# Optimal Marketing Strategy for Two Parallel Flights Competitive with Strategic Passengers 

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#### Abstract

A case study is done on two competing airline companies operating the same airline. This paper analyzes the sales strategy of airline ticket in competitive market. Based on strategic behavior, we established a two parallel flights competition game model, and discussed the effect of various factors on expected revenue. Then analyzed the impact of the purchase preferences on airline revenue. The results show that the market has only normal sales period and the ticket price is inequality for different purchase preferences. We also find that when the two airline companies are local monopolies, the part-covered sale strategy slightly outperforms the whole-covered sale strategy.


Keywords: Transportation economy; Strategic passenger behavior; Game; Competition

## 1. Introduction

With the rapid development of civil aviation enterprises in recent years, the competition between airlines is becoming more and more intense. Therefore, how to make the best sales strategy is of great significance for improving airline revenue in the fierce market competition. To achieve the goal of improve airline revenue, airlines tend to adopt the price to dynamically adjust the market demand in traditional theory of revenue management. Jiang et al(2014)prove the result that partition-restricted strategy and bidding price strategy can be close to the optimal through using DLP model, PNLP model and RLP model. They show that even if the probability of upgrading purchase is small, the yield also increased significantly[1]. Currie et al (2008)study the problem of dynamic pricing of two flights. the equilibrium solution is obtained by setting the pricing decision into an optimal control problem, giving the condition of existence of unique Nash equilibrium, and using the variational method[2]. Li et al (2009)study the dynamic pricing of airlines in competitive environment. In the paper, taking the two airlines of the same route as the research object[3]. Lin et al(2005)by using the game theory to discuss the real-time dynamic pricing competition of two airlines. The expected benefits of the two airlines are analyzed in both real-time inventory transparency and opacity, and the Nash equilibrium is obtained under real-time inventory transparency[4]. Luo et al(2010)study the dynamic pricing decision of two parallel flights in a competitive market. They show that the benefits can be raised by joint dynamic pricing[5].

None of the above studies considered the passenger's strategic behavior. In reality, due to the rapid development of "Internet + ", the Internet and related App application software has been deeply rooted, many customers will be form the rational price of the future ticket price based on the current price and other relevant information, in order to determine the purchase time for maximizing the utility of the ticket. Therefore, their purchase behavior more and more strategic. At present, the airline sales strategy research about strategic passengers focused on how to combine price and response mechanisms to guide market demand and get the maximum expected revenue. For example, see [6-8]. Anderson et al(2003)consider the impact of the passenger's delaying behavior on the airline's revenue, noting that the passenger's strategic behavior should be taken into account in the dynamic pricing decision [9]. Yan et al(2015)study the dynamic pricing of airfare based on passenger's strategic behavior and introduce PM and DPM strategies in dynamic pricing in the paper. They show that both strategies can effectively mitigate the strategic behavior and improve the profit[10]. Levin et al(2009)assume that the airlines and passengers have strategic behavior, prove that the existence of a perfect balance of sub-game, and get a balanced optimal conditions through the structure the dynamic game model between the companies and between the company and passengers, and Point out that if neglecting the passenger's strategic behavior when making the dynamic pricing decision, it will have a significant impact on the airline revenue[11]. Peng et al(2011)study the multi-period dynamic pricing problem of airlines considering strategic passengers. By constructing a stochastic optimization
model, they discuss the relationship when the airline sets the optimal price[12]. Bi et al(2014)study the dynamic pricing problem based on strategic passenger and duopoly competition environment. They analyze the Nash equilibrium in the presence of strategic behavior by constructing the two-period dynamic pricing model, and point out that it is possible to make the two airlines get the best income by properly adjusting the different coefficients of the ticket[13].
But the above models do not take into account passenger preferences. In fact, passengers have a purchase preference, they may like an airline's unique service or tickets, such as [14]. In this paper, the Hotelling model is used to describe the preference of passengers to airlines, and analyzes the impact of optimal pricing, sales scope of airlines and purchase intention on airline revenue, in order to provide guidance for airlines to increase revenue in the fierce market competition.
The remainder of the paper is organized as follows. In the next section, we formulate our model and its associated dynamic programming value function, moreover, we consider the different strategies of the airline. The impact of purchase intention on airline revenue is discussed in Section 3. We conclude with a brief discussion of the results, as well as directions for future research in Section 4.

