Geometric Nonlinear Analysis of Cable Stayed Bridge Based on the Effect of Stay Cable Sag

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Abstract: Based on the background of Chongqing Rail Transit Line 6 CaiJia Jialing River Bridge engineering, aiming at higher complexity of statically indeterminate structure for rail cable-stayed bridge, this paper details of the cable-stayed bridge nonlinear problems, and discuss the cable-stayed bridge iterative method of nonlinear analysis; Finally, according to the two models of a linear and only-considering part of the cable sag effect of geometric nonlinearity, we focus on the cable sag effect and calculate the response phase into the bridge structure. The results show that : the track cable-stayed structure under constant loads showed significant nonlinear, it is essential to consider the effect of geometric nonlinearity in the structural analysis and design.

Keywords: Nonlinear; Cable Stayed Bridge; Stay Cable Sag

1. Introduction

Cable-stayed bridge is a flexible high-order statically indeterminate structure ,compared with the traditional structural analysis such as continuous beam or continuous rigid frame structure analysis, even under normal design loads, its section stress will also exhibit strong geometric nonlinearity in the elastic range. With the cable-stayed bridge span's increasing, this kind of nonlinear is increasingly significant. Therefore, we should consider the effect of geometric nonlinearity when we conduct the analysis of cable-stayed bridge.

2. Geometric Nonlinear Analysis Method of Cable-stayed Bridge

Cable-stayed bridge geometric nonlinearity generated mainly in the following three sources: 1) cable-stayed sag effect; 2) coupling beams bending moment and axial force effect; 3) large displacement effects.

2.1. Stayed Cable Sag Effect

Cable-stayed produce sagging under its own gravity, Part of its ends displacement generated by material deformation, the other part is affected by cable sag, all above lead to the nonlinear relation between cable tension and displacement. With the increasing of tensioning force, the sag decrease gradually, and the axial stiffness increase continually. Therefore, when using a straight bar element finite element model, we must consider the impact of the stay cables sag . The method which use the equivalent elastic modulus to consider the sag effect conveniently and effectively has been widely used. It was first proposed by Ernst, and the formula of calculating equivalent elastic modulus is:

$$E_{eq} = E / \left[1 + w^2 L^2 AE / (12T^3) \right] \qquad (1)$$

In this formula: E is the effective elastic modulus of the material of the cable-stayed , L is the horizontal length of stay cables , w is the unit length of stay cables gravity , A is the cross-sectional area of cable-stayed , T is the cable-stayed tensile force .

Formula (1) gives the tangent modulus, when cable force range from T_1 to T_2 ,we need to use formula(2) to calculate the secant modulus of cable :

$$E_{eq} = \frac{E}{1 + w^2 L^2 AE(T_1 + T_2) / (24 T_1^2 T_2^2)} \qquad (2)$$

In the initial iteration, we use the equivalent tangent modulus of elasticity, and then use the equivalent secant modulus of elasticity of the cable to back to the generation of correction on cable unit so that it can improve the calculation accuracy efficiently.

After the elastic modulus of the cable-stayed correction, you can build the cable unit matrix as the straight bar unit under the local coordinate system:

$$k_c = \frac{AE_{eq}}{L_c} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad (3)$$

Typically, such a correction of elastic modulus can meet the accuracy requirements of engineering, but when the cable length is relatively large, we should use catenary element method.

2.2. Effects of Bending Moments and Axial Forces Beams Coupled

In the cable-stayed bridge, because of the cable force, the main beam and the bridge towers and other components are under the combined action of the moment and axial force. These components can also present nonlinear characteristics even in the case of materials meeting the hooke's law. Under the load, the components will produce larger deflection. Because of the exist of axial force, components will have a huge additional bending moment and the additional bending moment will aggravate the deformation of the structure. This is the result of the react of bending moment and axial force.

Processing method of beam-column effect is considering the impact of the geometric stiffness matrix or introducing the concept of stability function, or considering the geometric stiffness matrix and stability function at the same time so that it can implement the Linear computing of nonlinear equations.

2.3. Large Deformation Effect

Under the load, the geometric position of the cable-stayed bridge upper structure changes significantly. From the perspective of finite element, the node coordinates change a lot with the increasing of the load, so do the each unit length Angle and other geometry features, and structural stiffness matrix becomes a function of geometric deformation. Therefore the balance equation is no longer a linear relationship, superposition principle in small deformation assumption is no longer applicable as well. For large deformation effect of cable-stayed bridge , according to the theory of finite displacement, the initial displacement should be considered when calculating the stress and reaction force.

