Nonlinear Dynamic Analysis of Reinforced Concrete Arch Bridge Under the Seismic Load

Yu WANG
School of Civil Engineering & Architecture, Chongqing Jiaotong University, Chongqing 400074, CHINA

Abstract: Taking one reinforced concrete arch bridge of Chongqing as an example, considering the nonlinear analysis of structure, the bridge model is established by finite element software -. SAP2000 Then modal analysis and time history analysis is carried out Studying the vibration period, frequency and. deformation conditions of the model in each mode. Using nonlinear time-history analysis to study vibrational state and frequency response spectrum. According to the internal force and deformation diagrams, study the stress features of the bridge key sections. Some conclusions are deduced.

Keywords: Nonlinear; Dynamic Characteristics; Time-history Analysis; Seismic Wave

1. Introduction

Nonlinear analysis is the analysis type that the structure attribute exhibit a non-linear behavior with the changes of time, deformation, load non-linear change. The nonlinear behavior related to material properties, load and the specified analysis parameter. It can be divided into material nonlinear, geometry nonlinear and boundary nonlinear. Nonlinear analysis begins with a previous nonlinear analysis and continues. Its initial state include all the load effect analyzed previously, such as deformation, stress etc. Therefore, some nonlinear analysis cases can be connected together to realize the complex loading sequences.

2. The General Situation of the Bridge

The bridge in Chongqing is a two-way 4-lane prestressed reinforced box arch bridge, used as one of the main traffic thoroughfares by passengers and vehicles. The total length of this bridge is 133.20m with a 75m reinforced concrete box main span, and the approach bridges at both ends are made of 0.9m pre-stressed hollow slab girders with a span of 20m; the width of the bridge is 10m, made up of 0.5m (sidewalk) + 9m (vehicle road) + 0.5m (sidewalk); and a 1.36% longitudinal slope is designed. The upper section of the main span is made of pre-stressed concrete hollow slab with the height of 22.5cm and a span of 9×9 m =81m. "U" gravity-type abutments are used at both ends. C50 concrete, C40 concrete and C30 concrete are used for main arch, girder and piers, uprights together with other ancillary facilities. 3D model of the bridge established by software SketchUp is shown in Figure 1.

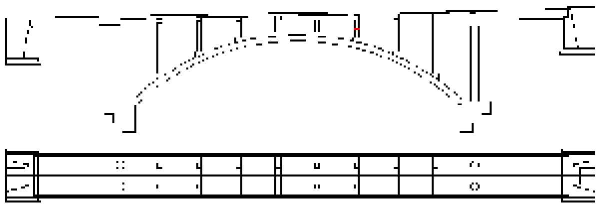


Figure 1. Longitudinal diagram and the top view of the whole bridge

3. The Establishing and Simplification of Model

The finite element model is a simulation of the original object, but not all the characteristics of the original object can be expressed. Arch bridge consisted of arch, beam, column, pier and ancillary facilities. In the analysis of the dynamic characteristics, different simulation methods are used in each component part to make the model can accurately reflect the geometry, material properties of structural members, and boundary connecting conditions of each member.

Reasonable simplification to the model represents that the model is very close to the prototype object. In this paper, using the finite element software SAP2000 to establish the bridge model, the main beam, main arch, pier, pile are simplified as abeam element. Which uses the following simplified.

Because there is no information of reaction materials in the drawings, this paper simplifies the materials of main arch, main girder and other ancillary facilities as C50 concrete, C40 concrete and C30 concrete respectively.

Because there is no information of the cross section of approach bridge in the drawings, according to the relationship of geometric proportion , approximate sections which come from stretching the section of cross beam of the main bridge vertically for 4 times are used instead of the section information of the cross beam on the approach bridge.

Considering the engineering example is the bridge which deforms under the dead weight and pre-stressed, this paper ignores the pre-stress.

The forms of load application in this paper referred to the new relevant standard specification for bridge.

