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Primary Learning Guidance of Linear Elastic Fracture Mechanics

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Abstract: The traditional strength theory ignores the internal defects of the component ,such as porosities, cracks etc. Thus it easily leads to low stress fracture accidents, which has its own very limited. Fracture mechanics is an emerging subject, starting from the actual defect of the substance itself. And it can analyze the actual carrying capacity very well for avoiding accidents. Through studying fracture mechanics, it can be better to guide the engineering practice. However, when facing the complicated mathematical formula derivation and strange elastic mechanics in the process of learning fracture mechanics, many beginners can't understand, let alone use the knowledge of fracture mechanics to guide the engineering practice. As the basis of learning fracture mechanics, linear elastic fracture mechanics is the key. Therefore it is necessary to guide the study of linear elastic fracture mechanics.

Keywords: Fracture Mechanics; Linear Elastic; Guidance

1. Introduction

As civil engineers, when studying fracture mechanics, we should learn how to put fracture mechanics into engineering practice, in order to solve the practical engineering problems. And we shouldn't spend much time in learning the tedious mathematical formula derivation. However, most beginners will focus on it, what is more, some people even will feel giddy when seeing the mathematical formula. Finally they have to give up learning fracture mechanics. They reckon that the difficulty for studying is the mathematics, and who doesn't understand the fracture mechanics is because of its bad math ability. But the real reason is that they don't know how to learn, how to get the entry. Aiming at the beginners' misunderstanding which appears among the process of learning fracture mechanics, this paper is based on the linear elastic fracture mechanics, and provides guidance for beginners to learn it better.

2. The Development of Linear Elastic Fracture Mechanics

In 1913, according to the principle of stress concentration, Inglis reckoned that the actual strength of the material was far below than the theoretical strength. What was the reason was that it existed some drawbacks within the solid materials. In 1920, from the view of the elastic body's energy balance, Griffith researched the problems of the crack propagation of brittle materials by using glass and ceramics. He put forward the energy criterion for the crack propagation of brittle materials, which became the basis for the linear elastic fracture mechanics. In 1948, America scientist Irwin proposed a revision to the Griffith's theory, and introduced the crack's energy release rate, which provided an important criterion for the crack's critical balance. In 1957, according to Westergaard stress function, Irwin solved the tensile stress' problems of the space plate with a penetrating crack. And the concept of stress intensity factor K was formally put forward. The stress intensity factor of crack tip is a criterion for measuring stress field intensity, which exists a critical value denoted as Kcr. For the same kind of material, the critical value of stress intensity factor has a minimum value named (Kcr) min. Then, on the basis of this minimum value ,it forms the notion of fracture toughness written as KI c, KI c=(Kcr) min. And, the fracture toughness is a index to measure the ability of resisting fracture, which relates to the temperature , the type of material, its shape and size. The fracture toughness of various materials can be measured by experiment. Thus, according to the stress intensity factor theory and energy theory as the foundation, linear elastic fracture mechanics is established.

3. The Stress Intensity Factor Theory

3.1. The Basis of Fracture Mechanics

For the complex forms of various cracks, the total can always be decomposed into one or more combined from type I crack, type II crack and type III crack. Type I crack belongs to the opening crack, which is as shown in Figure 1(a); Type II crack belongs to the sliding crack, as shown in Figure 1(b); Type III crack belongs to the tearing crack, as shown in Figure 1(c). Among the three basic types of crack, type I crack is the most common, basic and dangerous, so we regard it as the foundation to discuss the calculation of stress intensity factor.

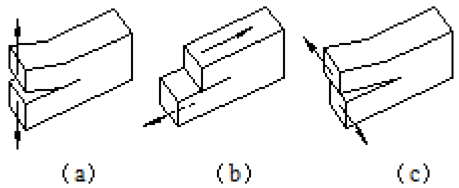

Figure 1. The basic types of crack

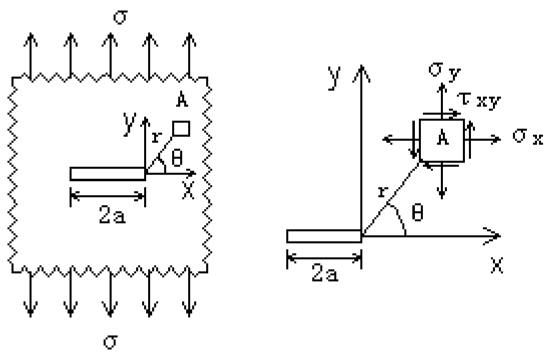

Figure 1. An infinite plate with a center penetrated crack

Figure 2 is an infinite plate with a center penetrated crack, and the crack's length is 2a. And, As shown in Figure 2, take a micro surface "A". And, by using the knowledge of elastic mechanics, it could get the expressions of stress component of the crack tip [1]:

$$\left.\begin{aligned} \sigma_x &= \sigma\sqrt{pa}\left[\frac{1}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)\right] \\ \sigma_y &= \sigma\sqrt{\pi a}\left[\frac{1}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)\right] \\ t_{xy} &= \sigma\sqrt{pa}\left(\frac{1}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2}\right) \end{aligned}\right\} \quad (1)$$

