Dual Channel Oligopoly Competition Strategy of Opaque Sale Based on the Hotelling Model

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Abstract: Opaque sale is a popular selling strategies for selling leftover capacity in airline ticket sales and is rapid development in recent years. This paper builds two oligopoly competition models, one is about single opaque sales channel, the other is dual channel (direct channel and opaque sales channel). Designed to find out which sales channel is more conducive to improving the revenue of the two airlines. We find that two sales channels exist when the passenger are less willing to buy. And In the case of opaque sales, sell from dual channel can effectively increase airlines' equilibrium profit. At the same time, the intermediary is motivated to implement this strategy. The increase in revenue of airlines and intermediaries illustrates the driving role of dual channel joint sales in driving market demand. Theoretical references can be provided for the pricing in competitive and channel selection of airlines.

Keywords: Opaque sale; Passenger strategic behavior; Revenue management

1. Introduction

As a typical perishable product, airline tickets are typically limited in quantity and sales time [1]. When the plane takes off, the unsold tickets lose their value forever. So as to achieve a win-win situation for both enterprises and customers, airlines often balance supply and demand by cutting prices if a large number of tickets are not sold when near the end of the sales period. But depreciate sales promotion behavior will make airline pricing decision uncertainty in the future, because the strategic behavior of passengers choosing to wait for the price to be bought after a price reduction. According to the characteristics of perishable product sales and passengers' strategic behavior, much attention has recently been paid to distribution. by use of opaque sales. This selling mechanism in which some attributes of product are hidden from the consumer and are revealed only after the purchase has been made [2]. For example, the opaque air ticket, launched by Fly.com and Vayama, is provided by SkyTeam, Oneworld and the Star Alliance member airlines, passengers buying an opaque ticket does not know which airline will provide the service before buying the ticket. In recent years, research on revenue management shows that segmentation enables airlines to generate incremental revenue and protecting the brand image of the enterprise by selling distressed inventory cheaply without disrupting existing distribution channels or retail pricing structures [3].

Different pricing body of existing literature on opaque sales is mainly divided into two categories. The first kind of literature discusses the equilibrium pricing and effectiveness analysis of opaque sales under the consumer bidding strategy. As Wang et al. (2009) showed that hotels increase revenue by adopting a consumer bidding strategy when business-type customer demand is uncertain[4]. Gal-Or (2011) found that when opaque intermediaries is an airlineservice, the airline can take consumer bidding strategies to better extract intermediaries surplus [5]. The essence of the consumer bidding opaque sales is that product or service provider decides whether to accept the consumer's offer and provide products or services based on the set reserve price. Due to the incomplete information characteristics of opaque sales, and the "delay behavior" [6] in the transaction process caused by the consumer bidding strategy, there are also literatures discussing the service provider pricing strategy, which is different from the consumer bidding strategy. Consumers must accept the price given by the product or service provider if they want to purchase a product or service. For the first time, Fay (2008) studied theservice provider pricing strategy, research shows that when consumers value products differently, it is more profitable to use opaque sales channels than traditional channels [7]. Jerath et al. (2010) concluded that intermediaries can increase airline revenue by using opaque sales when the value of the consumer is low [8]. Zhaofang Mao (2016) introduces two common strategies of opaque distribution mode, this study finds that when the consumer has a

large ratio of utility discount to the service provider pricing strategy and consumer bidding strategy, the former is better than the latter [9].
It can be seen that the existing literature has studied the influence of opaque sales on the market and consumers in terms of consumer bidding strategyand service provider pricingstrategy, and discussed the differences and similarities between opaque sales and traditional sales. However, under the background of competition, how to develop effective competition strategy to improve revenue is an urgent problem for airlines. On this issue, Huanget al. (2014) established a stylized economic model about two competitive sellers, a series of customers and an intermediary which use the consumer bidding strategies, study that seller's dynamic decision to choose direct sales or choose opaque sales [10]. Caiet al. (2013) shows that opaque sales are beneficial tocompeting suppliers and retailers who use the service provider pricing strategy by promoting consumer segmentation through probabilistic products generated by retailers that mix multiple competing suppliers [11].The above literature discusses the competition among sellers under opaque sales, but does not explore the competition strategy between sellers and intermediaries.
This paper based on the opaque distribution model of service provider pricing, the Hotelling model is used to establish a single opaque sales channel and the oligopolistic competition model of direct and opaque sales. The equilibrium pricing and competition strategies of two airlinesare discussed. In the case of the existence of the opaque distribution model, this paper shows that the revenue of the airlines and intermediaries is higher than the single opaque sales strategy under the dual channel sales strategy, and it can assist decision-makers to formulate reasonable competition strategies in the complex market environment so as to maximize profits.

2. Symbols and Assumptions

Suppose there are two competing airlines A and B in the market, they are located at the beginning and end of the Hotelling line with a length of 1.The number of non-difference tickets is $\frac{K}{2}$, and selling products through intermediaries with opaque sales. Among them, opaque sales channels are used to sell tickets of the two airlines, and some descriptive characteristics of the tickets, such as the source of the tickets and the stopping place, are concealed before the tickets are sold. Intermediary opaque sales channels are operated by third-party intermediary agents. Similar with Gal-or (2011) [5] and Lei (2017) [5], it is assumed that airlines and intermediaries enter into “share agreement”whereby the airline consigns the remaining tickets to theintermediaries for sale. The profit δ from sales are obtained by two airlines and the rest $1-\delta$ are held by intermediaries.
The number of passengers J is evenly distributed on the Hotelling line, and they location in the market is represented by x_i .Similar to Jerath et al. (2010) [9], passengers have different preferences between airlines, the parameter t denotes the strength of brand preference in the market. Apassenger at x_i incurs a disutility x_i when buy a ticket from airlines i .Each passenger has a general known valuation V for the ticket and purchases at most one unit. The passenger has a strategic behavior, that is, during the inquiry process, the purchase will only occur if the purchase utility is non-negative. If the utility in all cases is non-negative, you will choose a higher-utility period and channel to purchase, the utility function of passengers is linear. Based on market information, rational expectations are generated for prices, and the expected utility of current purchases and future purchases is determined to make optimal purchase time and purchase channel decisions.
According to the above assumptions and revenue management principles, airlines have two kinds of game states. First, the regional monopoly, that is, passengers on the left side of x_A all purchase tickets from airline A, and passengers on the right side of x_B all purchase tickets from airline B. Second, competing with each other, that is, passengers on the left side of x_A will purchase tickets on Airline B, and passengers on the right side of x_B will purchase tickets on Airline A.
Consider the following two-period model:
First-period: Both airlines sell directly, and simultaneously announce the prices of tickets in their respective direct sales channels is p_i , then passengers decide whether to buy now or wait to buy in the future.
Second-period: Airlines have two sales strategies: one is a single opaque sales channel sales, that is, the airline delegates all remaining air tickets to the intermediary I. The second is the oligopolistic competition market where airline direct sales and opaque sales coexist, that is, the airline has entrusted some of the remaining tickets to intermediary, and the rest is direct from the airline. The essence of both sales strategies is whether airlines enter the market in second-period. Intermediary began selling the remaining tickets and pricing them p_I on their own channels, according to the “share agreement” rule, the x of the profits from sales is distributed to the airline, and the two airlines allocate this according to the market sales share. Passenger consider the opaque channel from expectations about the probabilities that the ticket they will obtain will be from airline i (denoted by γ_i , $i = A, B$),then two airline revenue distribution ratio is $\gamma_i\delta$ and $\gamma_A + \gamma_B = 1$ .Passengers decide whether to buy or

leave according to the price announced by airlines and intermediary.