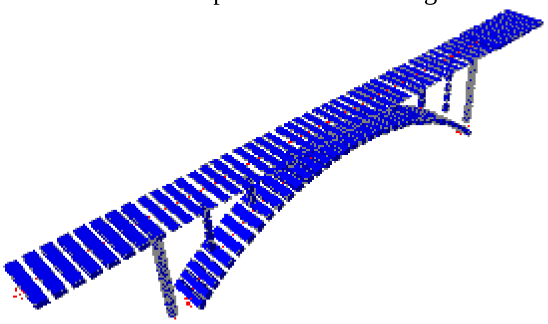


Figure 2. The finite element model, joints, members and constraints of the bridge

For the reinforced concrete arch bridge in this paper, C50 concrete, C40 concrete and C30 concrete are used for main arch, girder and piers, uprights together with other ancillary facilities. The selection of material and the material parameters of each main part of the bridge is shown in table 1. (all material parameters referred to the code for design of concrete structure).

4. Modal Characteristics based on the Linear Dynamic Analysis

Modal analysis which is the basis of dynamic analysis of structure is used to determine the structural vibration mode. The characteristic of natural vibration of structure can better be appreciated by modal analysis. Each structural modal has specific natural frequency, damping ratio and Modal Shape. If we are clear about each main modal characteristics of structure over a impressionable range of frequencies through modal analysis method, it can be predicted that the actual vibration response of this structure under the action of external or internal various vibration sources in the frequency band. Therefore, the modal analysis is an important method for the dynamic design of structure and failure diagnosis of equipment.

Table 1. The material parameters of each part

Item	Material	Modulus of elasticity(Gpa)	Density(kg/m3)	Poisson ratio	Thermal expansion coefficient
Main arch	C50	35	2500	0.1667	1.0×10-5
Main girder	C40	35	2500	0.1667	1.0×10-5
Pier	C30	30	2500	0.1667	1.0×10-5
Column	C30	30	2500	0.1667	1.0×10-5
Ancillary facilities	C30	30	2500	0.1667	1.0×10-5

In the software SAP2000, there are two methods of solving structural modal , namely the feature vector and Ritz vector analysis method. Vibration mode and frequency of undamped free vibration can be determined by the vector analysis of characteristic value. When determining the vibration mode and frequency of the structure system by vector analysis of characteristic value , the eigenvalue equation is:

$$(K - l M)\{j\} = 0$$

The stiffness matrix K is just as same as it in static analysis. Mass matrix M which is a diagonal matrix uses the lumped mass method and only considers the linear degrees of freedom, without considering the rotational degrees of freedom.

Ritz vector analysis seeks the vibration mode excited by the specific load. In the time-history analysis and response spectrum analysis based on the mode superposition method, the Ritz vector analysis provides a more accurate foundation of vibration mode than the feature vector. The reason for the more accurate result provided by the Ritz vector analysis is that it considers the spatial distribution of dynamic loads. But the method of characteristic value vector for the free vibration ignores this important information.

This paper simulated the 30th order modes of the bridge. Because this paper studies the nonlinear behaviors of main arch, only two typical modal patterns of the main arch deformed obviously are shown below.

Figure 3 for the 18th order mode, shows the vertical vibration of main girder and longitudinal vibration of pier. Its period and frequency are 0.14510s and 6.89194Hz, respectively.

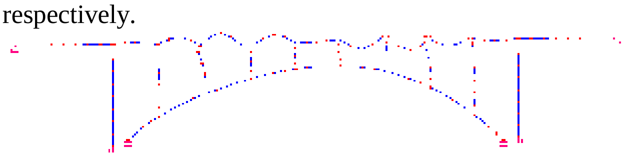


Figure 3. The 18th order mode

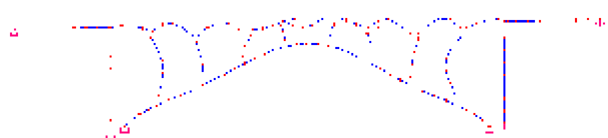
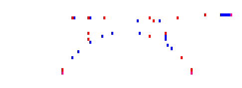
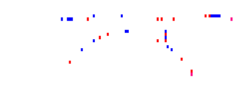
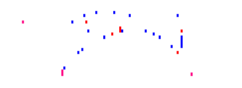
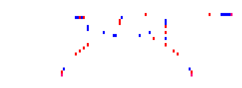
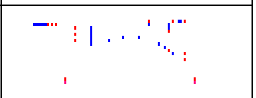
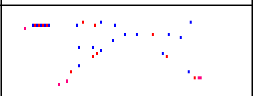


Figure 4. The 23th order mode

Figure 4 for the 23th order mode, shows the vertical vibration of main girder and longitudinal vibration of pier. Its period and frequency are 0.07600s and 13.15724Hz, respectively.

