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Phase Analysis Method Of Wavelet Transform For Simultaneous Extrusion

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Abstract: Synchronous extrusion wavelet transform has achieved good results in instantaneous frequency identification, modal parameter extraction, random noise suppression and other fields. In this paper, aiming at the inaccuracy of phase analysis of traditional signal analysis technology, the instantaneous phase of signal is extracted by using synchronous extrusion wavelet transform, and the phase analysis of single frame fringe is verified by computer simulation and preliminary experiment, which proves the feasibility of the method.

Keywords: Synchronous extrusion; Wavelet transform; Instantaneous phase; Modal parameter

1. Introduction

Fourier transform profilometry (FTP) is the most classical single frame fringe phase analysis method [1]. Fourier transform establishes the relationship between signal spatial / temporal domain and frequency domain. In the optical three-dimensional sensing of projecting sinusoidal fringe pattern, the height fluctuation of object surface is modulated to sinusoidal fringe image, resulting in the deformation of fringe image, which is also a non-stationary signal. The traditional signal analysis technology can not analyze the accurate phase distribution when the frequency components of the fringe image signal are overlapped due to the discontinuity or occlusion of the object height distribution. In order to overcome the problem of signal aliasing, the time-frequency analysis technology of signal is introduced into the field of fringe image analysis, mainly including Wigner Ville distribution, Gabor expansion, short-time Fourier transform, Hilbert Huang transform (HHT), wavelet transform, etc. However, the accuracy of phase analysis is low at the edge of image and frequency mutation. Daubechies et al. Put forward a new method of combining wavelet transform and Reconstruction - synchrosqueezing wavelet transform. This method can effectively reconstruct the time-frequency map after wavelet transform and obtain the time-frequency curve with high frequency accuracy. Moreover, any signal can be decomposed into linear superposition of several approximate harmonics. Even if the function waveform is non harmonic, the instantaneous frequency of the signal can be obtained by the synchronous extrusion algorithm. As a new time-frequency analysis method based on wavelet transform, this paper attempts to introduce the synchronous extrusion wavelet transform into the field of three-dimensional measurement of structured light. The feasibility of the method is verified by computer simulation and preliminary experiments.

2. Simultaneous Extrusion Wavelet Transform

As a powerful tool of non-stationary signal analysis, time-frequency analysis can give the instantaneous frequency and phase of each time, which has become a hot spot of modern signal processing and time-varying nonlinear system research [2]. Based on the wavelet transform, the continuous wavelet transform (CWT) is applied to the signal $x(t)$ to obtain the wavelet coefficient $W_x(a,b)$. Given the wavelet generating function ψ , the continuous wavelet transform of signal $x(t)$ is:

$$W_x(a,b)=\int_{-\infty}^{+\infty} x(t)\frac{1}{\sqrt{a}}\overline{y\left(\frac{t-b}{a}\right)}dt \qquad (1)$$

In the formula, a and b are scale factor and translation factor respectively, $y\left(\frac{t-b}{a}\right)$ is the conjugate of wavelet function $y\left(\frac{t-b}{a}\right)$. By using the mapping relationship between wavelet scale factor and signal frequency, the wavelet coefficients can be conveniently displayed on the time-frequency plane. The traditional wavelet transform method adopts various methods to extract wavelet ridges according to the wavelet transform time-frequency map to realize the instantaneous frequency. Extraction, and research shows that for the simplest harmonic function, the wavelet coefficient itself is the oscillation function of the time function in the time-frequency plane. When $x(t)=A\cos(wt)$, Assuming that the wavelet function has fast decay, $\hat{\psi}(x)$ tends to zero in the negative frequency domain and is concentrated near $x = w_0$. According to Plancherel's theorem, continuous wavelet transform of harmonic $x(t)$ can be obtained:

$$\begin{aligned} W_x(a,b) &= \frac{1}{2p}\int \overset{\$}{x}(x)\sqrt{a}\,\overline{\overset{\$}{y}(ax)}e^{ibx}dx \\ &= \frac{A}{4p}\int[d(x-w)+d(x+w)]\sqrt{a}\,\overline{\overset{\$}{y}(ax)}e^{ibx}dx \quad (2) \\ &= \frac{A}{4p}\sqrt{a}\,\overline{\overset{\$}{y}(aw)}e^{ibw} \end{aligned}$$