From the expressions of stress component, we can get the message that the coefficient "$s\sqrt{pa}$" is independent of the infinitesimal surface's position ,and it only relates to the load and the crack's width. Thus, the stress intensity factor of type I crack is "$K_I = s\sqrt{pa}$". At the crack tip, namely "r =0", no matter how small the load is, we could know that the stress components tend to infinity from the expressions of stress component. This shows that the crack tip is a singular point, and exists stress singularity. Stress singularity is the case that the stress will tend to infinity when solving the stress function. From the material's own strength, it is impossible to withstand so large a stress. Because, it would appear such a situation that as long as the component with crack, it will inevitably lead to damage of components, which obviously doesn't conform to the actual situation. So stress singularity does not appear in the actual structure. Therefore, the fracture criterion that accords with the actual situation of component should be "$K_I = K_{Ic}$", instead of a traditional design thought "$\sigma_{max} \le [\sigma]$". It means that when the stress intensity factor of crack tip is larger than the fracture toughness of material, the crack will just extend. When considering the impact of the geometry and load form on the stress strength, it introduces a coefficient of correction "y", so the stress intensity factor is "$K_I = ys\sqrt{pa}$". When fracture mechanics applies to analyze the component's strength, it can transform "$K_I = ys\sqrt{pa}$" into "$s = K_{Ic}(y\sqrt{pa})^{-1}$", and then it will calculate the breaking strength.

2.2. Westergaard Stress Function for Stress Component of Type I Crack

According to the elastic mechanics' knowledge, for finding the solution of plane problem, the general idea is divided into two steps. First, find a stress function "$j(x,y)$", and make it meet the compatibility equation and the equilibrium differential equation; And then, take advantage of the boundary conditions to solve the stress function. For calculating the stress intensity factor, it can through the method of complex variable function, and directly utilize the calculated stress function to solve the stress components. Now, it directly gives the expression of Muskhelishvili stress function[2]:

$$U(x,y) = x\mathrm{Re}\,j(z) + y\mathrm{Im}\,j(z) + \mathrm{Re}\,g(z) \quad (2)$$

Remarks: "Re" means calculate the real part of the complex variable function; "Im" means calculate the imaginary part of complex variable function.

The complex variable "Z" can be expressed as "$z = x + iy$" and its conjugate complex variables is "$\bar{z} = x - iy$". Any complex variable function can be written as "$f(z) = \mathrm{Re}\,f(z) + i\mathrm{Im}\,f(z)$", and its conjugate complex variable function is "$\overline{f(z)} = \mathrm{Re}\,f(z) - i\mathrm{Im}\,f(z)$".

$$f(z) + \overline{f(z)} = 2\mathrm{Re}\,f(z) \quad (3)$$

$$\begin{aligned} \bar{z}f(z) &= (zx - iy)f(z) \\ &= (x-iy)[\mathrm{Re}\,f(z) + i\mathrm{Im}\,f(z)] \\ &= [x\mathrm{Re}\,f(z) + y\mathrm{Im}\,f(z)] + i[x\mathrm{Im}\,f(z) - y\mathrm{Re}\,f(z)] \end{aligned}$$

And then $\bar{z}f(z) = \mathrm{Re}[\bar{z}f(z)] + i\mathrm{Im}[\bar{z}f(z)]$

By the real part equal of $\bar{z}f(z)$, we have

$$\mathrm{Re}[\bar{z}f(z)] = x\mathrm{Re}\,f(z) + y\mathrm{Im}\,f(z) \quad (4)$$

It follows from (4) that

$$\mathrm{Re}[\bar{z}j(z)] = x\mathrm{Re}\,j(z) + y\mathrm{Im}\,j(z) \quad (5)$$

Substituting (5) into (2), we have

$$U(x,y) = \mathrm{Re}[\bar{z}j(z)] + \mathrm{Re}\,g(z) \quad (6)$$

If

$$\left.\begin{aligned} j(z) &= \frac{1}{2} Z_{\mathrm{I}}(z) \\ g(z) &= Z_{\mathrm{I}}(z) - \frac{1}{2} z Z_{\mathrm{I}}(z) \end{aligned}\right\} \quad (7)$$

Remarks: “ $Z_{\mathrm{I}}(z)$ ” and “ $Z_{\mathrm{I}}(z)$ ” respectively are an integral and twice integral function of “ $Z_{\mathrm{I}}(z)$ ”.