Table 2. The period, frequency and deformation of main arch in vibration

Order	Period	Frequency	Deformation
9	0.1862	5.3717	
10	0.1843	5.4259	
18	0.1451	6.8919	
19	0.1201	8.3253	
20	0.1001	9.9898	
23	0.0760	13.1572	

5. Nonlinear dynamic analysis

The majority of dynamic responses of structures, nonlinear deformation occurs when the structure or all members exceed the yield limit. At this time, the acting forces on members and corresponding deformation are nonlinearly related. With the responses passing from linear to nonlinear, the computational complexity will increase. The nonlinear problems of structural dynamics are solved with iterative solution.

For a single degree of freedom system with nonlinear stiffness, the differential equations of motion at the moment t_k can be expressed as follows:

$$m\ddot{x}_k + c\dot{x}_k + F_s(x_k) = F_k$$

In the formula above, $F_s(x_k)$ represents the nonlinear restoring force in step k. At the moment t_{k+1}, there are the equation as follows:

$$m\ddot{x}_{k+1} + c\dot{x}_{k+1} + F_s(x_{k+1}) = F_{k+1}$$

Substracts the two equations above could get:

$$m(\ddot{x}_{k+1}-\ddot{x}_k)+c(\dot{x}_{k+1}-\dot{x}_k)+F_s(x_{k+1})-F_s(x_k)=F_{k+1}-F_k$$

Make : $\Delta\ddot{x}=\ddot{x}_{k+1}-\ddot{x}_k$, $\Delta\dot{x}=\dot{x}_{k+1}-\dot{x}_k$, $\Delta F_s=F_s(x_{k+1})-F_s(x_k)$, $\Delta F=F_{k+1}-F_k$,

Incremental equations can be expressed as :

$$m\Delta\ddot{x}+c\Delta\dot{x}+\Delta F_s=\Delta F$$

If in the period of time from t_k to t_{k+1}, the change relationships between the forces and displacements are linear (the linear stiffness), and the equation as follows still hold :

$$F_s(x_{k+1})=F_s(x_k)+\frac{F_s(x_{k+1})-F_s(x_k)}{x_{k+1}-x_k}(x_{k+1}-x_k)$$

The equation above could define the secant stiffness:

$$k_s=\frac{F_s(x_{k+1})-F_s(x_k)}{x_{k+1}-x_k}=\frac{\Delta F_s}{\Delta x}$$

The incremental equilibrium differential equation above can be expressed as:

$$m\Delta\ddot{x}+c\Delta\dot{x}+k_s\Delta x=\Delta F$$

It can be seen: if tangent stiffness is known, then the incremental differential equation can be calculated precisely. But the displacement x_{k+1} at the moment t_{k+1} is unknown, a tangent stiffness is also unknown in actual solution. Therefore, k_s will require the corresponding assumption in analysis solution.

5.1. Definition of the time-history function

Time-history analysis method (immediate dynamic analysis) is a method that with the input of ground motion or artificial ground motion on the structure motion equation and solved by the time-domain analysis or frequency-domain analysis to obtain the structural earthquake response in the whole time-history , and one of the analysis methods of code for seismic design of most countries .

The same as the linear time-history analysis, nonlinear time-history analysis needs to define the time-history curve. The method of definition is the same as the method of linear time-history analysis . If nonlinear analysis of the structure under rarely occurred earthquake is necessary, then need to select seismogram, you can use the common forms of seismic wave in the programs on line and the seismogram in several kinds of ground states commonly used in the standards of China, and obtain the seismic time-history curve under rarely occurred earthquake by controlling the peak value.

Analyze the vibrational state and the frequency response spectrum of the bridge based on the data files of seismic wave in Wenchuan County, Sichuan Province China in 2008. As shown in figure 5, define a time-history function called WENCHUAN. Import the seismic wave data WENCHUAN from the file in the program.

