

Load Flow Analysis and Matlab Study in Gauss-Seidel Method

Qi Liu, Liyi Wang

Department of Electrical Information, Shandong University of Science and Technology, Jinan, 250031, China

Abstract: In this study, Gauss-Seidel algorithm about iteration and flow analysis was used to determine the critical values of each node and the power flows on each transmission line in a power system. This essay used MATLAB to program and Power World to simulator, compared the theoretical values with the outcome in simulation and discussed the iteration numbers in both method and found that the values were correct. The iteration number of Gauss-Seidel algorithm in program under this certain system was 34 and it just cost 0.0246s, which proved Gauss-Seidel algorithm is valid and reliable.

Keywords: Load flow; Gauss-seidel method; Matlab

1. Introduction

The aim of this study is to use Gauss-Seidel algorithm in MATLAB to analyze the load flows of a power system with 5 buses and use the results to access the reliability and accuracy of this method under the given conditions.

2. Theory on Load Flows Analysis

A load flow (sometimes known as a power flow) is power system jargon for the steady-state solution of an electrical power network. [1] Load flow calculation is the most basic and widely used electrical calculation in electrical systems, especially in real-time monitoring, grid planning and design of grid operation status. Through this calculation, the operation state of the network under certain operating conditions and network structure can be determined which is including the voltage value, voltage angle, active power and reactive power of each bus, the power losses, active and reactive power flowing on the transmission line.

For the simple open network or ring network, there are two conditions: the load power and the generator voltage are known, or the load power and the load voltage are known. The power flows of both conditions can be analyzed through calculating the power losses and voltage dropping on the transmission line. [2] However, for the large power system which has so many buses and complex structure and connection, it needed to be solved as an equivalent model with certain parameters and analyzed in computer algorithms which can perform many times iteration in a very short time. Since the 50s in 20 century, computer algorithms are used widely in electrical engineering. The popular algorithms are: P-Q decomposition method, Gauss-Seidel method and Newton-raphson method. This essay used Gauss-Seidel method to do iterations and load flow analysis

3. Mathematical Model and Constraints of Power Flow Calculation

The network in the load flow analysis is consist of transformers, transmission lines, capacitors, reactors etc. To do a load flow analysis, the first step is setting the equivalent mathematical model with certain parameters of these plants. This linear model is usually series and parallel circuits. Node method is often used in this model.

The basic formula of load flow analysis is:

$$\dot{I}_i = \sum_{j=1}^n Y_{ij} \dot{V}_j \quad (i=1,2,\dots,n) \quad (1)$$

Where, \dot{I}_i is the injection current, \dot{V}_j is the bus voltage, Y is the admittance matrix of the nodes, n is the number of buses in the network. For n-node system, the general formula is:

$$P_i + jQ_i = \dot{V}_i \sum_{j=1}^n Y_{ij}^* \dot{V}_j \quad (i=1,2,\dots,n) \quad (2)$$

Where, P_i is the injection active power of the node, and Q_i is the injection reactive power of the node.

For each node in the model, there are 4 unknown values to determine which are the voltage value, voltage angle, active power and reactive power. But the known values of the nodes are different. Hence the buses are divided into 3 types:

PQ node, the active power and reactive power are certain.

PV node, the active power and voltage value are certain.

Slack bus, the voltage value and voltage angle are certain.

The calculation results solved by the equations represent the results of the mathematical model, but whether this result can be practically applied in engineering applications still needs to be tested, because the operation of the power system must meet certain technical and economic requirements. These requirements are constraints on certain variables in the flow calculation which are:

All the bus voltage values must satisfy:

$$V_{imin} \leq V_i \leq V_{imax} \quad (i=1,2,\dots,n) \quad (3)$$

because all electrical equipment in the electrical system must operate near the rated voltage.

All the active power and reactive power of each node must satisfy:

$$\begin{aligned} P_{Gmin} \leq P_{Gi} \leq P_{Gmax} \quad (i=1,2,\dots,n) \\ Q_{Gmin} \leq Q_{Gi} \leq Q_{Gmax} \end{aligned} \quad (4)$$

Several voltages of each node must satisfy:

$$|\theta_i - \theta_j| < |\theta_i - \theta_j|_{max} \quad (5)$$

To sum up, the equivalent mathematical model shows that formulae of load flow analysis are a set of nonlinear equations with given constraints. if the results can meet the requirements, then output them, if not, the operations of the system needs to be modified and all the calculations should be start again.

4. Formation and Modification of Admittance Matrix

To analyze the load flows, the first step is formatting the admittance matrix, this matrix has the following characteristics:

The order of the node admittance matrix is equal to the number of nodes other than the reference node in the power network;

The node admittance matrix is a sparse matrix;

The node admittance matrix is a symmetric square matrix;

The self-admittance calculation formula of the node admittance matrix is as follows:

$$Y_{ii} = \sum_{j \in i} y_{ij} \quad (6)$$

The mutual admittance calculation formula of the admittance matrix is as follows:

$$Y_{ij} = Y_{ji} = -y_{ij} = -\frac{1}{z_{ij}} \quad (7)$$

If consider the transformer, the admittance matrix can be modified by using the following formulae:

$$\begin{cases} \Delta Y_{ii} = \frac{1}{kz} + \frac{k-1}{kz} = \frac{1}{z} \\ \Delta Y_{jj} = \frac{1}{kz} + \frac{1-k}{k^2z} = \frac{1}{k^2z} \\ \Delta Y_{ij} = \Delta Y_{qp} = -\frac{1}{kz} \end{cases} \quad (8)$$

Where, ΔY_{ii} is the change of the self-admittance of node i, ΔY_{jj} is the change of the self-admittance of node j, and ΔY_{ij} is the increased mutual admittance between i and j. k is the turns ratio and z is the impedance of the transformer.

