x), x \notin \underline{P}_b\left(Cl^{\geq}_n\right) \,\&\, c\left(x, Cl'^{\geq}_n\right)\geq b\right\}$$
$$-\left\{x \mid x^+ \notin D'^{+}_P(x), x \in \underline{P}_b\left(Cl^{\geq}_n\right) \,\&\, c\left(x, Cl'^{\geq}_n\right)< b\right\}$$

Proof: If $n \leq n'$, then $Cl'^{\geq}_n = Cl^{\geq}_n \cup \left\{x^+\right\}$.

If $x = x^+$,

$$c\left(x^+, Cl'^{\geq}_n\right)=\frac{\left|D^{+}_P(x^+)\cap Cl'^{\geq}_n\right|}{\left|Cl'^{\geq}_n\right|}\geq b.$$

Then $x^+ \in \underline{P}_b\left(Cl'^{\geq}_n\right)$ and

$\underline{P}_b\left(Cl'^{\geq}_n\right)=\underline{P}_b\left(Cl^{\geq}_n\right)\cup\left\{x^+\right\}$.

$\forall x \in U$,

If $x^+ \in D'^{+}_P(x)$,

$$c\left(x, Cl'^{\geq}_n\right)=\frac{\left|D'^{+}_P(x)\cap Cl'^{\geq}_n\right|}{\left|Cl'^{\geq}_n\right|}=\frac{\left|D^{+}_P(x)\cap Cl^{\geq}_n\right|+1}{\left|Cl^{\geq}_n\right|+1}\geq c\left(x, Cl^{\geq}_n\right).$$

Then if $x \notin \underline{P}_b\left(Cl^{\geq}_n\right)$ and $c\left(x, Cl'^{\geq}_n\right)\geq b$, we have $\underline{P}_b\left(Cl'^{\geq}_n\right)=\underline{P}_b\left(Cl^{\geq}_n\right)\cup\{x\}$.

If $x^+ \notin D'^{+}_P(x)$,

$$c\left(x, Cl'^{\geq}_n\right)=\frac{\left|D'^{+}_P(x)\cap Cl'^{\geq}_n\right|}{\left|Cl'^{\geq}_n\right|}=\frac{\left|D^{+}_n(x)\cap Cl^{\geq}_n\right|}{\left|Cl^{\geq}_n\right|+1}< c\left(x, Cl^{\geq}_n\right).$$

So if $x \in \underline{P}_b\left(Cl^{\geq}_n\right)$ and $c\left(x, Cl'^{\geq}_n\right)< b$, then $\underline{P}_b\left(Cl'^{\geq}_n\right)=\underline{P}_b\left(Cl^{\geq}_n\right)-\{x\}$.

Otherwise, $\underline{P}_b\left(Cl'^{\geq}_n\right)=\underline{P}_b\left(Cl^{\geq}_n\right)$.

Proposition 3 If $n > n'$, $Cl'^{\geq}_n = Cl^{\geq}_n$, then we have

$$\overline{P}_b\left(Cl'^{\geq}_n\right)=\overline{P}_b\left(Cl^{\geq}_n\right)\cup\left\{x^+ \mid c\left(x^+, Cl^{\geq}_n\right)\geq 1-b\right\};$$

Proposition 4 If $n \leq n'$, $Cl'^{\geq}_n = Cl^{\geq}_n \cup \left\{x^+\right\}$,

$$\overline{P}_b\left(Cl'^{\geq}_n\right)=\overline{P}_b\left(Cl^{\geq}_n\right)\cup\left\{x^+ \mid c\left(x^+, Cl'^{\geq}_n\right)\geq b\right\}$$
$$\cup\left\{x \mid x^+ \in D'^{+}_P(x), x \notin \overline{P}_b\left(Cl^{\geq}_n\right) \,\&\, c\left(x, Cl'^{\geq}_n\right)\geq 1-b\right\}$$
$$-\left\{x \mid x^+ \notin D'^{+}_P(x), x \in \overline{P}_b\left(Cl^{\geq}_n\right) \,\&\, c\left(x, Cl'^{\geq}_n\right)< 1-b\right\}$$