5.2. The definition of time-history working condition

The same as the linear time-history analysis, after defining the time-history function curve, need to define the working condition of nonlinear time-history analysis. There will be a dialog box for defining the working conditions of time-history analysis, after adding a new working condition and choosing TIME-HISTORY in the pull-down menu of the type of working conditions.

The dialog box of definition for the working condition of nonlinear analysis is the same with the dialog box of linear time-history analysis, shown in the figure 6 below.

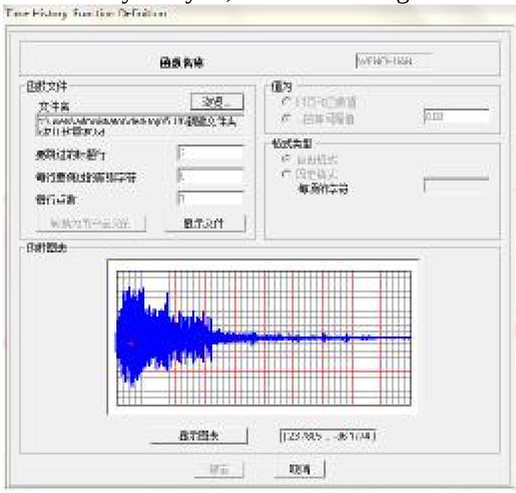


Figure 5. The definition of time-history function

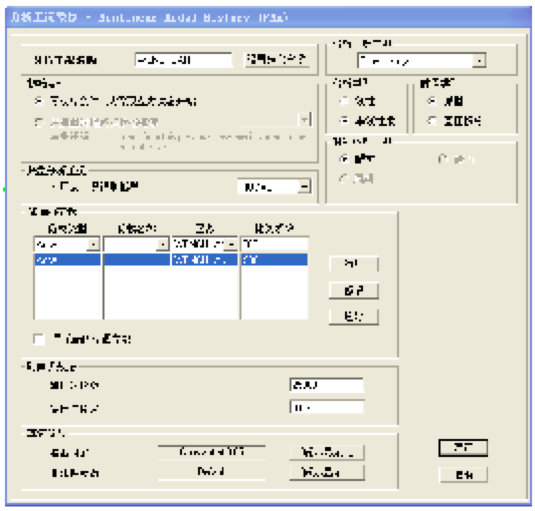


Figure 6. The definition of time-history working condition

5.3. Fast Nonlinear Analysis (FNA) Method

In the definition of nonlinear time-history analysis, we need to choose the type of nonlinear analysis in the option of analysis types. The same as the linear time-history analysis, it is necessary to select the type of time-history analysis. In the nonlinear dynamic analysis, the structural properties of some units, or an effect of the structure may be nonlinear along with the change of time. For every moment, classical mechanics equilibrium equation of structural system is still valid, so the traditional nonlinear method still solves problems through the equilibrium equation at each moment of the time-history integration. The same as the linear time-history analysis, the integration methods of nonlinear time-history analysis for equilibrium equation at each moment can be divided into two categories, modal integration and direct integration. For the direct integration, the integral methods that nonlinear time-history analysis commonly used are the same with linear analysis. For modal integration method, the program SAP2000 adopted a new method called fast nonlinear analysis method (FNA).

Fast nonlinear analysis method (FNA) is an effective way for nonlinear analysis. In this method that the nonlinear is considered as an external load, the modal equation which considers the nonlinear load and modification, is produced. Similar to the structural linear modal equation, the modal equation can take a Vibration mode decomposition which is similar to linear. Then based on the approximate solution showed by Taylor series, using precise piecewise polynomial integration, the modal equation are solved with iterative solution . Calculate the nonlinear force vector based on the deformation of the nonlinear element and the history of changed speed came from the analysis previously, and produce the modal force vector to form the new iteration modal equation and solve it. These are the basic ideas and steps of FNA in the program SAP2000 .

5.4. The nonlinear type of SAP2000

Currently, according to their natures, the nonlinear properties the program SAP2000 can considered, could be divided into four types: geometric nonlinearity, material nonlinearity, boundary nonlinearity and nonlinear of the link units. These covers the several nonlinear types what the structure analysis needs to consider. Not all types of nonlinear time-history analysis could consider these nonlinear types. The nonlinear types that different time-history types could consider are different.