If consider the capacitor, the following equation can be used:

$$(MVAR)_{charging} = \frac{\sqrt{3}|V|I_{charging}}{10^3} = \omega C_n |V|^2 \quad (9)$$

5. Gauss-Seidel Iteration Method

If there are n simultaneous nonlinear algebraic equations,

$$\begin{cases} f_1(x_1, x_2, \dots, x_n) = 0 \\ f_2(x_1, x_2, \dots, x_n) = 0 \\ \vdots \\ f_n(x_1, x_2, \dots, x_n) = 0 \end{cases} \quad (10)$$

The above equations can be rewritten to the following formula.

$$\begin{cases} x_1 = g_1(x_1, x_2, \dots, x_n) \\ x_2 = g_2(x_1, x_2, \dots, x_n) \\ \vdots \\ x_n = g_n(x_1, x_2, \dots, x_n) \end{cases} \quad (11)$$

After K+1, iterations

$$\begin{cases} x_1^{(k+1)} = g_1(x_1^{(k)}, x_2^{(k)}, \dots, x_n^{(k)}) \\ x_2^{(k+1)} = g_2(x_1^{(k+1)}, x_2^{(k)}, \dots, x_n^{(k)}) \\ \vdots \\ x_n^{(k+1)} = g_n(x_1^{(k+1)}, x_2^{(k+1)}, \dots, x_{n-1}^{(k+1)}, x_n^{(k)}) \end{cases} \quad (12)$$

Formula (11) can be abbreviated as

$$x_i^{(k+1)} = g_i(x_1^{(k+1)}, x_2^{(k+1)}, \dots, x_{i-1}^{(k+1)}, x_i^{(k)}, \dots, x_n^{(k)}) \quad (i=1,2,\dots,n) \quad (13)$$

This method is called Gauss-Seidel method. Its characteristic is that the latest value obtained by iteration calculation is immediately used to calculate the new value of the next variable.

6. Gauss-Seidel Load Flows Algorithm

Power flow calculation of Gauss-Seidel method is based on nodal admittance matrix. If there are n nodes in the system, the number of PQ nodes is m. The number of PV nodes is n-(m+1). The slack bus does not participate in the iteration.

According to formula, the formula can be interpreted as

$$\dot{V}_i = \frac{1}{Y_{ii}} \left(\frac{P_i - jQ_i}{V_i} - \sum_{\substack{j=1 \\ j \neq i}}^n Y_{ij} V_j^* \right) \quad (14)$$

changing formula to the iterative scheme of Gauss-Seidel method.

$$\dot{V}_i^{(k+1)} = \frac{1}{Y_{ii}} \left(\frac{P_i - jQ_i}{V_i^{(k)}} - \sum_{j=1}^{i-1} Y_{ij} V_j^{*(k+1)} - \sum_{j=i+1}^n Y_{ij} V_j^{*(k+1)} \right) \quad (15)$$

When applying the above iteration formula, the power of PQ node is given. If the initial value of the node voltage is given, the iterative calculation can be carried out. For PV nodes, the active power and amplitude of voltage are

given. The reactive power of the node only gives the initial value at the beginning of the iteration. Thereafter the iteration value must be calculated in the iteration process. Therefore, In each iteration, the following calculations must be done for PV nodes.

The node voltage obtained by formula is not necessarily equal to the given voltage amplitude. To satisfy this given condition, the phase angle of the node voltage will be preserved. The magnitude of the node voltage will be taken as a given value directly.

$$\dot{V}_i^{(k)} = V \angle \theta_i^{(k)} \quad (16)$$

And reactive power of nodes can be calculated by the following formula.

$$Q_i^{(k)} = -\text{Im} \left\{ \dot{V}_i^{(k-1)} \left[\sum_{j=1}^{i-1} Y_{ij} \dot{V}_j^{(k)} + \sum_{j=i+1}^n Y_{ij} \dot{V}_j^{(k-1)} \right] \right\} \quad (17)$$

After the above settlement, the new value of node voltage can be calculated by formula (15).

The criterion for convergence of iteration is

$$\max \left\{ \left| \dot{V}_i^{(k+1)} - \dot{V}_i^{(k)} \right| \right\} < \varepsilon \quad (18)$$

At the end of the iteration, the power of the slack bus and the power distribution in the network need to be calculated.