## 2. Model Descriptions

Two competing airlines, A and B , each hold a quantity $\frac{N}{2}$ of inventory. We assume that there is no vertical differentiation between tickets of the two airlines, i.e., one ticket is not inherently superior to the other. This is a simplification of reality (in practice direct flights offered by one airline might be sold along with indirect flights offered by the other airline). The entire selling horizon is divided into two periods: the regular sales period (sold at normal price) and the price reduction period (sold at discounted price). The airline maximizes revenue by setting a price.
There are $J$ passengers on the market. Each passenger has a valuation $V$ for the ticket and purchases at most one unit, they have strategic behavior. Since the airlines sell tickets over two periods, possibly at di erent prices, the passengers strategically time their purchases based on their valuation, inventory availability, and the airlines' pricing strategies, in order to maximize their utility. In other words, a passenger might decide to purchase tickets in the price reduction period rather than the regular sales period, based on his expectation of prices and availability in the price reduction period.
Passengers have di erent preferences between airlines. The reason might be loyalty to the airline, preference for a brand or simply an established relationship with the company. This paper uses the Hotelling model to charac-
terize passenger preferences for airlines. We assume that the two competing airlines A and B are located at each end of a Hotelling line of length 1 and a continuum of passengers is spread on the horizontal line over the interval $[0,1]$ with uniform density. A population of $J$ passengers is spread uniformly over the entire line. The greater the value of any point on the Hotelling line, the greater the preference for airline A. The utility of the passengers depends on two parts: the price of the ticket and the passenger's preference for the airline. The ticket price is determined by the market equilibrium endogenously. The brand preference of every passenger is completely characterized by his location $x \in[0,1]$ on the line. Thus, although all the passengers have the same valuation $V$ for the ticket, they have varying preferences towards the competing airlines, which influences the utility a passenger derives when he purchases a ticket from an airline. Figure 1 captures the Hotelling game distribution of airlines and strategy passengers in our model.


Figure 1. The Hotelling Game Distribution of Airlines and Strategy Passengers

The following notation is used throughout the paper. $p_{i}$ : the price of the airline $i(i=\mathrm{A}, \mathrm{B})$.
$t$ : the strength of brand preference in the market, $t>0$.A passenger at x incurs a disutility $t x$ when buying a ticket from airline A and a disutility $t(1-x)$ when buying a ticket from airline B.
$\pi_{i}$ : the total revenue of the airline $i$.
$\theta$ : the utility discount for the passenger of buying a ticket on the price reduction period, which can be interpreted as degree of strategy. Indeed, the value $\theta=0$ means that the passenger completely disregards the possibility of purchasing on the price reduction period. In contrast, the value $\theta=1$ means that the passenger values the current purchase the same as a purchase on the price reduction period and will exhibit fully strategic behavior. It is similar to Liu et al(2013)[15] and Levin et al(2010)[16].
Let us formally make some assumption for our model.
The information is complete, that is, airlines and passengers can clearly know all the information on the market. A rational expectation equilibrium, i.e., the expected value matches the actual value. Because all passengers have the same expectations for the behavior of other passengers and airlines, and this assumption makes all participants' expectations consistent with the equilibrium results. Sriram[6] and Jerath[14] have adopted such assumptions. The parameter $\frac{v}{t}$ denotes the purchase intention of the passenger, which reflects the degree of competition between the two airlines. With the increase in the purchase
intention, the market has become increasingly competitive. Assuming $\frac{V}{t} \geq \frac{1}{2}$, so that when the airline ticket is priced at zero, the utility of the passenger is not negative.