Further, suppose:

(1) Requirement determination. With the development of forecasting technology, the results of data forecasting have become more and more accurate, so it is possible to know clearly the demand of tickets before selling them.

(2) All participants are rational and complete, and airlines and passengers are well aware of the values of all parameters. Further, a rational expectation equilibrium exists if and only if expectations of prices (of revenue maximizing airline) are consistent with realizations and expectations over product availability are consistent with realizations. Employing the rational expectations equilibrium concept as in Yu et al (2017) [13] and Xi-Mei et al (2018) [14].

(3) The utility of passenger's intertemporal purchase is discounted, which is defined as θ, $\theta \in (0,1)$, and reflect the degree of passengers' strategy as in Hu et al (2016) [15]and Chen and Yan (2017) [16].

(4) This paper will encounter the ratio $\frac{V}{t}$ frequently in the analysis to follow. This ratio can be interpreted as a "brand preference adjusted valuation" for a product and it reflects the degree of competition between the airline. When $\frac{V}{t} \geq \frac{1}{2}$, passengers can gain non-negative utility by purchasing free tickets from airlines.

For the convenience of the following discussion, the decision of an airline under a single opaque sales channel is defined as MOS strategy (Monopoly Opaque Selling) and decision variables under this policy are represented by the "M".Then define the decision under the dual channel of airline direct sales and opaque sales as COS strategy(Competitive Opaque Selling) and decision variables under this policy are represented by the "C". This paper respectively exploring the equilibrium pricing strategies of the two airlines based on the relationship between supply and demand, the following analysis divides the market state into oversupply and undersupply.Variables j represent the state of supply in the marketand $j = L, H$ .Among them, "L" is oversupply and "H" is undersupply.

3. MOS Strategy

Under the MOS strategy, first-period is sold directly by the airline and second-period is sold separately by the opaque intermediary. Passengers net utility is $V - p_A - tx$ which purchases a ticket from airline A, and if purchases from airline B, his net utility is $V - p_A - t(1-x)$. Hence, any passenger who is considering buying an opaque product has an ex-ante expected utility given by $\max\{\theta\beta\left[V - p_{Ij}^M - \gamma_{Aj}^M t x_{Aj}^M - \gamma_{Bj}^M\left(1 - x_{Aj}^M\right)\right], 0\}$, we denote the probability that the passenger can obtain an opaque ticket by β.

Lemma 3.1: When airlines choose MOS strategy:

$$\gamma_{Aj}^M = \gamma_{Bj}^M = \frac{1}{2} \quad (1)$$

Proof: It's similar to Jerath et al (2010)[9].We will not go into details here. In fact, the symmetry of airlines A and B, it is easy to conclude that the opaque sales ticket is equal to the probability of the two airlines.

Lemma 3.1 is a significant result because it show that, if the airlines have equal capacities, then it is rational for passengers to expect that in the opaque channel, half of the tickets come from one airline and the other half from the other. So the rational expectations utilityof which purchases the opaque ticket is $\theta\beta\left[V - p_{Ij}^M - \frac{1}{2}\right]$.When markets reach equilibrium, expected utility of passengers is $\theta\beta\left[V - p_{Ij}^M - \frac{1}{2}\right] = 0$. Sointermediary prices at $p_{Ij}^M = V - \frac{1}{2}$.

3.1. Low demand（$K > J$）

whenthe tickets oversupply, all passengers can buy tickets and $\beta = 1$. The solution to the game is formalized in Proposition 3.1

Proposition 3.1 When demand is deterministic and there is ample capacity($K > J$),there is $\frac{V}{t} \in \left[\frac{1}{2}, \frac{4-\delta}{2(2-\delta)}\right)$, airlines will implement MOS strategy. The price charged by the airlines to all passenger will then be $p_{AL}^M = p_{BL}^M = t\left[\frac{V}{2t} + \frac{\delta}{4}\left(\frac{V}{t} - \frac{1}{2}\right)\right]$, and the maximized profit is given by

$$\pi_{AL}^M = \pi_{BL}^M = t\left[\left(\frac{1}{4} - \frac{\delta}{2} + \frac{3\delta^2}{16}\right)\frac{V^2}{t^2} + \left(\frac{3\delta}{4} - \frac{3\delta^2}{16}\right)\frac{V}{t} + \left(\frac{3\delta^2}{64} - \frac{\delta}{4}\right)\right]J .$$

Intermediary prices at $p_{IL}^M = V - \frac{1}{2}$,and attains the revenue

$$\pi_{IL}^M = t\left[\frac{(\delta-2)(1-\delta)}{2}\frac{V^2}{t^2} + \frac{(3-\delta)(1-\delta)}{2}\frac{V}{t} + \frac{(\delta-4)(1-\delta)}{8}\right]J$$

But when $\frac{V}{t} \in \left[\frac{4-\delta}{2(2-\delta)}, +\infty\right)$,the two airlines compete with each other, and airlines pursuing profit maximization will not implement opaque sales. At this time, when

$\frac{V}{t} \in \left[\frac{4-\delta}{2(2-\delta)}, \frac{3}{2}\right)$,the maximized profit is given by $\pi_{AL}^{M} = \pi_{BL}^{M} = \frac{t}{2}\left(\frac{V}{t} - \frac{1}{2}\right) J$. Otherwisethe maximized profit is given by $\pi_{AL}^{M} = \pi_{BL}^{M} = \frac{t}{2} J$ when $\pi_{AL}^{M} = \pi_{BL}^{M} = \frac{t}{2} J$.