Geometric nonlinearity mainly refers to the effect of P-△, the analysis of geometric large deformation and nonlinear related to geometry properties of structure. Traditional analysis of the linear static and dynamic are based on the assumption of small deformation. It is applicable to general structure system, but generally not the large span or flexible structure system. The main task of geometric nonlinearity is to consider the actual large deformation when differ greatly in the assumption and the situation of actual structure. Material nonlinearity mainly refers to the structural nonlinearity caused by the properties of material of the structure. For the steel and concrete commonly used in building structure, their stress and strain is linear in a certain range of stress. This is also a basis of routine analysis and design of structure. Beyond that range, stress will exhibit a strong nonlinear property, so the characteristic of bearing capacity of the structural materials is characterized by nonlinear properties in general. Sometimes, nonlinear of structural materials includes the structural material units of single - pull or single - pressure considered in the structural analysis. Boundary nonlinearity refers to the contact problem of the boundary, such as common problems of crack and the connection problems of boundary. It can be achieved by unit slot or hook unit in SAP2000. Nonlinear of main link unit refers to the nonlinear properties of additional damper and vibration isolator considered in the structure design. These structural units are not only characterized by nonlinear properties, and could consider the properties of plastic adaption in cyclic loading and energy dissipation of units through the definition of hysteresis loop. When selecting the working condition of the nonlinear time-history analysis for modal integration, the default option of the program is to consider the nonlinear of the nonlinear link element, and could not be modified.

When choosing nonlinear time-history analysis of the direct integration, the default option of program is to consider the material nonlinearity and nonlinear of the link units, and could not be modified. In the two integration methods, the material properties of time correlation in the list of nonlinear properties are always not selected. Because the content which involves a non-linear change related to the age of hardening of the material properties of the concrete in construction stage, is meaningless in the time-history analysis, this property would not be considered in the nonlinear time-history analysis.

5.5. The analysis of deflection of main arch

Deflection related to structural safety reflects the overall characteristics of the structure. For the main girder, the analysis of deflection includes the vertical displacement, the longitudinal displacement and the lateral displacement of main girder. The normal driving and driving comfort of the vehicle could be affected by oversize displacement of main girder. When the displacement exceeds a certain limit, the early warning could be given in time to ensure the safety of the bridge operation.

According to the global coordinate system to determine the symbol of displacement. The front side is positive and vice is negative.

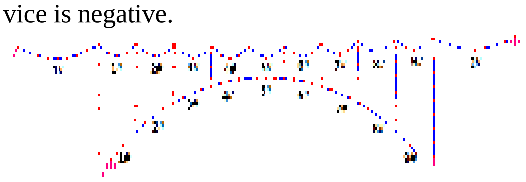


Figure 7. Deformation under the dead load

The deformation under the dead load is showed in figure 7. By the figure we can clearly see that the deformation might happen in the main part of the bridge is large. The deformation of the approach bridge is smaller than that of main part of bridge. To made it clear that deformation of all parts of the structure, the main arch and main bridge will be numbered from left to right in turn for 1# ~ 9#. As shown in figure 7, the approach bridges on the left and right are 1# and 2# respectively. The maximum deflection of the deformation of mid-span is in node 482 (main section 4 # of the main bridge). The maximum deflection is about -2.67 cm.

The direct integration method is the nonlinear time-history analysis in the Wenchuan earthquake effect. The maximum and the minimum displacements of each section in this working condition are shown in table 3 and table 5, respectively. In the nonlinear behavior of the Wenchuan earthquake effect, the values of a quantity reflect the node number with larger nodal displacement and the value of deflection. The values are compared with the results of the linear time-history analysis, as shown in table 4 and table 6.