## 3. Equilibrium Analysis

According to the traditional theory of revenue management, the airlines face the mutual choice of the maximization of the revenue and the ratio of the passenger to the seat. Thus, in the Hotelling model, the sales strategy of the airlines in the two demand states has the following possibilities:
In the case of low demand, airlines have the ability to meet the needs of the entire market, and will leave some tickets, e.g., $N>J$.In this case there are two kinds of market conditions: First, the regional monopoly market, that is, the passenger on the left side of $x_{A}$ on the Hotelling line buys a ticket from airline A , the passenger on the right side of $x_{B}$ buys the ticket from airline B , and the passenger between $x_{A}$ and $x_{B}$ does not purchase, where $x_{A}$ and $x_{B}$ represent the locations of the farthest passengers who bought tickets from firms A and B , respectively, on the Hotelling line. Second, the competitive market, that is, the passengers on the right side of $x_{B}$ may be purchased from airline A , and the passengers on the left side of $x_{A}$ may also purchase from airline B.
In the case of high demand, airline cannot meet the needs of the entire market, i.e., $N<J$, but can be sold all the tickets. In this case there are also two kinds of market conditions: First, the demand-contented regional monopoly market, that is, the maximum demand just equals the market supply. In this case, the airlines can choose one of the following two strategies for selling ticket: the part-covered sale strategy, namely, the airline sells part of the ticket at high prices; the whole-covered sale strategy, namely, the airline sells all the tickets at low prices. Second, the out-of-stock regional monopoly market. In this case, the airlines have limited capacity, so that the airlines are unable to cover the full market.
In the paper, we consider the decision of airline $A$ in detail, and the analysis will be identical for airline B. Thus airline B is no longer repeated. Let $x_{i}^{1}$ denotes the optimal sales range for airline $i$ in the regular sales period and the same net utility point of the two sales cycles for the passenger. The parameter $x_{i}^{2}$ denotes the optimal sales range for airline $i$ in the price reduction period. Airlines A sets revenue maximizing prices $p_{A}^{1}$ in the regular sales period and accrue profits $\pi_{A}^{1}=p_{A}^{1} x_{A}^{1} J$, and sets revenue maximizing prices $p_{A}^{2}$ in the price reduction period and accrue profits $\pi_{A}^{2}=p_{A}^{2}\left(x_{A}^{2}-x_{A}^{1}\right) J$.

Lemma : When the passenger has a strategic behavior, there is only the regular sales period in any market condition.
Proof: The expected total revenue for the two sales cycles for airline A is:

$$
\begin{equation*}
\pi_{A}=\left[p_{A}^{1} x_{A}^{1}+p_{A}^{2}\left(x_{A}^{2}-x_{A}^{1}\right)\right] J \tag{1}
\end{equation*}
$$

Derive the equation (1) to obtain:

$$
\begin{equation*}
\frac{\partial \pi_{A}^{A}}{\partial x_{A}^{1}}=p_{A}^{1}-p_{A}^{2} \geq 0 \tag{2}
\end{equation*}
$$

From (2) we can see that $\pi_{A}$ is a power multiplication function with respect to $x_{A}^{1}$. Thus, with the increase of $x_{A}^{1}$, airline A revenue increases, that is, when $x_{A}^{1}=x_{A}^{2}$, the value of $\pi_{A}$ is the largest. Therefore, there is only the regular sales period in any market condition.
This completes the proof.
Lemma 1 shows that when there is only a regular sales period and the passenger has strategic behavior and preference, the airline will gain the maximum revenue. In the later section, let $x_{A}$ denotes the scope of market sales. Next, this paper analyzes the equilibrium state in the market and the scope of sales under the regional monopoly.

### 3.1. Low Demand ( $N>J$ )

In this case, the demand is less than the supply, all passengers can purchase tickets and the net utility is $V-p_{A}-t x_{A}$. Theorem 1. When the supply exceeds demand, the equilibrium results under different purchase intentions are shown in Table 1.