Proof:First, discuss the regional monopoly of the two airlines. According to the definition of regional monopoly, airlines have sales of remaining tickets after direct sale of first-period. For airline A located at x_{AL}^{M} must be indifferent between buying from the opaque intermediary, and passengers get the same benefits when they buy a ticket from an airline or an intermediary. So when $V - p_{AL}^{M} - tx_{AL}^{M} = \theta\beta \left[V - p_{IL}^{M} - \gamma_{AL}^{M} t x_{AL}^{M} - \gamma_{BL}^{M} t\left(1 - x_{AL}^{M}\right)\right]$, according to the analysis of lemma 1, the right side of the above equation is equal to 0 and $p_{AL}^{M} = V - tx_{AL}^{M}$,the profit of airlines is given by

$$\pi_{AL}^{M} = p_{AL}^{M} x_{AL}^{M} J + \delta \gamma_{AL}^{M} p_{IL}^{M}\left(x_{BL}^{M} - x_{AL}^{M}\right) J$$

$$= \left(V - tx_{AL}^{M}\right) x_{AL}^{M} J + \frac{\delta}{2}\left(V - \frac{t}{2}\right)\left(x_{BL}^{M} - x_{AL}^{M}\right) J$$

when $\frac{\partial \pi_{AL}^{M}}{\partial x_{AL}^{M}} = 0$, we have $x_{AL}^{M} = 1 - x_{BL}^{M} = \frac{V}{2t} - \frac{\delta}{4}\left(\frac{V}{t} - \frac{1}{2}\right)$, so the price charged by the airlines to all passenger will then be $p_{AL}^{M} = t\left[\frac{V}{2t} + \frac{\delta}{4}\left(\frac{V}{t} - \frac{1}{2}\right)\right]$, and the airline maximized profit is given by

$$\pi_{AL}^{M} = t\left[\left(\frac{1}{4} - \frac{\delta}{2} + \frac{3\delta^2}{16}\right)\frac{V^2}{t^2} + \left(\frac{3\delta}{4} - \frac{3\delta^2}{16}\right)\frac{V}{t} + \left(\frac{3\delta^2}{64} - \frac{\delta}{4}\right)\right] J$$

Intermediary attains the revenue is

$$\pi_{IL}^{M} = (1-\delta) p_{IL}^{M}\left(x_{BL}^{M} - x_{AL}^{M}\right) J$$

$$= t\left[\frac{(\delta-2)(1-\delta)}{2}\frac{V^2}{t^2} + \frac{(3-\delta)(1-\delta)}{2}\frac{V}{t} + \frac{(\delta-4)(1-\delta)}{8}\right] J$$

Under the regional monopoly of the airlineneed to meet $x_{AL}^{M} < \frac{1}{2}$ and $\frac{V}{t} < \frac{4-\delta}{2(2-\delta)}$

When $\frac{V}{t} \geq \frac{4-\delta}{2(2-\delta)}$, airlines that seek maximum profits will not unilaterally deviate from equilibrium of $x_{AL}^{M} = 1 - x_{BL}^{M} = \frac{1}{2}$. We have $V - p_{AL}^{M} - tx_{AL}^{M} = 0$, so $p_{AL}^{M} = p_{BL}^{M} = V - \frac{t}{2}$, and we get the profit $\pi_{AL}^{M} = \pi_{BL}^{M} = \frac{t}{2}\left(\frac{V}{t} - \frac{1}{2}\right) J$ .Passengers purchase indifference when airlines compete with each other. We get the $V - p_{AL}^{M} - t\tilde{x}_{AL}^{M} = V - p_{BL}^{M} - t\left(1 - \tilde{x}_{AL}^{M}\right)$, then $\tilde{x}_{AL}^{M} = \frac{1}{2} + \frac{\left(p_{BL}^{M} - p_{AL}^{M}\right)}{2t}$. When the income function and the derivative is equal to 0, $\tilde{x}_{AL}^{M} = \frac{1}{2}$, $p_{AL}^{M} = p_{BL}^{M} = t$, $\pi_{AL}^{M} = \pi_{BL}^{M} = \frac{t}{2} J$. We must have $V - p_{AL}^{M} - tx_{AL}^{M} \geq 0$, so $\frac{V}{t} \geq \frac{3}{2}$. Also when $\frac{V}{t} \in \left[\frac{4-\delta}{2(2-\delta)}, \frac{3}{2}\right)$, we get the price of $p_{AL}^{M} = p_{BL}^{M} = V - \frac{t}{2}$; On the other hand, when $\frac{V}{t} \in \left[\frac{3}{2}, +\infty\right)$, there have $p_{AL}^{M} = p_{BL}^{M} = t$,Balanced income is $\pi_{AL}^{M} = \pi_{BL}^{M} = \frac{t}{2} J$ .In both cases, airlines that pursue profit maximization will implement direct sales.

3.2. High demand（$K < J$）

whenthe tickets undersupply, the farthest sales distance of airline A is $\frac{K}{2J}$, further the $\frac{K}{2J} < \frac{1}{2}$, it is not possible for some passenger to get tickets. The solution to the game is formalized in Proposition 3.2.

Proposition 3.2 When demand is deterministic and there is ample capacity($K < J$), there is $\frac{V}{t} \in \left[\frac{1}{2}, \frac{4\frac{K}{J} - \delta}{2(2-\delta)}\right)$, airlines will implement MOS strategy. The price charged by the airlines to all passenger will then be $p_{AH}^{M} = p_{BH}^{M} = t\left[\frac{V}{2t} + \frac{\delta}{4}\left(\frac{V}{t} - \frac{1}{2}\right)\right]$, and the maximized profit is given by

$$\pi_{AH}^{M} = \pi_{BH}^{M} = t\left[\left(\frac{J}{4} + \frac{3\delta^2 J}{16} - \frac{\delta J}{2}\right)\frac{V^2}{t^2} + \left(\frac{\delta K}{2} - \frac{3\delta^2 J}{16} + \frac{\delta J}{4}\right)\frac{V}{t} + \left(\frac{3\delta^2 J}{64} - \frac{\delta K}{4}\right)\right] J .$$

Intermediary prices at $p_{IH}^{M} = V - \frac{1}{2}$,and attains the revenue

$$\pi_{IH}^{M} = t\left[\left(\frac{3\delta J}{2} - \frac{\delta^2 J}{2} - J\right)\frac{V^2}{t^2} + \left(K - \delta K - \delta J + \frac{\delta^2 J}{2} + \frac{J}{2}\right)\frac{V}{t} + \left(\frac{\delta K}{2} - \frac{K}{2} + \frac{\delta J}{8} - \frac{\delta^2 J}{8}\right)\right] J$$

But when $\frac{V}{t} \in \left[\frac{4\frac{K}{J} - \delta}{2(2-\delta)}, +\infty\right)$,the two airlines compete with each other, and airlines pursuing profit maximization will not implement opaque sales. At this time,the maximized profit is given by $\pi_{AH}^{M} = \pi_{BH}^{M} = t\left(\frac{V}{t} - \frac{K}{2J}\right)\frac{K}{2}$.