2. Delete an object x^-, $U' = U - \left\{x^-\right\}$.

Proposition 5 If $n > n'$, $Cl'^{\geq}_n = Cl^{\geq}_n$, we have

$$\underline{P}_b\left(Cl'^{\geq}_n\right)=\underline{P}_b\left(Cl^{\geq}_n\right)-\left\{x^- \mid x^- \in \underline{P}_b\left(Cl^{\geq}_n\right)\right\}$$

Proposition 6 If $n \leq n'$, $Cl'^{\geq}_n = Cl^{\geq}_n - \left\{x^-\right\}$, then

$$\underline{P}_b\left(Cl'^{\geq}_n\right)=\underline{P}_b\left(Cl^{\geq}_n\right)-\left\{x^- \mid x^- \in \underline{P}_b\left(Cl^{\geq}_n\right)\right\}$$
$$-\left\{x \mid x^- \in D^{+}_P(x), x \in \underline{P}_b\left(Cl^{\geq}_n\right) \,\&\, c\left(x, Cl'^{\geq}_n\right)< b\right\}$$
$$\cup\left\{x \mid x^- \notin D^{+}_P(x), x \notin \underline{P}_b\left(Cl^{\geq}_n\right) \,\&\, c\left(x, Cl'^{\geq}_n\right)\geq b\right\}$$

Proposition 7 If $n > n'$ and $Cl'^{\geq}_n = Cl^{\geq}_n$,

$$\overline{P}_b\left(Cl'^{\geq}_n\right)=\overline{P}_b\left(Cl^{\geq}_n\right)-\left\{x^- \mid x^- \in \overline{P}_b\left(Cl^{\geq}_n\right)\right\}$$

Proposition 8 If $n \leq n'$ and $Cl'^{\geq}_n = Cl^{\geq}_n - \left\{x^-\right\}$, then

$$\overline{P}_b\left(Cl'^{\geq}_n\right)=\overline{P}_b\left(Cl^{\geq}_n\right)-\left\{x^- \mid x^- \in \overline{P}_b\left(Cl^{\geq}_n\right)\right\}$$
$$-\left\{x \mid x^- \in D^{+}_P(x), x \in \overline{P}_b\left(Cl^{\geq}_n\right) \,\&\, c\left(x, Cl'^{\geq}_n\right)< 1-b\right\}$$
$$\cup\left\{x \mid x^+ \notin D^{+}_P(x), x \notin \overline{P}_b\left(Cl^{\geq}_n\right) \,\&\, c\left(x, Cl'^{\geq}_n\right)\geq 1-b\right\}$$

3.2. Updating the set approximations of downward union

1. Insert an object x^+, $U' = U \cup \left\{x^+\right\}$

Proposition 9 If $n < n'$ and $Cl'^{\leq}_n = Cl^{\leq}_n$,

$$\underline{P}_b\left(Cl'^{\leq}_n\right)=\underline{P}_b\left(Cl^{\leq}_n\right)\cup\left\{x^+ | c\left(x^+, Cl'^{\leq}_n \geq b\right)\right\}$$

Proposition 10 If $n \geq n'$ and $Cl'^{\leq}_n = Cl^{\leq}_n \cup \left\{x^+\right\}$,

$$\underline{P}_b\left(Cl'^{\leq}_n\right)=\underline{P}_b\left(Cl^{\leq}_n\right)\cup\left\{x^+ \mid c\left(x^+, Cl'^{\leq}_n\right)\geq b\right\}$$
$$\cup\left\{x \mid x^+ \in D'^{-}_P(x), x \notin \underline{P}_b\left(Cl^{\leq}_n\right) \,\&\, c\left(x, Cl'^{\leq}_n\right)\geq b\right\}$$
$$-\left\{x \mid x^+ \notin D'^{-}_P(x), x \in \underline{P}_b\left(Cl^{\leq}_n\right) \,\&\, c\left(x, Cl'^{\leq}_n\right)< b\right\}$$

Proposition 11 If $n < n'$, $Cl'^{\leq}_n = Cl^{\leq}_n$,

$$\overline{P}_b\left(Cl'^{\leq}_n\right)=\overline{P}_b\left(Cl^{\leq}_n\right)\cup\left\{x^+ \mid c\left(x^+, Cl^{\geq\leq}_n\right)\geq 1-b\right\}$$