Table 1. The Equilibrium State of the Market when the Supply Exceeds Demand

| purchase <br> intentions <br> $\frac{V}{t}$ | Prices <br> $P_{A}=P_{B}$ | Market Coverage <br> $x_{A}=1-x_{B}$ | Profits <br> $\pi_{A}=\pi_{B}$ |
| :---: | :---: | :---: | :---: |
| $\left[\frac{1}{2}, 1\right)$ | $\frac{V}{2}$ | $\frac{V}{2 t}$ | $\frac{J V^{2}}{4 t}$ |
| $\left[1, \frac{3}{2}\right)$ | $V-\frac{t}{2}$ | $\frac{1}{2}$ | $\left(V-\frac{t}{2}\right) \frac{J}{2}$ |
| $\left[\frac{3}{2},+\infty\right)$ | $t$ | $\frac{1}{2}$ | $\frac{J t}{2}$ |

Proof: First, consider the case in which the market is in the regional monopoly. In the case, the airlines are acting as local monopolies. We consider the decision of airline A in detail, and the analysis will be identical for airline B. In this section, there are two sales strategies, that is, the part-covered sale strategy and the whole-covered sale strategy. we discuss the expected revenue under different market coverage and compare them to get the optimal sales strategy.

The part-covered sale strategy: If airline A chooses the price $p_{A}$, the right-most passenger to buy from the airline will be at $x_{A}$ such that $V-p_{A}-t x_{A}=0$, i.e., the utility of the passenger at $x_{A}$ is zero. By the boundary condition, the optimal price be $p_{A}^{*}=V-t x_{A}$, and the demand be $x_{A} J$. Thus, the profit for the airline will be $\pi_{A}=p_{A} x_{A} J=\left(V-t x_{A}\right) x_{A} J$. Then, let $\frac{\partial \pi_{A}}{\partial x_{A}}=0$, which get the optimal market coverage be $x_{A}^{*}=\frac{V}{2 t}$. Then the maximized profit is given by $\pi_{A}^{*}=\frac{J V^{2}}{4 t}$. However, to ensure that the airlines are local monopolies, we need to ensure that at the optimum $x_{A}<\frac{1}{2}$, which yields $\frac{V}{t}<1$.
The whole-covered sale strategy: In the case, the market coverage of airline A be $x_{A}=\frac{1}{2}$. Similarly, by the boundary condition, the optimal price be $p_{A}=V-\frac{t}{2}$. Then the maximized profit is given by $\pi_{A}^{\prime}=\left(\frac{V}{2}-\frac{t}{4}\right) J$.
Now, comparing the profits of the two strategies:

$$
\begin{equation*}
\frac{1}{t}\left(\pi_{A}-\pi_{A}^{\prime}\right)=\frac{t}{4}\left(\frac{V^{2}}{t^{2}}\right)-\frac{J}{2}\left(\frac{V}{t}\right)+\frac{J}{4} \tag{3}
\end{equation*}
$$