Proof:First, discuss the regional monopoly of the two airlines.According to the definition of regional monopoly, airlines have sales of remaining tickets after direct sale of first-period. For airline A located at x_{AH}^{M} must be indifferent between buying from the opaque intermediary and there is $x_{AH}^{M}<\frac{K}{2J}$, passengers get the same benefits when they buy a ticket from an airline or an intermediary. So when

$$V-p_{AH}^{M}-tx_{AH}^{M}=\theta\beta\left[V-p_{IH}^{M}-\gamma_{AH}^{M}tx_{AH}^{M}-\gamma_{BH}^{M}t\left(1-x_{AH}^{M}\right)\right]$$

which $\beta=\frac{K-x_{AH}^{M}J-\left(1-x_{BH}^{M}\right)J}{\left(x_{BH}^{M}-x_{AH}^{M}\right)J}$. According to the analysis of lemma 1, the right side of the above equation is equal to 0 and $p_{AH}^{M}=V-tx_{AH}^{M}$,the profit of airlines is given by

$$\pi_{AH}^{M}=p_{AH}^{M}x_{AH}^{M}J+\delta\gamma_{AH}^{M}p_{IH}^{M}\left(\frac{K}{J}-x_{AH}^{M}-\left(1-x_{BH}^{M}\right)\right)J$$

$$=\left(V-tx_{AH}^{M}\right)x_{AH}^{M}J+\frac{\delta}{2}\left(V-\frac{t}{2}\right)\left(\frac{K}{J}-x_{AH}^{M}-\left(1-x_{BH}^{M}\right)\right)J$$

when $\frac{\partial\pi_{AH}^{M}}{\partial x_{AH}^{M}}=0$, we have $x_{AH}^{M}=1-x_{BH}^{M}=\frac{V}{2t}-\frac{\delta}{4}\left(\frac{V}{t}-\frac{1}{2}\right)$, so the price charged by the airlines to all passenger will then be $p_{AH}^{M}=t\left[\frac{V}{2t}+\frac{\delta}{4}\left(\frac{V}{t}-\frac{1}{2}\right)\right]$, and the airline maximized profit is given by

$$\pi_{AH}^{M}=t\left[\left(\frac{J}{4}+\frac{3\delta^{2}J}{16}-\frac{\delta J}{2}\right)\frac{V^{2}}{t^{2}}+\left(\frac{\delta K}{2}-\frac{3\delta^{2}J}{16}+\frac{\delta J}{4}\right)\frac{V}{t}+\left(\frac{3\delta^{2}J}{64}-\frac{\delta K}{4}\right)\right]J$$

Intermediary attains the revenue is

$$\pi_{IH}^{M}=\left(1-\delta\right)p_{IH}^{M}x_{IH}^{M}J$$

$$=t\left(1-\delta\right)\left(\frac{V}{t}-\frac{1}{2}\right)\left\{K-\left[\frac{V}{t}-\frac{\delta}{2}\left(\frac{V}{t}-\frac{1}{2}\right)\right]J\right\}$$

$$=t\left[\left(\frac{3\delta J}{2}-\frac{\delta^{2}J}{2}-J\right)\frac{V^{2}}{t^{2}}+\left(K-\delta K-\delta J+\frac{\delta^{2}J}{2}+\frac{J}{2}\right)\frac{V}{t}+\left(\frac{\delta K}{2}-\frac{K}{2}+\frac{\delta J}{8}-\frac{\delta^{2}J}{8}\right)\right]J$$

Under the regional monopoly of the airlineneed to meet $x_{AH}^{M}<\frac{K}{2J}$ and $\frac{V}{t}<\frac{4\frac{K}{J}-\delta}{2\left(2-\delta\right)}$.

When $\frac{V}{t}\in[\frac{4\frac{K}{J}-\delta}{2\left(2-\delta\right)},+\infty)$,airlines that seek maximum profits will not unilaterally deviate from equilibrium of $x_{AH}^{M}=1-x_{BH}^{M}=\frac{K}{2J}$. So we get the $V-p_{AH}^{M}-tx_{AH}^{M}=0$, further the airline price by $p_{AH}^{M}=p_{BH}^{M}=t\left(\frac{V}{t}-\frac{K}{2J}\right)$. Balanced income is $\pi_{AH}^{M}=\pi_{BH}^{M}=t\left(\frac{V}{t}-\frac{K}{2J}\right)\frac{K}{2}$, in this case, airlines that pursue profit maximization will implement direct sales.

4. COS Strategy

Under the COS strategy, first-period is sold directly by the airline and second-period is sold separately by the airline and opaque intermediary. Passengers net utility is $V-p_{A}-tx$ which purchases a ticket from airline A, and if purchases from airline B, his net utility is $V-p_{A}-t\left(1-x\right)$. Hence, any passenger who is considering buying an opaque product has an ex-ante expected utility given by $\max\{\theta\beta[V-p_{Ij}^{C}-\gamma_{Aj}^{C}tx_{Aj}^{C}-\gamma_{Bj}^{C}\left(1-x_{Aj}^{C}\right)],0\}$.

Lemma 4.1：When airlines choose COS strategy：

$$\gamma_{Aj}^{C}=\gamma_{Bj}^{C}=\frac{1}{2} \quad (2)$$

Proof: It's similar to Jerath et al (2010)[9]. We will not go into details here.

With the same reason, Lemma 4.1 is a significant result because it showed that $\theta\beta\left[V-p_{Ij}^{C}-\frac{1}{2}\right]=0$, so $p_{Ij}^{C}=V-\frac{1}{2}$.

4.1. Low demand（$K>J$）

It's similar to section 3.1, this section analyzes the equilibrium decision of COS strategy under oversupply. The solution to the game is formalized in Proposition 4.1

Proposition 4.1When demand is deterministic and there is ample capacity （ $K>J$ ）, there is $\frac{V}{t}\in\left[\frac{1}{2},\frac{(\theta+3)-2\delta}{4(1-\delta)}\right)$, airlines will implement COS strategy. The price charged by the airlines in first-period to all passenger will then be $p_{AL}^{C}=p_{BL}^{C}=t\left\{\frac{V}{t}-\frac{(1+\theta)\left[(1-\delta)\frac{V}{t}+\frac{\delta}{2}\right]}{\theta+3}\right\}$, second-period is $p_{AL}^{C}{}'=p_{BL}^{C}{}'=t\left[\frac{V}{t}-\frac{2(1-\delta)\frac{V}{t}+\delta}{\theta+3}\right]$, and the maximized profit is given by

$$\pi_{AL}^{C}=\pi_{BL}^{C}=\frac{t}{\theta+3}\left\{\left(\delta^{2}-2\delta+1\right)\frac{V^{2}}{t^{2}}+\left[\frac{(\theta+3)\delta}{2}+\delta-\delta^{2}\right]\frac{V}{t}+\frac{\delta^{2}}{4}-\frac{(\theta+3)\delta}{4}\right\}J\cdot$$