3. The Method of Cable-stayed Bridge Nonlinear Static Analysis and Calculation

Currently, The main solution of geometric nonlinear equations is increment iterative method and incremental-iterative algorithm. In this paper, we use iterative method. iterative method is the method that applies all the entire external loads to the structure at once, figure out the node displacement by the tangent stiffness before deformation, then calculate the structural stiffness according to the deformation, finally figure out the force of the end point. Due to the different structural stiffness before and after deformation, there are some unbalanced load on the node. In order to satisfy the node balance, we take these unbalanced load as node load and apply these to nodes, then calculate the relative displacement of the node after deformation. We repeat this iterative process until the unbalanced loads are less than the allowable value. The iterative process is shown in Figure 1.

For general stiffness questions， the balanced equation is:

$$[K]\{d\}-\{R\}=0 \quad (4)$$

For nonlinear problems, the balanced equation is:

$$[j(\{d\})]=[K(\{d\})]\{d\}-\{R\}=0 \quad (5)$$

If the approximate solution of $[j(\{d\})]=0$ is reached in the certain convergence condition (based on the displacement), we could obtain the result by Taylor formula：

$$[j(\{d_{n+1}\})]=[j(\{d\})]+(\frac{d[j(\{d\})]}{dd})\Delta d_{n+1}=0 \quad (6)$$

In this formula:

$$d_{n+1}=d_n+\Delta d_{n+1} \quad (7)$$

Then

$$\frac{d[j(\{d_n\})]}{dd}=\frac{d[F(\{d_n\})]}{dd}=[K_T(\{d_n\})] \quad (8)$$

We could obtain the Iterative formula:

$$[K_T(\{d_n\})]\Delta d_{n+1}=\{R\}j\{F(\{d\})\}=\Delta\{R_n\} \quad (9)$$

In this formula, $\Delta\{R_n\}$ is the unbalanced loads, and this method is namely iterative method. Rail traffic Cable-stayed bridge with double cable plane concrete Twin Towers.

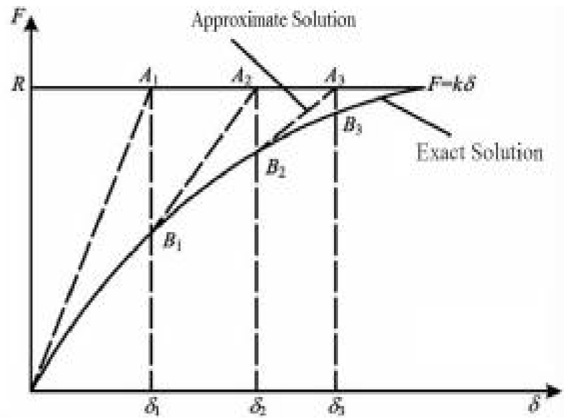

Figure 1. The iteration method of schematic diagram

4. Calculation Example

4.1. Project Background

Caijia Jialing River Bridge located in Chongqing Metro Line 6 is a cable-stayed bridge, used as one of the main rail transportation. The total length of this bridge is 1250m with a 60m+135m+250m+135m+60m cable-stayed bridge with double cable plane main span, it has two 183 meters high diamond tower. The width of the bridge is 15m, contains two tracks(4.7m); The whole structure includes 56 pairs of cables. The main girder of the main bridge is designed as a constant height concrete box section with single room and single box. The south approach spans is composed of 2×45m continuous beam bridge, the north bridge approach is 3×60+3×60+3×50m continuous rigid frame bridge.