From (3) we can see that the function value is not negative. Therefore, in the case of regional monopoly market, the airline can get greater benefits when chooses the partcovered sale strategy.
When $\frac{V}{t} \geq 1$, the above equilibrium does not hold, since the airlines are not local monopolies (the optimal coverage for each airline will be $>\frac{1}{2}$ ). We propose that for $1 \leq \frac{V}{t}<\frac{3}{2}$ both airlines charge prices $p_{A}^{*}=p_{B}^{*}=V-\frac{t}{2}$ in equilibrium, cover half the market and make profits $\pi_{A}^{*}=\pi_{B}^{*}=\left(V-\frac{t}{2}\right) \frac{J}{2}$. We now show that neither airline wants to deviate unilaterally from this equilibrium. Suppose airline A raises its price slightly and charges $p_{A}^{+}=V-\frac{t}{2}+\varepsilon t$ where $\varepsilon>0$, while airline B still charges $p_{B}=V-\frac{t}{2}$. Then, airline A covers $x_{A}=\frac{1}{2}-\varepsilon$ and makes a profit $\pi_{A}^{+}=\left(\frac{1}{2}-\varepsilon\right)\left(V-\frac{t}{2}+\varepsilon t\right) J$. However, under the condition $\frac{V}{t} \geq 1$, this profit is lower than the equilibrium profit, so that the airline does not have an incentive to raise its price above the equilibrium price. Now, consider the case in which the airline lowers its price slightly and
charges $p_{A}^{-}=V-\frac{t}{2}-\varepsilon t$. The point $x$ at which the indifferent passenger is located is then found by solving the condition $V-p_{A}-t \bar{x}=V-p_{B}-t(1-\bar{x})$, which yields $\bar{x}=\frac{1+\varepsilon}{2}$, and the profit for airline A is given by $\pi_{A}^{-}=\frac{1}{2}(1+\varepsilon)\left(V-\frac{t}{2}-\varepsilon t\right) \frac{J}{2}$. However, under the condition $\frac{V}{t}<\frac{3}{2}$, this profit is always lower than the equilibrium profit, so that the airline does not have an incentive to lower its price below the equilibrium price. Hence, the equilibrium proposed above is indeed an equilibrium for the range $1 \leq \frac{V}{t}<\frac{3}{2}$.
Now consider the case in which the two airlines are in direct competition. Assume that the indi erent passenger is located at $x$. Since this passenger is indi erent to buying from A or B , the following condition holds for him: $V-p_{A}-t \bar{x}=V-p_{B}-t(1-\bar{x})$, which gives $\bar{x}=\frac{1}{2}+\frac{p_{B}-p_{A}}{2 t}$. The profits for airlines A and B are given, respectively, by $\pi_{A}=p_{A} \bar{x} J$ and $\pi_{B}=p_{B}(1-\bar{x}) J$. Maximizing the profits jointly, we obtain $p_{A}^{*}=p_{B}^{*}=t$, $\bar{x}=\frac{1}{2}$ and $\pi_{A}^{*}=\pi_{B}^{*}=\frac{J t}{2}$. Under our assumption that the outside utility of a passenger is zero, we need to ensure that $V-p_{A}-t \bar{x}=V-p_{B}-t(1-\bar{x}) \geq 0$, which gives the condition $\frac{V}{t} \geq \frac{3}{2}$.
This completes the proof.

### 3.2. High Demand ( $N<J$ )

In this case, the supply is less than the demand, since each airline has only $\frac{N}{2}$ unit tickets, so it is impossible to cover the entire market, i.e., $x_{A}=x_{B}=\frac{N}{2 J}<\frac{1}{2}$.
Theorem 2. When the supply is less than the demand, the equilibrium results under different purchase intentions are shown in Table 2.

Table 2. The Equilibrium State of the Market when the Supply is Less than the Demand

| purchase <br> intentions <br> $\frac{V}{t}$ | Prices <br> $P_{A}=P_{B}$ | Market Coverage <br> $x_{A}=1-x_{B}$ | Profits <br> $\pi_{A}=\pi_{B}$ |
| :---: | :---: | :---: | :---: |
| $\left[\frac{1}{2}, \frac{N}{J}\right)$ | $\frac{V}{2}$ | $\frac{V}{2 t}$ | $\frac{J V^{2}}{4 t}$ |


| $\left[\frac{N}{J},+\infty\right)$ | $V-t \frac{N}{2 J}$ | $\frac{N}{2 J}$ | $\left(V-\frac{N t}{2 J}\right) \frac{N}{2}$ |
| :---: | :---: | :---: | :---: |