Intermediary prices at $p_{IL}^{C}=V-\frac{1}{2}$, and attains the revenue

$$\pi_{IL}^{C}=\frac{t}{\theta+3}\left[-4(1-\delta)^{2}\frac{V^{2}}{t^{2}}+(\theta+3)(1-\delta)\frac{V}{t}+\left(4\delta^{2}-6\delta-2\right)\frac{V}{t}-\frac{(\theta+3)(1-\delta)}{2}+\delta(1-\delta)\right]J$$

But when $\frac{V}{t}\in\left[\frac{(\theta+3)-2\delta}{4(1-\delta)},+\infty\right)$, the two airlines compete with each other, and airlines pursuing profit maximization will not implement opaque sales. It's similar to section 3.1.

Proof:First, discuss the regional monopoly of the two airlines. For airline A the passenger located at x_{iL}^{C} in first-periodmust be indifferent between buying from the airlinein first-period and located at $x_{iL}^{C'}$ in second-period must be indifferent between buying from the opaque intermediary. From that we have $V-p_{AL}^{C}-tx_{AL}^{C}=\theta\left(V-p_{AL}^{C\,'}-tx_{AL}^{C}\right)$, $p_{AL}^{C}=\theta p_{AL}^{C\,'}+(1-\theta)V-(1-\theta)tx_{AL}^{C}$, and $\theta\left(V-p_{AL}^{C\,'}-tx_{AL}^{C\,'}\right)=\theta\beta\left[V-p_{IL}^{C}-\gamma_{AL}^{C}tx_{AL}^{C\,'}-\gamma_{BL}^{C}t\left(1-x_{AL}^{C\,'}\right)\right]$,

$p_{AL}^{C\,'}=V-tx_{AL}^{C\,'}$. So

$$p_{AL}^{C}=V-(1-\theta)tx_{AL}^{C}-\theta tx_{AL}^{C\,'}$$

the profit of airlines is given by

$$\pi_{AL}^{C}=p_{AL}^{C}x_{AL}^{C}J+p_{AL}^{C\,'}\left(x_{AL}^{C\,'}-x_{AL}^{C}\right)J+2\delta\gamma_{AL}^{C}p_{IL}^{C}\left(\frac{1}{2}-x_{AL}^{C\,'}\right)J$$

$$=\left[(\theta-1)tx_{AL}^{C\,2}-tx_{AL}^{C\,'2}+(1-\theta)tx_{AL}^{C}x_{AL}^{C\,'}+\left(V-\delta V+\frac{\delta t}{2}\right)x_{AL}^{C\,'}+\frac{\delta V}{2}-\frac{\delta t}{2}\right]J$$

when $\frac{\partial\pi_{AL}^{C}}{\partial x_{AL}^{C}}=0$, we have $x_{AL}^{C}=\frac{(1-\delta)\frac{V}{t}+\frac{\delta}{2}}{\theta+3}$ and $x_{AL}^{C\,'}=\frac{2(1-\delta)\frac{V}{t}+\delta}{\theta+3}$, so the price charged by the airlines in first-period to all passenger will then be

$$p_{AL}^{C}=V-\frac{(1+\theta)[(1-\delta)\frac{V}{t}+\frac{\delta}{2}]t}{\theta+3}=t\left\{\frac{V}{t}-\frac{(1+\theta)\left[(1-\delta)\frac{V}{t}+\frac{\delta}{2}\right]}{\theta+3}\right\}$$

and first-period is

$$P_{AL}^{C\,'}=V-\frac{[2(1-\delta)\frac{V}{t}+\delta]t}{\theta+3}=t\left[\frac{V}{t}-\frac{2(1-\delta)\frac{V}{t}+\delta}{\theta+3}\right]$$

The airline maximized profit is given by

$$\pi_{AL}^{C}=\frac{t}{\theta+3}\left\{\left(\delta^{2}-2\delta+1\right)\frac{V^{2}}{t^{2}}+\left[\frac{(\theta+3)\delta}{2}+\delta-\delta^{2}\right]\frac{V}{t}+\frac{\delta^{2}}{4}-\frac{(\theta+3)\delta}{4}\right\}J$$

Intermediary attains the revenue is

$$\pi_{IL}^{C}=2\pi(1-\delta)\,p_{IL}^{C}\left(\frac{1}{2}-x_{AL}^{C\,'}\right)J$$

$$=\frac{t}{\theta+3}\left[-4(1-\delta)^{2}\frac{V^{2}}{t^{2}}+(\theta+3)(1-\delta)\frac{V}{t}+\left(4\delta^{2}-6\delta-2\right)\frac{V}{t}-\frac{(\theta+3)(1-\delta)}{2}+\delta(1-\delta)\right]J$$

Under the regional monopoly of the airlineneed to meet $x_{AL}^{C\,'}<\frac{1}{2}$ and $\frac{V}{t}<\frac{(\theta+3)-2\delta}{4(1-\delta)}$

When $\frac{V}{t}\in\left[\frac{(\theta+3)-2\delta}{4(1-\delta)},+\infty\right)$, airlines that pursue profit maximization will implement direct sales and will not unilaterally deviate from section 3.1 equilibrium.

4.2. High demand（$K<J$）

It's similar to section 3.2, this section analyzes the equilibrium decision of COS strategy under undersupply. The solution to the game is formalized in Proposition 4.2.

Proposition 4.2 When demand is deterministic and there is ample capacity ($K<J$), there is $\frac{V}{t}\in\left[\frac{1}{2},\frac{\frac{(\theta+3)K}{2J}-\delta}{2(1-\delta)}\right)$, airlines will implement COS strategy. The price charged by the airlines in first-period to all passenger will then be $p_{AH}^{C\,'}=p_{BH}^{C\,'}=t\left[\frac{V}{t}-\frac{2(1-\delta)\frac{V}{t}+\delta}{\theta+3}\right]$, second-period is $p_{AL}^{C\,'}=p_{BL}^{C\,'}=t\left[\frac{V}{t}-\frac{2(1-\delta)\frac{V}{t}+\delta}{\theta+3}\right]$, and the maximized profit is given by

$$\pi_{BH}^{C}=\frac{t}{\theta+3}\left\{(1-\delta)^{2}\frac{V^{2}}{t^{2}}+\left[\frac{(\theta+3)\delta K}{2J}-\delta^{2}+\delta\right]\frac{V}{t}+\left[\frac{\delta^{2}}{4}-\frac{(\theta+3)\delta K}{4J}\right]\right\}J\cdot$$

Intermediary prices at $p_{IH}^{C}=V-\frac{1}{2}$, and attains the revenue

$$\pi_{IH}^{C}=\frac{t}{\theta+3}\left\{-4(1-\delta)^{2}\frac{V^{2}}{t^{2}}+\left[\frac{(\theta+3)(1-\delta)K}{J}+4\delta^{2}-6\delta+2\right]\frac{V}{t}-\frac{(\theta+3)(1-\delta)K}{2J}+\delta(1-\delta)\right\}J$$

But when $\frac{V}{t}\in\left[\frac{\frac{(\theta+3)K}{2J}-\delta}{2(1-\delta)},+\infty\right)$, the two airlines compete with each other, and airlines pursuing profit maximization will not implement opaque sales. It's similar to section 3.2.