In the formula, $\overset{\$}{\psi}(x) = \frac{1}{\sqrt{2p}}\int_{-\infty}^{+\infty}\overset{\$}{y}(t)e^{ibt}dt$ and $\overset{\$}{x}(x)$ are the Fourier transform of wavelet generating function y and signal $x(t)$ respectively. See fig. 1 for the wavelet diagram of harmonic signal $x(t) = A\cos(wt)(A=1, w=4p)$. It can be seen from formula (3) and fig. 1 that if $\overset{\$}{\psi}(x)$ tends to zero in the negative frequency domain and $x = w_0$ is concentrated nearby, then the wavelet coefficients are also concentrated in $a = \frac{w_0}{w}$ in the time-frequency plane and distributed in a certain range of color scale [3]. In addition, the wavelet coefficient itself is the oscillation function of the time function b, so the wavelet ridge extracted directly from the wavelet metric graph can not directly reflect its relationship with the instantaneous frequency.

In order to calculate the instantaneous frequency of the signal accurately, although the wavelet coefficient $W_f(a,b)$ is distributed on all scales, the oscillation characteristic of the wavelet coefficient on b points to the initial frequency w_0 no matter what the value of a is. Therefore, it is suggested to make a preliminary estimation of the instantaneous frequency through derivation. The instantaneous frequency $w_x(a,b)$ of the signal x:

$$w_x(a,b) = \begin{cases} \dfrac{-i\partial_b W_x(a,b)}{W_x(a,b)} & |W_x(a,b)| > 0 \\ \infty & |W_x(a,b)| = 0 \end{cases} \quad (3)$$

According to formula (3), the instantaneous frequency $w_x(a,b)$ can be estimated, and then $(a,b)\xrightarrow{yields}[\omega_x(a,b),b]$ mapping can be established. The wavelet coefficient $W_x(a,b)$ is transformed from time scale plane to time frequency plane, and then it becomes $W_x[\omega_x(a,b),b]$.

The simultaneous extrusion wavelet transform obtains the value $T_x(\omega_l,b)$ of the simultaneous extrusion transform by the wavelet coefficient $W_x[\omega_x(a,b),b]$ on the extrusion time-frequency in the interval w_t near any center frequency $\left[\omega_l - \frac{1}{2}w,\ \omega_l + \frac{1}{2}w\right]$. In practical calculation, since a, b, w are all discrete, set $a_i - a_{i-1} = \Delta a$, the transformation value of synchronous extrusion can be expressed as:

$$T_x(\omega_l,b) = \sum_{a_i:|\omega_x(a,b)-\omega_l|\le\ w/2} W_x(a,b)a_i^{-3/2}(\Delta a)_i \quad (4)$$

On the surface, stwt is similar to other TFR algorithms, but other TFR algorithms, such as STFT, wavelet transform, Wigner Ville distribution, can't capture the features in a small range [4]. The wavelet transform of synchronous extrusion can clearly describe and extract the components of time-varying spectrum, and the transform is reversible, so the initial signal $x(b)$ can be reconstructed by $T_x(\omega_l,b)$ inverse transform. The inverse transform of wavelet transform for simultaneous extrusion is:

$$x(b) = \mathrm{Re}\left[C_j^{-1}\sum_l T_x(\omega_l,b)(\Delta w)\right] \quad (5)$$

In the formula, $\mathrm{C}_\varphi^{-1} = \int_0^{+\infty}\varphi^*(\varepsilon)\frac{\mathrm{d}\varepsilon}{\varepsilon}$, Re is the realistic part, $\varphi^*(\varepsilon)$ is the Fourier transform conjugated by the mother wavelet function. The inverse transform can obtain the approximate complete reconstruction of the original signal [5].