Proof: In the case, we analyze the high demand case. The total capacity of the two airlines $(N)$ is less than the total demand ( $J$ ) and airlines act as local monopolies. Again, we consider airline A and the analysis is identical for airline $B$.
The demand-contented regional monopoly market. Similar to the above proof, there are two sales strategies, that is, the part-covered sale strategy and the whole-covered sale strategy. We discuss the expected revenue under different market coverage and compare them to get the optimal sales strategy.
The part-covered sale strategy: Let $x_{A}$ be the position where the net utility value of the passenger is zero from airline $A$. the right-most passenger to buy from the airline will be at $x_{A}$ such that $V-p_{A}-t x_{A}=0$. The price charged by the airline to all passengers will then be $p_{A}^{*}=V-t x_{A}$, and the demand will be $x_{A} J$. Thus, the profit for the airline will be $\pi_{A}=p_{A} x_{A} J=\left(V-t x_{A}\right) x_{A} J$. Then, let $\frac{\partial \pi_{A}}{\partial x_{A}}=0$, which get the optimal market coverage be $x_{A}^{*}=\frac{V}{2 t}$. Then the maximized profit is given by $\pi_{A}^{*}=\frac{J V^{2}}{4 t}$. However, to ensure that the airlines do not stock out, we need to ensure that at the optimum $x_{A} \leq \frac{N}{2 J}$, which gives $\frac{V}{t} \leq \frac{N}{J}$.
The whole-covered sale strategy: In the case, the market coverage of airline A be $x_{A}=\frac{N}{2 J}$. Similarly, by the boundary condition, the optimal price be $p_{A}=V-\frac{N t}{2 J}$. Then the maximized profit is given by $\pi_{A}^{\prime}=\left(\frac{V}{2}-\frac{N t}{4 J}\right) N$.
Now, comparing the profits of the two strategies:

$$
\begin{equation*}
\left.\frac{1}{t}\left(\pi_{A}-\pi_{A}\right)=\frac{J}{4} \frac{V^{2}}{t^{2}}\right)-\frac{N}{2}\left(\frac{V}{t}\right)+\frac{N^{2}}{4 J} \tag{4}
\end{equation*}
$$

From (4) we can see that the function value is not negative. Therefore, in the case of the demand-contented regional monopoly market, the airline can get greater benefits when chooses the part-covered sale strategy.
The out-of-stock regional monopoly market: For $\frac{V}{t}>\frac{N}{J}$ each airline will charge the price $p_{A}^{*}=p_{B}^{*}=V-\frac{N t}{2 J}$, cover $x_{A}^{*}=1-x_{B}=\frac{N}{2 J}$ and make profits $\pi_{A}^{*}=\pi_{B}^{*}=\left(V-\frac{N t}{2 J}\right) \frac{N}{2}$. Note that the airline cannot lower its price below this level, since it does not have the capacity to serve the expanded market. It can be easily shown, using an $\varepsilon^{\varepsilon}$-deviation
argument as in the proof of theorem 1, that the airline does not have an incentive to lower its price below this level.
This completes the proof.
From and can be obtained directly below the corollary:
Corollary In both cases of the regional monopoly when the supply exceeds demand and the demand-contented regional monopoly when the supply is less than the demand, the airline can obtain more revenues by taking the part-covered sale strategy.
Next, we present a numerical experiment to illustrate the above results. We tested sets of examples with $J=1$, $t=1$. For $N<J$ the total capacity of the two airlines be $N=0.8$. The results are shown in Fig. 2 and Fig.3.


Figure 2. The Revenue Comparison of two Strategies for Over-supply


Figure 3. The Revenue Comparison of two Strategies for Less-supply

As can be seen from Figure 2 and Figure 3: In the market whether the regional the monopoly of over-supply or the demand-contented regional monopoly of less-supply, the airline can get greater profits by taking the part-covered sale strategy, that is, using a higher ticket price to replace the ratio of the passenger to the seat. The corollary is verified. At the same time, the numerical experiment shows that the difference between the expected revenues of the two sales strategies increases as the purchase intention decreases.
Through the above analysis we can see, for $\frac{V}{t} \in\left[\frac{1}{2}, 1\right)$ or $\frac{V}{t} \in\left[\frac{1}{2}, \frac{N}{J}\right)$, the airline should take the part-covered sale
strategy; for $\frac{V}{t} \in[1, \infty)$ or $\frac{V}{t} \in\left[\frac{N}{J},+\infty\right)$, the airline should take the whole-covered sale strategy.