Proof: (1) discuss the regional monopoly of the two airlines. For airline A the passenger located at x_{iH}^{C} in first-period must be indifferent between buying from the airline in first-period and located at $x_{iH}^{C'}$ in second-period must be indifferent between buying from the opaque intermediary, so we have $x_{AH}^{C} < x_{AH}^{C'} < \frac{K}{2J}$, $\beta = \frac{K - x_{AH}^{C}{'} J - \left(1 - x_{BH}^{C}{'}\right) J}{\left(x_{BH}^{C}{'} - x_{AH}^{C}{'}\right) J}$. From that we have

$$V - p_{AH}^{C} - tx_{AH}^{C} = \theta\left(V - p_{AH}^{C}{'} - tx_{AH}^{C}\right)$$

$$p_{AH}^{C} = \theta p_{AH}^{C}{'} + (1-\theta)V - (1-\theta)tx_{AH}^{C},$$

and

$$\theta\left(V - p_{AH}^{C}{'} - tx_{AH}^{C}{'}\right) = \theta\beta\left[V - p_{IH}^{C} - \gamma_{AH}^{C} tx_{AH}^{C}{'} - \gamma_{BH}^{C} t\left(1 - x_{AH}^{C}{'}\right)\right]$$

$$p_{AH}^{C}{'} = V - tx_{AH}^{C}{'}$$

So $p_{AH}^{C} = V - (1-\theta)tx_{AL}^{C} - \theta tx_{AH}^{C}{'}$, the profit of airlines is given by

$$\pi_{AH}^{C} = p_{AH}^{C} x_{AH}^{C} J + p_{AH}^{C}{'}\left(x_{AH}^{C}{'} - x_{AH}^{C}\right) J + \delta\gamma_{AH}^{C} p_{IH}^{C}\left[\frac{K}{J} - x_{AH}^{D}{'} - \left(1 - x_{BH}^{D}{'}\right)\right] J$$

$$= \left[(\theta-1)tx_{AH}^{C\,2} - tx_{AH}^{C\,'2} + (1-\theta)tx_{AH}^{C} x_{AH}^{C}{'} + \left(V - \delta V + \frac{\delta t}{2}\right) x_{AH}^{C}{'} + \frac{\delta KV}{2} - \frac{\delta Kt}{2}\right] J$$

when $\frac{\partial \pi_{AH}^{C}}{\partial x_{AH}^{C}} = 0$, we have $x_{AH}^{C} = \frac{(1-\delta)\frac{V}{t} + \frac{\delta}{2}}{\theta+3}$ and $x_{AH}^{C}{'} = \frac{2(1-\delta)\frac{V}{t} + \delta}{\theta+3}$, so the price charged by the airlines in first-period to all passenger will then be

$$p_{AH}^{C} = V - \frac{(1+\theta)[(1-\delta)\frac{V}{t} + \frac{\delta}{2}]t}{\theta+3} = t\left\{\frac{V}{t} - \frac{(1+\theta)\left[(1-\delta)\frac{V}{t} + \frac{\delta}{2}\right]}{\theta+3}\right\}$$

and first-period is

$$P_{AH}^{C}{'} = V - \frac{[2(1-\delta)\frac{V}{t} + \delta]t}{\theta+3} = t\left[\frac{V}{t} - \frac{2(1-\delta)\frac{V}{t} + \delta}{\theta+3}\right]$$

The airline maximized profit is given by

$$\pi_{AH}^{C} = p_{AH}^{C} x_{AH}^{C} J + p_{AH}^{C}{'}\left(x_{AH}^{C}{'} - x_{AH}^{C}\right) J + 2\delta\gamma_{AH}^{C} p_{IH}^{C}\left(\frac{K}{2J} - x_{AH}^{C}{'}\right) J$$

$$= \frac{t}{\theta+3}\left\{(1-\delta)^2 \frac{V^2}{t^2} + \left[\frac{(\theta+3)\delta K}{2J} + \delta - \delta^2\right]\frac{V}{t} + \frac{\delta^2}{4} - \frac{(\theta+3)\delta K}{4J}\right\} J$$

Intermediary attains the revenue is

$$\pi_{IH}^{C} = 2(1-\delta) P_{IH}^{C}\left(\frac{K}{2J} - x_{AH}^{C}{'}\right) J$$

$$= 2t(1-\delta)\left(\frac{V}{t} - \frac{1}{2}\right)\left[\frac{K}{2J} - \frac{2(1-\delta)\frac{V}{t} + \delta}{\theta+3}\right] J$$

$$= \frac{t}{\theta+3}\left\{-4(1-\delta)^2 \frac{V^2}{t^2} + \left[\frac{(\theta+3)(1-\delta)K}{J} + 4\delta^2 - 6\delta + 2\right]\frac{V}{t} - \frac{(\theta+3)(1-\delta)K}{2J} + \delta(1-\delta)\right\} J$$

Under the regional monopoly of the airlineneed to meet $x_{AH}^{C}{'} < \frac{K}{2J}$ and $\frac{V}{t} < \frac{\frac{(\theta+3)K}{2J} - \delta}{2(1-\delta)}$

When $\frac{V}{t} \in \left[\frac{\frac{(\theta+3)K}{2J} - \delta}{2(1-\delta)}, +\infty\right)$, airlines that pursue profit maximization will implement direct sales and will not unilaterally deviate from section 3.2equilibrium.