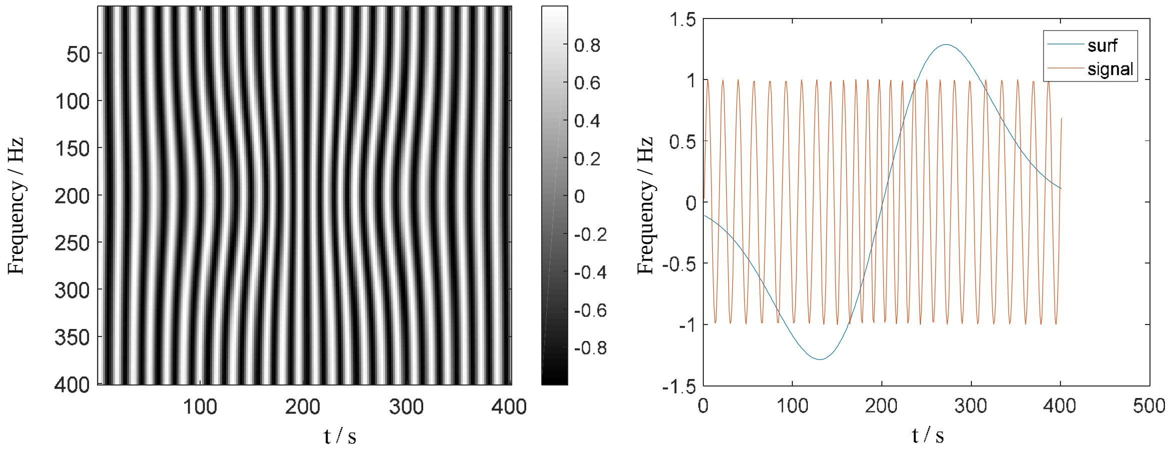
3. Computer Simulation

In this paper, the step of extracting the instantaneous phase of signal is based on the idea of TIME-SPECTRUM rearrangement, to get the value of wavelet transform of synchronous extrusion, then calculate the analytical signal corresponding to the real signal, and finally extract the instantaneous attribute.

In order to verify the phase analysis theory of wavelet transform, the computer simulation is carried out in this paper. The simulated modulation phase is represented by the function $z = 3xe^{(-x^2+y^2)}$, the size of the fringe pattern of the fringe image is 400*400, and the height of the object surface is modulated by the sine function.

$$a = b*sin(2p\ fx+z) \quad (6)$$

In the formula, a is a line in the fringe image, b is the sine function of the image, f is the fringe frequency $f = {}^1\!/_{16}$, x is the spatial coordinate, and z is the height of the object surface. As shown in fig. 1.

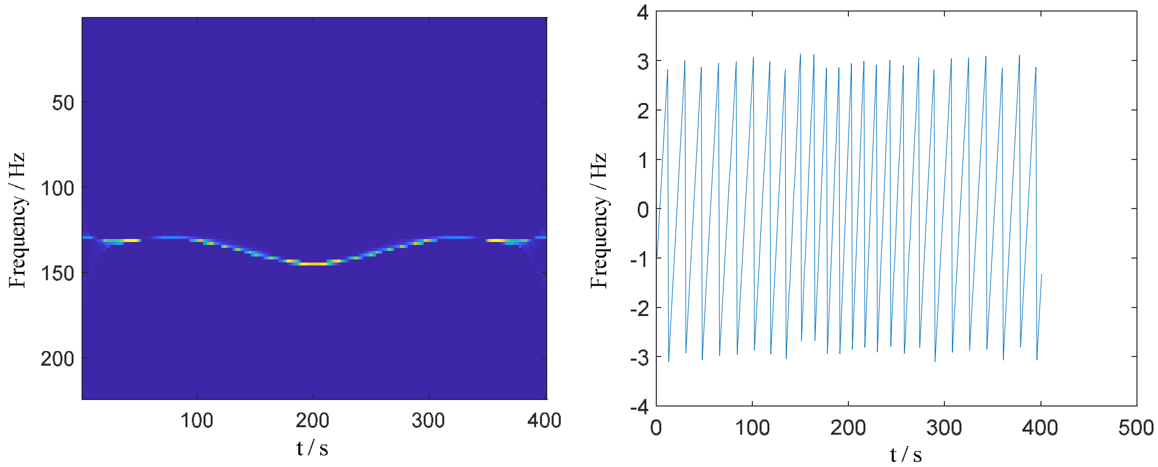


(a) Deformed fringe image

(b) Line 200 gray image

Figure 1. The deformed fringe image of the object height modulated by the frequency and the gray level of the 200th line