Substituting (7) into (6), we have

$$U(x,y) = \mathrm{Re}[\frac{1}{2}\bar{z}Z_{\mathrm{I}}(z)] + \mathrm{Re}[Z_{\mathrm{I}} - \frac{1}{2}zZ_{\mathrm{I}}(z)]$$

$$= \mathrm{Re}\,Z_{\mathrm{I}}(z) + \frac{1}{2}\mathrm{Re}\left(\bar{z} - z\right)Z_{\mathrm{I}}(z)$$

$$= \mathrm{Re}\,Z_{\mathrm{I}}(z) + \mathrm{Re}(-iy)Z_{\mathrm{I}}(z)$$

$$= \mathrm{Re}\,Z_{\mathrm{I}}(z) + y\,\mathrm{Im}\,Z_{\mathrm{I}}(z)$$

Then

$$U(x,y) = \mathrm{Re}\,Z_{\mathrm{I}}(z) + y\,\mathrm{Im}\,Z_{\mathrm{I}}(z) \quad (8)$$

According to the elastic mechanics, the expressions of stress components are as follows:

$$\left.\begin{aligned} s_x &= \partial^2 U(x,y)/\partial y^2 \\ s_y &= \partial^2 U(x,y)/\partial x^2 \\ t_{xy} &= -\partial^2 U(x,y)/\partial x\partial y \end{aligned}\right\} \quad (9)$$

From Cauchy - Riemann equation, we have

$$\partial(\mathrm{Re}U)/\partial x = \partial(\mathrm{Im}U)/\partial y$$

$$\partial(\mathrm{Re}U)/\partial y = -\partial(\mathrm{Im}U)/\partial x$$

It is easy to see that

$$s_x = \partial^2 U/\partial y^2$$

$$= \frac{\partial}{\partial y}\left[\frac{\partial}{\partial y}(\mathrm{Re}\,Z_{\mathrm{I}} + y\,\mathrm{Im}\,Z_{\mathrm{I}})\right]$$

$$= \frac{\partial}{\partial y}\left[-\mathrm{Im}\,Z_{\mathrm{I}} + \mathrm{Im}\,Z_{\mathrm{I}} + y\,\mathrm{Re}\,Z_{\mathrm{I}}\right]$$

$$= \frac{\partial}{\partial y}(y\,\mathrm{Re}\,Z_{\mathrm{I}})$$

$$= \mathrm{Re}\,Z_{\mathrm{I}} - y\,\mathrm{Im}\,Z_{\mathrm{I}}'$$

Similarly, we have

$$s_y = \mathrm{Re}\,Z_{\mathrm{I}} + y\,\mathrm{Im}\,Z_{\mathrm{I}}'$$

$$t_{xy} = -y\,\mathrm{Re}\,Z_{\mathrm{I}}'$$

Therefore, we have

$$\left.\begin{aligned} s_x &= \mathrm{Re}\,Z_{\mathrm{I}}(\mathrm{z}) - y\,\mathrm{Im}\,Z_{\mathrm{I}}'(\mathrm{z}) \\ s_y &= \mathrm{Re}\,Z_{\mathrm{I}}(\mathrm{z}) + y\,\mathrm{Im}\,Z_{\mathrm{I}}'(\mathrm{z}) \\ t_{xy} &= -y\,\mathrm{Re}\,Z_{\mathrm{I}}'(\mathrm{z}) \end{aligned}\right\} \quad (10)$$

“ $Z_{\mathrm{I}}(z)$ ” is the famous stress function “Westergaard stress function”. For the general case, it should introduce a constant “A”, whose value will depend on the boundary conditions. Therefore, the general expressions of stress component are as follows[3]:

$$\left.\begin{aligned} s_x &= \mathrm{Re}\,Z_{\mathrm{I}}(\mathrm{z}) - y\,\mathrm{Im}\,Z_{\mathrm{I}}'(\mathrm{z}) + A \\ s_y &= \mathrm{Re}\,Z_{\mathrm{I}}(\mathrm{z}) + y\,\mathrm{Im}\,Z_{\mathrm{I}}'(\mathrm{z}) - A \\ t_{xy} &= -y\,\mathrm{Re}\,Z_{\mathrm{I}}'(\mathrm{z}) \end{aligned}\right\} \quad (11)$$

4. The Energy Theory

In addition to the stress intensity factor theory, the energy theory is another fracture criterion of material, which plays an important role in the development of fracture mechanics. According to the viewpoint of the energy theory, the external work done on an object equals to the increased elastic energy and the surface energy increased from the crack surfaces. The increased surface energy comes from the elastic energy released by an object. In other words, if the released elastic energy is smaller than the increased surface energy, the crack will not be extended. In order to utilize the energy theory to analyze the crack propagation better, it introduces the energy release rate “GI”. As a criterion of crack propagation, “KI” and “GI” has the following relationship[4]:

$$G_{\mathrm{I}} = \frac{1}{E_1}K_{\mathrm{I}}^2$$

Remarks:

$$E_1 = \begin{cases} E & \textit{Thestate of plane stress} \\ \dfrac{E}{1-m_1^2} & \textit{Thestate of plane strain} \end{cases}$$

Thus, as the basic theory of linear elastic fracture mechanics, the stress intensity factor theory and the energy theory are unified.

5. Summary

Fracture mechanics is broad and profound, and this article only introduces some knowledge about linear elastic fracture mechanics. Hope to provide guidance for the beginners to learn fracture mechanics, and cultivate their interest in learning.

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