Proposition 12 If $n \geq n'$, $Cl'^{\leq}_n = Cl^{\leq}_n \cup \left\{x^+\right\}$,

$$\overline{P}_b\left(Cl'^{\leq}_n\right)=\overline{P}_b\left(Cl^{\leq}_n\right)\cup\left\{x^+ \mid c\left(x^+, Cl'^{\leq}_n\right)\geq b\right\}$$
$$\cup\left\{x \mid x^+ \in D'^{-}_P(x), x \notin \overline{P}_b\left(Cl^{\leq}_n\right) \,\&\, c\left(x, Cl'^{\leq}_n\right)\geq 1-b\right\}$$

$$-\left\{x \mid x^{+} \notin D_P^{-}(x), x \in \overline{P}_b\left(Cl_n^{\le}\right) \& c\left(x, Cl_n'^{\le}\right) < 1-b\right\}$$

2.Delete an object x^{-} , $U' = U - \{x^{-}\}$.

Proposition 13 If $n < n'$ and $Cl_n'^{\le} = Cl_n^{\le}$,

$$\underline{P}_b\left(Cl_n'^{\le}\right) = \underline{P}_b\left(Cl_n^{\le}\right) - \left\{x^{-} \mid x^{-} \in \underline{P}_b\left(Cl_n^{\le}\right)\right\}$$

Proposition 14 If $n \ge n'$ and $Cl_n'^{\le} = Cl_n^{\le} - \{x^{-}\}$,

$$\underline{P}_b\left(Cl_n'^{\le}\right) = \underline{P}_b\left(Cl_n^{\le}\right) - \left\{x^{-} \mid x^{-} \in \underline{P}_b\left(Cl_n^{\le}\right)\right\}$$

$$-\left\{x \mid x^{-} \in D_P^{-}(x), x \in \underline{P}_b\left(Cl_n^{\le}\right) \& c\left(x, Cl_n'^{\le}\right) < b\right\}$$

$$\cup\left\{x \mid x^{-} \notin D_P^{-}(x), x \notin \underline{P}_b\left(Cl_n^{\le}\right) \& c\left(x, Cl_n'^{\le}\right) \ge b\right\}$$

Proposition 15 If $n < n'$ and $Cl_n'^{\le} = Cl_n^{\le}$,

$$\overline{P}_b\left(Cl_n'^{\le}\right) = \overline{P}_b\left(Cl_n^{\ge}\right) - \left\{x^{-} \mid x^{-} \in \overline{P}_b\left(Cl_n^{\le}\right)\right\}$$

Proposition 16 If $n \ge n'$ and $Cl_n'^{\le} = Cl_n^{\le} - \{x^{-}\}$,

$$\overline{P}_b\left(Cl_n'^{\le}\right) = \overline{P}_b\left(Cl_n^{\le}\right) - \left\{x^{-} \mid x^{-} \in \overline{P}_b\left(Cl_n^{\le}\right)\right\}$$

$$-\left\{x \mid x^{-} \in D_P^{-}(x), x \in \overline{P}_b\left(Cl_n^{\le}\right) \& c\left(x, Cl_n'^{\le}\right) < 1-b\right\}$$

$$\cup\left\{x \mid x^{+} \notin D_P^{-}(x), x \notin \overline{P}_b\left(Cl_n^{\le}\right) \& c\left(x, Cl_n'^{\le}\right) \ge 1-b\right\}$$

Since the proof of Proposition3-16 is similar to those of Proposition 1 or 2, we omit them.

4. Numerical Examples

The Dynamic information systems is shown as Table 1. Since the calculation processes are similar, here we only show how to update the upper and lower approximation of $Cl_2^{\ge}$ and $Cl_2^{\le}$. Firstly, the dominance and dominated classes are:

Table 1. Dynamic information systems

U	a_1	a_2	d	U	a_1	a_2	d	U	a_1	a_2	d
x_1	50	75	2	x_1	50	75	2	x_1	50	75	2
x_2	65	50	1	x_2	65	50	1	x_2	65	50	1
x_3	70	75	1	x_3	70	75	1	x_3	70	75	1
x_4	50	60	1	x_4	50	60	1	~~x_4~~	~~50~~	~~60~~	~~1~~
x_5	80	90	2	x_5	80	90	2	x_5	80	90	2
x_6	90	80	3	x_6	90	80	3	x_6	90	80	3
x_7	80	80	3	x_7	80	80	3	x_7	80	80	3
x_8	90	90	3	x_8	90	90	3	x_8	90	90	3
				x_9	90	80	2				