Take a row in the stripe image as an example, take out the 200th row of the image, as shown in the figure, the dotted line represents the height fluctuation of the object, and the solid line represents the density change of the stripe caused by the height change of the object The Morlet function is used as the parent wavelet to transform it into a synchronous extruding wavelet, and the transformation result is shown in fig. 2.



(a) Wavelet quantity graph

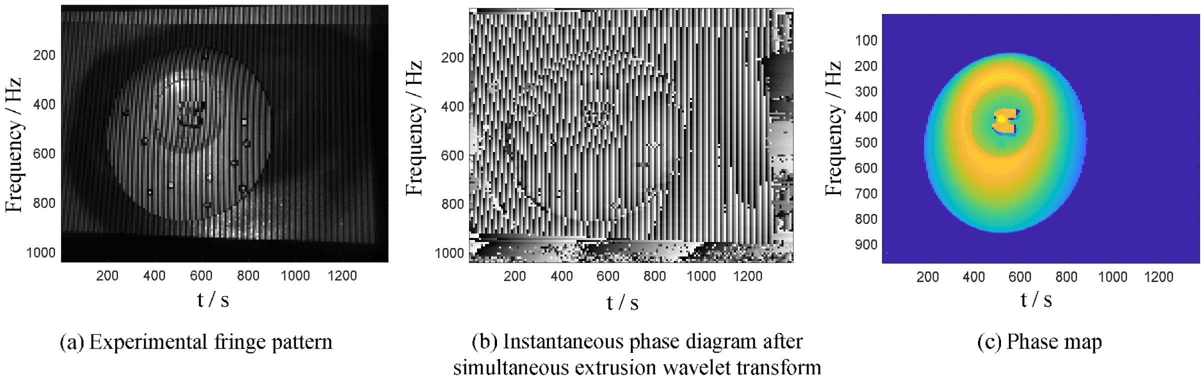
(b) Instantaneous phase diagram

Figure 2. Morlet wavelet's simultaneous squeezed wavelet map, instantaneous phase

According to Fig.2, compared with the continuous wavelet transform, the synchronous extrusion transform reduces the energy tailing effect and concentrates more energy.

4. Experimental Study

Through a preliminary experiment to verify the application of synchronous extrusion wavelet transform in the phase demodulation of fringe image, the fringe projection device of the experiment is lightcraft 4500 of Texas Instruments, the camera model is a black-and-white camera of basilera102f, the object is the display base, and the fringe is projected on the surface of the object, and the obtained fringe image is shown in fig. 3.



(a) Experimental fringe pattern

(b) Instantaneous phase diagram after simultaneous extrusion wavelet transform

(c) Phase map

Figure 3. Experimental drawing

According to fig. 3, there is an error in the phase of demodulation processing in the area where the fringe deformation is severe (the edge of the object) and the illumination is dark. In the area where the illumination is good, the phase is more accurate, and the phase diagram of the object is clear and complete.

5. Result

Due to the occlusion and discontinuity of the real object surface, its response is usually non-stationary signal. The traditional time-frequency analysis method is often difficult to analyze the instantaneous phase change of the surface profile. In this paper, the simultaneous extrusion wavelet transform is introduced into the instantaneous phase analysis of single fringe image in the field of three-dimensional measurement for the first time.

The time-frequency curve of the synchronous extrusion wavelet transform is finer and clearer, which shows that the energy is more concentrated, the phase is more accurate, and the contour change of the object can be recognized effectively. However, the phase error near the end point is relatively large, and the initial phase is also different, which needs further analysis and research.

6. Acknowledgment

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