Table 3. Nodal displacement of partial nodes in the nonlinear time-history analysis (maximum)

Node	Section	Value type	Deflection (cm)
5	1# section of the main arch	Max	149. 5946
9	2# section of the main arch	Max	847. 9591
13	3# section of the main arch	Max	1219. 055
17	4# section of the main arch	Max	1135. 799
21	5# section of the main arch	Max	1064. 682
25	6# section of the main arch	Max	1152. 842
29	7# section of the main arch	Max	1224. 033
33	8# section of the main arch	Max	839. 9547
37	9# section of the main arch	Max	141. 3237

Table 4. Nodal displacement of partial nodes in the linear time-history analysis (maximum)

Node	Section	Value type	Deflection (cm)
5	1# section of the main arch	Max	12.79885
9	2# section of the main arch	Max	81.1077
13	3# section of the main arch	Max	123.1685
17	4# section of the main arch	Max	115.2731
21	5# section of the main arch	Max	104.9252
25	6# section of the main arch	Max	114.8479
29	7# section of the main arch	Max	121.8628
33	8# section of the main arch	Max	79.78689
37	9# section of the	Max	12.61641

	main arch		

Table 5. Nodal displacement of partial nodes in the nonlinear time-history analysis (minimum)

Node	Section	Value type	Deflection (cm)
5	1# section of the main arch	Min	-144.60
9	2# section of the main arch	Min	-877.239
13	3# section of the main arch	Min	-1238.65
17	4# section of the main arch	Min	-1135.14
21	5# section of the main arch	Min	-1059.24
25	6# section of the main arch	Min	-1162.55
29	7# section of the main arch	Min	-1231.47
33	8# section of the main arch	Min	-824.504
37	9# section of the main arch	Min	-141.191

Table 6. Nodal displacement of partial nodes in the linear time-history analysis (minimum)

Node	Section	Value type	Deflection (cm)
5	1# section of the main arch	Min	-16.5532
9	2# section of the main arch	Min	-100.11
13	3# section of the main arch	Min	-153.215
17	4# section of the main arch	Min	-142.091
21	5# section of the main arch	Min	-127.955
25	6# section of the main arch	Min	-142.095
29	7# section of the main arch	Min	-152.89
33	8# section of the main arch	Min	-101.114
37	9# section of the main arch	Min	-17.2433

5.6. Internal force analysis of the main arch

The study of the internal force of bridge is mainly to analyze the stress and strain of the structure of the bridge. The purpose is to understand and master the stress and strain of the key sites and control points of the main girder. Therefore, the internal force envelope diagram of time-history analysis in earthquake provides many important basis for knowing the internal force of the structure.

Figure 8 and figure 9 are the envelope diagrams of bending moment and axial force in earthquake effect respectively. From the figures, the bending moment is concentrated in the main arch of mian bridge. Larger bending moments exist in the main arches, piers and columns of main bridge. There is no bending moment on the girder, because of the movements of the supports in the earthquake. The structure of the main bridge which is a simply supported beam arch bridge, is a statically determinate structure. The approach bridge is also a simply supported beam bridge belong to the statically determinate structure. The statically determinate structure can produce only displacement with no internal forces. The internal forces of each control section are showed in table 7 and table 9 respectively.

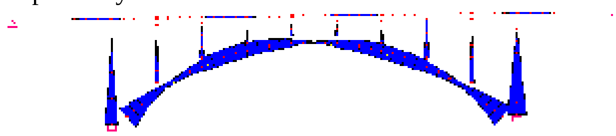


Figure 8. The envelope diagram of bending moment under the Wenchuan earthquake

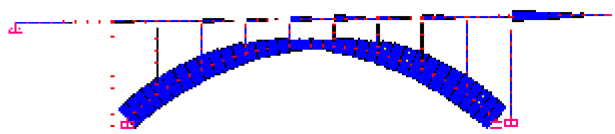


Figure 9. The envelope diagram of axial force under the Wenchuan earthquake

Table 7. The axial force and bending moment of main arch in the linear time-history analysis (maximum)

Member	Section	Value type	Axial force (KN)	Bending moment (KN·m)
4	1# section of the main arch	Max	7244.072	40848.31
8	2# section of the main arch	Max	6802.72	12389.51
12	3# section of the main arch	Max	6154.747	32099.28
16	4# section of the main arch	Max	4336.422	27192.63
20	5# section of the main arch	Max	2874.592	2667.525
24	6# section of the main arch	Max	4989.201	32467.87
28	7# section of the main arch	Max	7665.807	39608.6
32	8# section of the main arch	Max	8102.814	17128.6
36	9# section of the main arch	Max	8144.724	32720.97

Table 8. The axial force and bending moment of main arch in the linear time-history analysis (minimum)