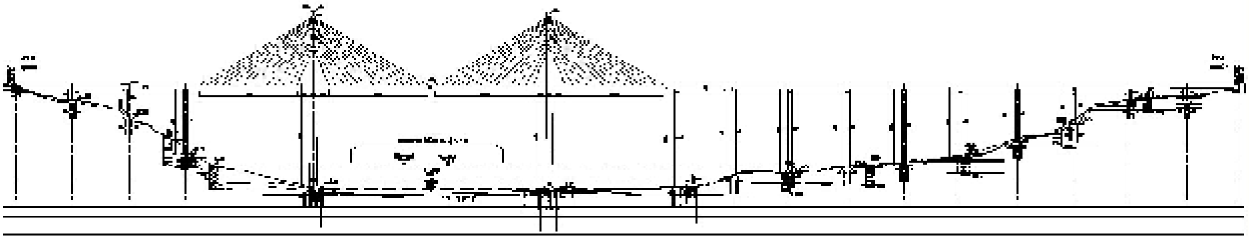

Figure 2. The overall layout of Cai Jia Jialing River Bridge

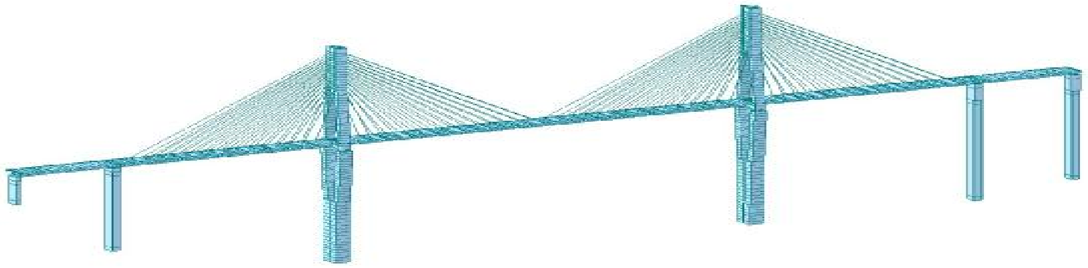

Figure 3. Finite element model of Caijia Jialing River Bridge

Table 1. Comparison of the analysis Results of displacement and internal force in constant load condition

Parameters	Condition I	Condition III	The relative effect(%)
horizontal displacement of the South Tower（cm）	2.27	2.51	10.4
the maximum vertical displacement of the main span (m)	12.35	14.47	17.2
bending moment of Southern tower bottom (kN˙m)	78935	84775	7.4
bending moment at the midpoint of the main span (kN˙m)	-28785	-32013	11.2
axial force at the bottom of Southern tower (kN)	-387523	-371854	4.0
axial force of the main beam at the bottom of Southern tower (kN)	-136323	-133596	2.0
bending moment of the main beam at the bottom of Southern tower (kN˙m)	130774	137804	5.4

4.2. Calculation Model

Calculation model was established by Midas / civil 2010 software(as shown in Figure 3), which is used to carry out nonlinear static analysis of Caijia Bridge. Loading conditions of nonlinear analysis is Constant load Aiming at this condition,this paper mainly select the displacement and internal force of several key sites for the key comparison,including horizontal displacement of the South Tower,t he maximum vertical displacement of the main span, bending moment of Southern tower bottom, bending moment at the midpoint of the main span, axial force at the bottom of Southern tower, axial force of the main beam at the bottom of Southern tower and bending moment of the main beam at the bottom of Southern tower.

This paper only set the following two conditions:

The calculation condition I: Linear analysis, without considering any kind of nonlinear factors in the analysis;

The calculation condition II: Analysis of the nonlinear effects of cable sag, Only revise the cable elastic modulus by Ernst formula, beam-column effect and large structure displacement effect were not considered.

Results in Table 1. In Table 1, the bending moment made beam flange under pressure is positive;axial tension is positive, contrary to the negative.

As is shown in Table 1, The horizontal displacement on top of tower changed from 2.27cm to 2.51cm,increased by 10.4%. The vertical displacement of the main girder of the main span is increased by 17.2%; bending moment at the tower bottom increases by 7.4%, the axial force is reduced by 4% at the same time, bending moment at the midpoint of the main span increased by 11.2%;the tail moment of girder is increased by 5.4%, the axial force in the same position is reduced by 2%.

Considering the nonlinear effects of cable sag effect, causes reduction of the structure stiffness.

Conclusion

Through the above analysis, the following conclusions can be obtained:

Considering the nonlinear effects of cable sag effects for cable element, resulting in overall structural stiffness decreased, so the top horizontal displacement and vertical displacement of the main span enlarge.

The bending moment and the deflection of the main span are influenced by geometric nonlinear effect strongly, however, less impact is occurred on the tower girder fixed position.

For the complex mechanical properties of rail cable-stayed bridge, the influence of geometric nonlinear should be fully considered when structural design is carried on.

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