From Proposition 3.1 to Proposition 4.2, it can be seen that the smaller purchase intention of the passenger (ie, when $\frac{V}{t}$ is small) ensures that the airline can implement a regional monopoly in the direct sales, and there are surplus tickets in the second-period for opaque sales. When the passengers have a greater willingness to purchase, the two airlines will maximize the revenue, and there will be no remaining air tickets and no possibility of opaque sales while fully competing. From the above analysis, the following inference can be directly obtained:

Inference: 1)There is existing $(\frac{V}{t})_1^* = \frac{\delta(1+3\theta)}{4(1-\theta) + 2\delta(1+3\theta)}$, when $\frac{V}{t} > (\frac{V}{t})_1^*$, Airlines in first-period are priced higher under the COS strategy than the MOS strategy; otherwise, the price under the MOS strategy is higher;

(2) The sales of airlines in first-period under the COS strategy are lower than the MOS strategy;

(3) There is existing $(\frac{V}{t})_2^* = \frac{\delta(\theta-5)}{4(1-\theta) + 2\delta(\theta-5)}$, when $\frac{V}{t} > (\frac{V}{t})_2^*$, the sales volume of the intermediary under the COS strategy is higher than the MOS strategy.

Proof: (1) According to the above proof, regardless of whether the market demand situation is high or low, under the MOS strategy, the airline A in first-period is priced as $p_{Aj}^{M} = t\left[\frac{V}{2t} + \frac{\delta}{4}\left(\frac{V}{t} - \frac{1}{2}\right)\right]$, and the airline A in first-period is priced as $p_{Aj}^{C} = t\left\{\frac{V}{t} - \frac{(1+\theta)\left[(1-\delta)\frac{V}{t} + \frac{\delta}{2}\right]}{\theta+3}\right\}$ under the MOS strategy. Cause the $p_{Aj}^{C} = p_{Aj}^{D}$, so

$$V - \frac{(1+\theta)[(1-\delta)\frac{V}{t} + \frac{\delta}{2}]t}{\theta+3} = t[\frac{V}{2t} + \frac{\delta}{4}(\frac{V}{t} - \frac{1}{2})]$$

From that wo get the $\frac{V}{t}=\frac{\delta(1+3\theta)}{4(1-\theta)+2\delta(1+3\theta)}$. So when $\frac{V}{t}>\frac{\delta(1+3\theta)}{4(1-\theta)+2\delta(1+3\theta)}$, airlines under the COS strategy in first-period are priced higher.

(2) Under the MOS strategy, the sales of airlines in first-period $x_{Aj}^{M}=1-x_{Bj}^{M}=\frac{V}{2t}-\frac{\delta}{4}\left(\frac{V}{t}-\frac{1}{2}\right)$ and under the MOS strategy in second-period is $x_{Aj}^{C}=\frac{(1-\delta)\frac{V}{t}+\frac{\delta}{2}}{\theta+3}$.

Cause the $\Delta x_{Aj}=x_{Aj}^{M}-x_{Aj}^{C}$, we have the $\Delta x_{Aj}=\frac{V}{2t}-\frac{\delta}{4}\left(\frac{V}{t}-\frac{1}{2}\right)-\frac{(1-\delta)\frac{V}{t}+\frac{\delta}{2}}{\theta+3}=t\left[\frac{\theta(2-\delta)+\delta+2}{4(\theta+3)}\frac{V}{t}-\frac{\delta}{2(\theta+3)}+\frac{\delta}{8}\right]>0$.

Explain that under the COS strategy, the sales volume of airline infirst-period under the COS strategy is lower than that in the MOS strategy.

(3) When cause $x_{IL}^{C}=x_{IL}^{M}$ and $x_{IH}^{C}=x_{IH}^{M}$, all obtained $\frac{V}{t}=\frac{\delta(\theta-5)}{4(1-\theta)+2\delta(\theta-5)}$, so Intermediaries sell more under the COS strategy in $\frac{V}{t}>\frac{\delta(\theta-5)}{4(1-\theta)+2\delta(\theta-5)}$.

5. Numerical Study

This section simulates the benefits of airlines and intermediaries under MOS strategy and DOS strategy through numerical examples (maple2016), and focuses on the effectiveness of the two strategies. The basic parameters are set as follows: the total number of passengers is standardized to 1, $t=1$,and when the supply is less than demand, $K=0.8$.Because of the symmetry, take airline A as an example.

5.1. The impact of different purchase intentions on airline decision making

When $\theta=0.3$, the relationship between passenger purchase intention and airline first period pricing, sales volume and intermediary sales volume under two strategies is simulated, as shown in Figure1, Figure 2 and Figure 3.

As can be seen from Figure 1,when $\frac{V}{t}$ is greater than a certain threshold, the price of the first period airline under the COS strategy is higher than that of the MOS strategy. Meanwhile, in Figure 2, when $\frac{V}{t}$ is larger than a certain threshold, the sales volume of the intermediary under the COS policy is higher. It indicates that passenger's purchase intention will affect the pricing and sales volume of the two strategies, and this effect is linear. And it can be seen from Figure 3 that the difference in sales volume under the two sales strategies is always greater than 0 no matter what value the income distribution and the degree of policy are taken, which verifies the inference.

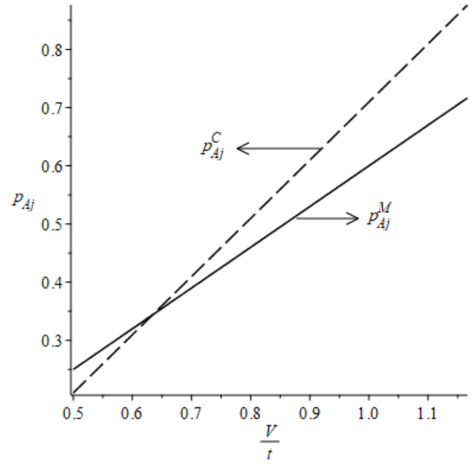

Figure 1. First-period pricing of airline A

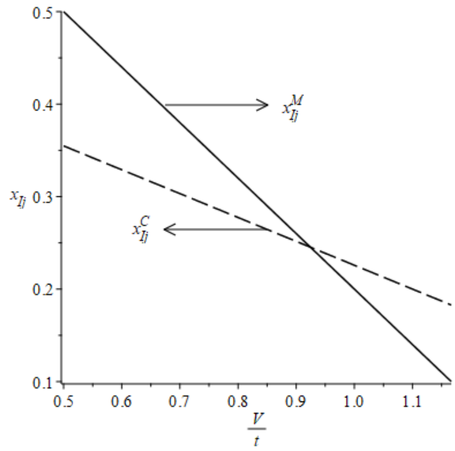

Figure 2. Intermediary coverage

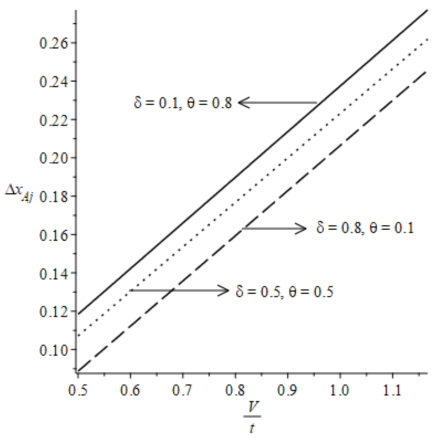

Figure 3. Influence of different parameters of airline in first-period

5.2. Effectiveness analysis of two strategies

Taking A as an example, and taking and $\theta = 0.9$ respectively, the results of the airline's Equilibrium income simulation under COS and MOS strategies are shown in Figure4-7.