$D_P^{+}(x_1) = \{x_1, x_3, x_5, x_6, x_7, x_8\}$, $D_P^{-}(x_1) = \{x_1, x_4\}$;

$D_P^{+}(x_2) = \{x_2, x_3, x_5, x_6, x_7, x_8\}$, $D_P^{-}(x_2) = \{x_2\}$;

$D_P^{+}(x_3) = \{x_3, x_5, x_6, x_7, x_8\}$, $D_P^{-}(x_3) = \{x_1, x_2, x_3, x_4\}$;

$D_P^{+}(x_4) = \{x_1, x_3, x_4, x_5, x_6, x_7, x_8\}$, $D_P^{-}(x_4) = \{x_4\}$;

$D_P^{+}(x_5) = \{x_5, x_8\}$, $D_P^{-}(x_5) = \{x_1, x_2, x_3, x_4, x_5, x_7\}$;

$D_P^{+}(x_6) = \{x_6, x_8\}$, $D_P^{-}(x_6) = \{x_1, x_2, x_3, x_4, x_6, x_7\}$;

$D_P^{+}(x_7) = \{x_5, x_6, x_7, x_8\}$, $D_P^{-}(x_7) = \{x_1, x_2, x_3, x_4, x_7\}$;

$D_P^{+}(x_8) = \{x_8\}$, $D_P^{-}(x_8) = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$

Assuming 1<2<3,the decision classes are:

$Cl_1 = \{x_2, x_3, x_4\}$, $Cl_2 = \{x_1, x_5\}$, $Cl_3 = \{x_6, x_7, x_8\}$

And the upward union and downward union are:

$Cl_2^{\ge} = Cl_2 \cup Cl_3$, $Cl_2^{\le} = Cl_1 \cup Cl_2$.

Let $b = 0.6$,

$$c\left(x_1, Cl_2^{\ge}\right) = \frac{\left|D_P^{+}(x_1) \cap Cl_2^{\ge}\right|}{\left|Cl_2^{\ge}\right|} = 1, c\left(x_2, Cl_2^{\ge}\right) = \frac{4}{5},$$

$$c\left(x_3, Cl_2^{\ge}\right) = \frac{4}{5}, c\left(x_4, Cl_2^{\ge}\right) = 1,$$

$$c\left(x_5, Cl_2^{\ge}\right) = \frac{2}{5}, c\left(x_6, Cl_2^{\ge}\right) = \frac{2}{5}, c\left(x_7, Cl_2^{\ge}\right) = \frac{4}{5},$$

$$c\left(x_8, Cl_2^{\ge}\right) = \frac{1}{5}.$$

The calculation of $c\left(x_i, Cl_2^{\le}\right)$ is similar to the above process, so we omit it.

b -upper/lower approximation of upward/downward union are:

$\underline{P}_b\left(Cl_2^{\ge}\right) = \{x_1, x_2, x_3, x_4, x_7\}$,

$\overline{P}_b\left(Cl_2^{\ge}\right) = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$

$\underline{P}_b\left(Cl_2^{\le}\right) = \{x_3, x_5, x_6, x_7, x_8\}$,

$\overline{P}_b\left(Cl_2^{\le}\right) = \{x_1, x_3, x_5, x_6, x_7, x_8\}$.

Case1. Insert object x_9. Because $x_9 \in Cl_2$,we have $Cl_2'^{\geq} = Cl_2^{\geq} \cup \{x_9\}, Cl_2'^{\leq} = Cl_2^{\leq} \cup \{x_9\}$

The dominance and dominated classes of x_9 are:

$D_P'^{+}(x_9) = \{x_6, x_8, x_9\}$,

$D_P'^{-}(x_9) = \{x_1, x_2, x_3, x_4, x_6, x_7, x_9\}$.