## 4. Purchase Intention Analysis

As can be seen from Table 1 and Table 2, the purchase intention of the passenger has an important effect on the profit. In this section, we analyze the relationship between purchase intention and profit in detail.
Theorem 3. In the case of oversupply, when $\frac{V}{t} \in\left[\frac{1}{2}, \frac{3}{2}\right)$, the optimal revenue of the airline is proportional to $\frac{v}{t}$; when $\frac{V}{t} \in\left[\frac{3}{2},+\infty\right)$, the optimal revenue of the airline has nothing to do with $\frac{V}{t}$.
Proof: For $\frac{1}{2} \leq \frac{V}{t}<1$, from the derivative of $\frac{V}{t}$ for the maximum expected yield function in the proof of theorem 1:

$$
\begin{equation*}
\frac{\partial \pi_{A}}{\partial \frac{v}{t}}=\frac{J V}{4}>0 \tag{5}
\end{equation*}
$$

For $1<\frac{V}{t}<\frac{3}{2}$, from the derivative of $\frac{V}{t}$ for the maximum expected yield function in the proof of theorem 1:

$$
\begin{equation*}
\frac{\partial \pi_{A}}{\partial \frac{V}{t}}=\frac{V t}{4\left(\frac{V}{t}\right)^{2}}>0 \tag{6}
\end{equation*}
$$

From Eq. (5) and Eq. (6) we can see, for $\frac{1}{2} \leq \frac{V}{t}<1$ and $1<\frac{V}{t}<\frac{3}{2}$, the optimal revenue of the airline is proportional to $\frac{V}{t}$.
When $\frac{V}{t}=1$, Equations (5) and (6) are equal. Thus, for $\frac{V}{t} \in\left[\frac{1}{2}, \frac{3}{2}\right)$, the expected yield function is continuous, so in this range the value of the revenue function is continuously monotonically increasing. At the same time, from the expected yield function of the competitive market in the proof of theorem 1 we know, the expected revenues of the airline remain unchanged. Therefore, when $\frac{V}{t} \geq \frac{3}{2}$, the optimal revenue of the airline has nothing to do with $\frac{V}{t}$.
This completes the proof.
From theorem 3 we can see that in the case of oversupply, when the market is in a competitive state, the airline can earn the maximum profit.
Theorem 4. In the case of less-supply, the optimal revenue of the airline is proportional to $\frac{V}{t}$.

Proof: Similar to the proof of theorem 3. The above theorem can be obtained by deriving the yield function for the case of less-supply.
Theorem 4 shows that in the case of less-supply, the optimal yield increases as $\frac{v}{t}$ increases. Therefore, the greater the purchase intention of the passengers, the greater the profits the airline earns.

## 5. Conclusions

In this paper, consider the assumption that the passenger has a strategic behavior and discuss the sales strategy of airline ticket. The Hotelling model is used to analysis the sales cycle, we find that no matter what the market in which state, there is only a regular sales period. On this basis, we model this strategic interaction between competing airlines and passengers, and the optimal price is obtained, respectively, for the oversupply and less-supply. Finally, we discuss the impact of passenger purchase intention on airline revenue. The results show that: (i) In the case of regional monopoly and demand-contented regional monopoly, the purchase intention of the passengers is low, and the revenues when the airline takes the part-covered sale strategy are always greater than or equal to the revenues when the airline takes the wholecovered sale strategy. At the same time, the difference between the expected revenues of the two sales strategies increases as the purchase intention decreases. (ii) In the case of competitive market and out-of-stock regional monopoly, the purchase intention of the passengers is high, and the airline can earn more revenue by taking the whole-covered sale strategy. (iii) When the market is in a regional monopoly, the revenue of the airline increases as the purchase intention of the passengers increases; and in the competitive market, the optimal revenue of the airline do not change as the purchase intention of the passengers increases.
We omitted several considerations from the model in order to obtain sharper insights, and these considerations pose several interesting questions for future research. First and foremost, study the marketing strategy when the strategic passenger is a certain percentage. Second, when the market is multi-cycle, discuss the airline's pricing and sales strategy.

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