Member	Section	Value type	Axial force (KN)	Bending moment (KN·m)
4	1# section of the main arch	Min	-6505.76	-33436
8	2# section of the main arch	Min	-6619.81	-13252.5
12	3# section of the main arch	Min	-5691.54	-42389

16	4# section of the main arch	Min	-3921.3	-32884.8
20	5# section of the main arch	Min	-2739.74	-3623.88
24	6# section of the main arch	Min	-4415.48	-27802.3
28	7# section of the main arch	Min	-6297.77	-30973.8
32	8# section of the main arch	Min	-7201.6	-12277.7
36	9# section of the main arch	Min	-7158.06	-43330.6

Table 9. The axial force and bending moment of main arch in the nonlinear time-history analysis (maximum)

Member	Section	Value type	Axial force (KN)	Bending moment (KN·m)
4	1# section of the main arch	Max	142000	691700
8	2# section of the main arch	Max	133300	285600
12	3# section of the main arch	Max	120600	627800
16	4# section of the main arch	Max	84990	496900
20	5# section of the main arch	Max	56340	109200
24	6# section of the main arch	Max	97790	684800
28	7# section of the main arch	Max	150200	751200
32	8# section of the main arch	Max	158800	230700
36	9# section of the main arch	Max	159600	737500

Table 10. The axial force and bending moment of main arch in the nonlinear time-history analysis (minimum)

Member	Section	Value type	Axial force (KN)	Bending moment (KN·m)
4	1# section of the main arch	Min	-127500	-579400
8	2# section of the main arch	Min	-129700	-348500
12	3# section of the main arch	Min	-111600	-836700
16	4# section of the main arch	Min	-76860	-578000
20	5# section of the main arch	Min	-53700	-108200
24	6# section of the main arch	Min	-86540	-577400
28	7# section of the main arch	Min	-123400	-593300
32	8# section of the main arch	Min	-141200	-193000
36	9# section of the main arch	Min	-140300	-958200

Conclusion

Through the above analysis, the following conclusions can be obtained:

Through the modal analysis, grasp the dynamic characteristics of the bridge, observe the vibration situation of bridge in different modals , and understand the stiffness characteristics of the whole bridge;

According to the results of time-history analysis, the units with larger micro-strain usually exist on the main bridge in earthquake. The vibration state and frequency response spectrum of the bridge are analyzed. It is a necessary complement for the vibration state and deformation of bridge in earthquake.

Tn earthquake-prone areas such as Yunnan, Guizhou, and Sichuan province, the stage of bridge design and operation could use the nonlinear dynamic analysis, especially the time-history analysis method. Through the nonlinear dynamics analysis, the adverse sections of bridge can be found effectively. The method should be the basis of structural seismic design.

Through the analysis of finite element model established in SAP2000, we have a deeply understanding of the situations of the overall deformation, internal force, the modal analysis and nonlinear seismic analysis of bridge, to a better grip on the key stressed parts of bridge. At the same time, based on the modal and nonlinear dynamic analysis of the structure, not only to better grasp the dynamic characteristics of the bridge, and deepened the understanding to this bridge.

References

[1] MA Lewei, XU Zhaodong. Structural dynamics[M]. Beijing: Science Press, 2007.

[2] ZHOU Yongjun, ZHAO Xiaoxing, SONG Yifan etal. Modal analysis and experiment test of Continuous beam Bridge[J]. Journal of Chang 'an University, 2007.

[3] CHEN Changhong. SAP2000 Analysis of Structural Engineering Case [M]. Beijing: Metallurgical industry press, 2010.

[4] D.Wu, S.Tesfamariam, S.F.Stiemer etal. Seismic fragility assessment of RC frame structure designed according to modern Chinese code for seismic design of buildings[J].Earthquake Engineering and Engineering vibration[C],2012.

[5] Chengyin Liu, Joshua Olund, Al. an Cardini, Paul D'Attillio, Eric Feldblum, Joho, DeWolf. Structural health monitoring of bridges in the State of Connecticut[J]. Earthquake Engineering and Engineering Vibration, 2008, 4(7):427-437.

[6] LI Jianliang, ZHAO Jing, LI Fuhai etal. Analysis of time-history analysis method on the seismic design[J]. The earthquake in Sichuan[C], 2011.