The updated dominance and dominated set are:

$D_P'^{+}(x_1) = \{x_1, x_3, x_5, x_6, x_7, x_8, x_9\}$, $D_P'^{-}(x_1) = \{x_1, x_4\}$;

$D_P'^{+}(x_2) = \{x_2, x_3, x_5, x_6, x_7, x_8, x_9\}$, $D_P'^{-}(x_2) = \{x_2\}$;

$D_P'^{+}(x_3) = \{x_3, x_5, x_6, x_7, x_8, x_9\}$, $D_P'^{-}(x_3) = \{x_1, x_2, x_3, x_4\}$;

$D_P'^{+}(x_4) = \{x_1, x_3, x_4, x_5, x_6, x_7, x_8, x_9\}$, $D_P'^{-}(x_4) = \{x_4\}$;

$D_P'^{+}(x_5) = \{x_5, x_8\}$, $D_P'^{-}(x_5) = \{x_1, x_2, x_3, x_4, x_5, x_7\}$;

$D_P'^{+}(x_6) = \{x_6, x_8, x_9\}$,

$D_P'^{-}(x_6) = \{x_1, x_2, x_3, x_4, x_6, x_7, x_9\}$;

$D_P'^{+}(x_7) = \{x_5, x_6, x_7, x_8, x_9\}$,

$D_P'^{-}(x_7) = \{x_1, x_2, x_3, x_4, x_7\}$;

$D_P'^{+}(x_8) = \{x_8\}$,

$D_P'^{-}(x_8) = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\}$.

According to Proposition 2,4,10,12, the updated b -upper/lower approximation of upward/downward union are：

$\underline{P}_b(Cl_2'^{\geq}) = \underline{P}_b(Cl_2^{\geq}) = \{x_1, x_2, x_3, x_4, x_7\}$

$\overline{P}_b(Cl_2'^{\geq}) = \overline{P}_b(Cl_2^{\geq}) \cup \{x_9\}$

$= \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_9\}$

$\underline{P}_b(Cl_2'^{\leq}) = \underline{P}_b(Cl_2^{\leq}) \cup \{x_9\}$

$= \{x_3, x_5, x_6, x_7, x_8, x_9\}$

$\overline{P}_b(Cl_2'^{\leq}) = \overline{P}_b(Cl_2^{\leq}) \cup \{x_9\} - \{x_1\}$

$= \{x_3, x_5, x_6, x_7, x_8, x_9\}$.

Case 2. Delete object x_4: because $x_4 \in Cl_1, Cl_2'^{\geq} = Cl_2^{\geq}$, $Cl_2'^{\leq} = Cl_2^{\leq} - \{x_4\}$.

Firstly, we updated dominance classes. Secondly, according to Proposition 5,7,13,15 the updated b -upper/lower approximation of upward/downward union are:

$\underline{P}_b(Cl_2'^{\geq}) = \underline{P}_b(Cl_2^{\geq}) - \{x_4\} = \{x_1, x_2, x_3, x_7\}$

$\overline{P}_b(Cl_2'^{\geq}) = \overline{P}_b(Cl_2^{\geq}) - \{x_4\} = \{x_1, x_2, x_3, x_5, x_6, x_7\}$

$\underline{P}_b(Cl_2'^{\leq}) = \underline{P}_b(Cl_2^{\leq}) = \{x_3, x_5, x_6, x_7, x_8\}$

$\overline{P}_b(Cl_2'^{\leq}) = \overline{P}_b(Cl_2^{\leq}) - \{x_1\} = \{x_3, x_5, x_6, x_7, x_8\}$.

Take $\underline{P}_b(Cl_2'^{\geq})$ and $\overline{P}_b(Cl_2'^{\geq})$ as an example. When adding/deleting an object, the computational complexity of traditional tough set method is $O(mn^2)$, which can be reduced to $O(mn)$ using the proposed incremental method. Therefore, the proposed method is obviously more efficient.

5. Conclusions

In dynamic environment, information is constantly updated, and how to effectively deal with this kind of information system is an important topic. In this paper, we proposed an incremental approach for updating the approximations of VPRS model based on dominance relations under the variation of the object set. We gave detailed theoretical results with proofs and a numerical example to support our incremental method. One of our future work is to conduct some experiments with real datasets and consider the variations of attribute sets.

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