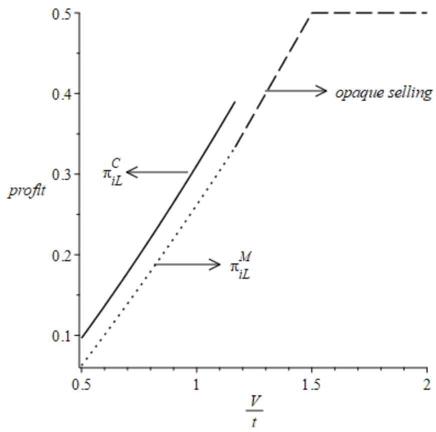

Figure 4. Airline A's income when $\theta = 0.5$

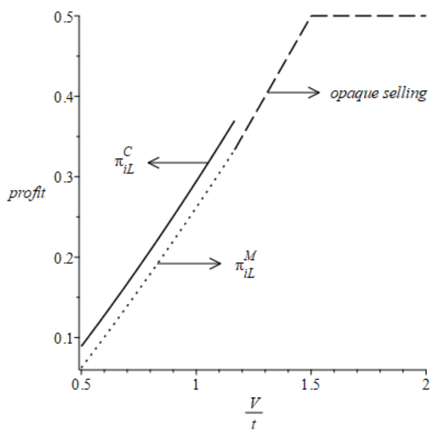

Figure 5. Airline A's income when $\theta = 0.9$

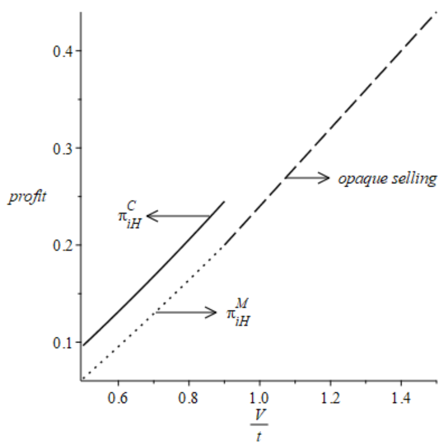

Figure 6. Airline A's income when $\theta = 0.5$

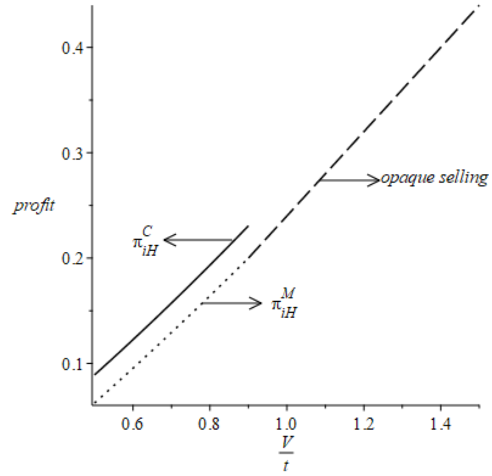

Figure 7. Airline A's income when $\theta = 0.9$

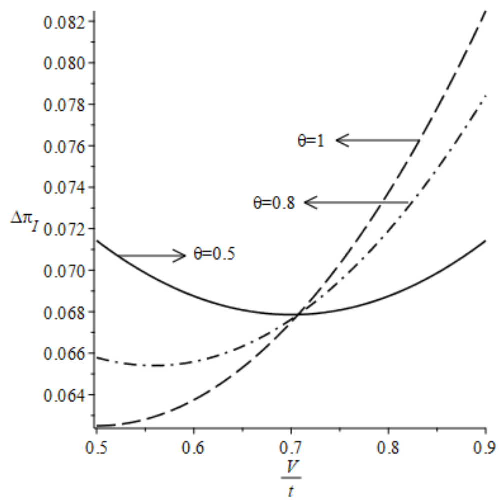

Figure 8. Impact of different strategies on intermediary earnings

It can be seen from Figure4 and Figure6 that, no mattersupply exceeds demand or supply is less than demand, with the increase of $\frac{V}{t}$, the revenue of airlines in the dual-channel sales mode is higher than that of a single opaque sales strategy under the specific degree of passenger strategy and the income distribution ratio between airlines and intermediaries. Combined withFigure5 and Figure7, it can be seen that with the increase of the degree of passengers' strategic, the revenue of airlines under the dual channels is closer to the single opaque sales channel which indicate that the greater the degree of passengers' strategic, the less effective the sales strategy of airlines participating in competition will be in promoting the increase of revenue. At the same time, it can be seen from Figure8 that, when the airline participates in the competition, the income difference between the intermediaries is more than 0 no matter the supply exceeds demand or supply is less than demand, which means that the intermediary has the motivation to implement the COS strategy. Therefore, for airlines and intermediaries, the dual channel oligopoly competition strategy are effective.

6. Conclusion

Aiming at the opaque distribution mode popular in airline ticket sales in recent years, this paper establishes the oligopoly competitive price models of single opaque marketing channel and the oligopoly competitive price models of direct marketing and opaque sales coexistence under the Seller pricing opaque sales model, obtains the corresponding market equilibrium price and equilibrium profit, and analyzes the conditions for the establishment of equilibrium. At the same time, taking the passenger's willingness to pay as a reference value, this paper discussed the income of airlines and intermediaries in the coexistence of opaque sales channels and direct sales channels of airlines.

It shows:

(1) opaque sales exist only when passengers are less willing to buy;

(2) The purchase intention of passengers has a greater impact on the pricing and sales volume of the two strategies, and the impact is linear;

(3) The dual-channel sales model can effectively increase the equilibrium revenue of airlines and intermediaries, and with the increase of the strategic degree of passengers, the revenues of airlines under the dual channels are closer to a single opaque sales channel.

Opaque sales of perishable products attract more price-sensitive customers with low prices, and maximize the sales of products whose value is dying. This reflects the short-term and intuitive advantages of improving revenue, and is now widely used in hotels, tickets and other perishable products. However, this paper only makes a preliminary study on its effectiveness. In the future, it can further expand on the heterogeneity of passenger purchase behavior and the difference of service quality of airlines, and deeply investigate the "nibbling effect" between customers and the influence on enterprise brands under this sales model.

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