The formula for calculating the power of transmission lines are following.

$$S_{ij} = P_{ij} + jQ_{ij} = \dot{V}_i^* I_{ij} = \dot{V}_i^* \sum_{k=0}^n Y_{ik} \dot{V}_k - \dot{V}_i^* \sum_{k=0}^n Y_{ij} \dot{V}_j \quad (19)$$

7. Method

7.1. Processing of program

According to the flow chart general Gauss-Seidel iteration method, combined with the information of the given system with 5 buses, we designed 5 main steps in our program in MATLAB. The base power is chosen as 100MVA, and to distinguish the imaginary figure with the node number i , $ii = \sqrt{-1}$ is defined in the program.

Firstly, the data of the buses and transmission lines were shown in two matrixes. Then the following procedures used data from whole rows and columns to solve them concentratedly. Secondly, the bus types were defined as:

- 1-PQ bus; 2-PV bus; 3-slack bus

And an iteration about the node number was used to calculate the numbers and store them in three matrixes to show different types of the nodes.

Then we created a new matrix as admittance matrix using the equation (8) to (10). The transformer is also considered in a judgement statement to distinguish if the transformer is on the transmission line or if it is considered.

The maximum number of iterations was 1000. The initial value of the nodes voltage was also be kept. The "sumyv" calculation is due to part of equation (1). First at all, the reactive power of PV node was calculated by formula (16). Then, the voltage of every node was calculated by formula (14). If the node was PV node, the node voltage would be modified by formula (15). The difference between new voltage and old voltage was compared with convergence error by formula (17). Through continuous iteration, the ideal result was obtained. Because the experimental requirements did not specify the boundaries of reactive power, the over-limit inspection of reactive power was not required.

In this part, a judgement statement was used to distinguish different types buses. Then an iteration was used to distinguish each node.

The matrix "output" is used to display the results of power analysis. The first two columns of matrix "output" are the number of nodes. Formula (18) can calculate the apparent power of i to j and the apparent power of j to i .

7.2. Simulation in power world

Then we simulated this system using Power World, the voltages with angle of the buses, the power flows on the transmission lines were known.

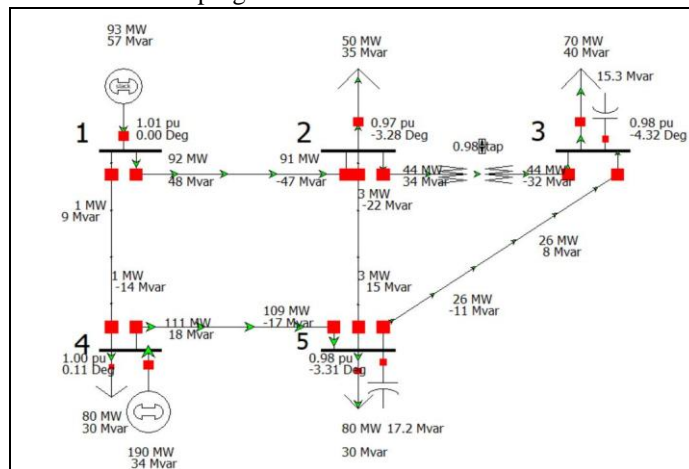


Figure 1. Simulation outcome in power world

8. Results and Discussion

The outcome of bus information matrix which is including the voltages with angles on each bus and the active power and reactive power of each bus in program is:

8.1. Node data

Table 2. The node data in the program

Bus Number	Voltage (p.u.)	Angle	Pg (mw)	Qg (mvar)	Pi (mw)	Qi (mvar)
1	1.01	0	100.5516	61.1201	0	0
2	0.9604	-3.7697	0	0	50	35
3	0.9671	-5.0669	0	0	70	40
4	1	0.1311	190	31.4224	60	50
5	0.9682	-3.9746	0	0	80	30

And the node data in Power World is:

Table 3. The node data in power world

Bus Number	Voltage(p.u.)	Angle	Pg(mw)	Qg(mvar)	Pi(mw)	Qi (mvar)
1	1.01	0	93	59	0	0
2	0.97	-3.28	0	0	50	35
3	0.98	-4.32	0	0	70	40
4	1	0.11	190	34	60	50
5	0.98	-3.31	0	0	80	30

What is shown is that the voltages are almost same, the difference between them is due to the accuracy of Power World which uses two decimal places. The values of power are similar as well.

8.2. Load flows

The load flow distribution in the MATLAB is shown in the following table.

Table 4. The load flow distribution in the matlab

Node i	Node j	P(i,j) (mw)	Q(i,j) (mvar)	P(j,i) (mw)	Q(j,i) (mvar)
1	2	105.1861	54.3659	103.6195	-57.4815
1	4	0.5655	6.7541	0.5331	-16.7385
2	3	53.9172	44.5329	53.9172	-42.5167
2	5	-0.2838	-22.0493	-0.3149	9.1582
3	5	-31.0442	2.5166	31.2018	-9.3667
4	5	130.5388	28.1626	128.3903	-29.7912

The load flow distribution in the Power World is shown in the following table.

Table 5. The load flow distribution in the power world

Node i	Node j	P(i,j) (mw)	Q(i,j) (mvar)	P(j,i) (mw)	Q(j,i) (mvar)
1	2	92	48	91	-47
1	4	1	9	1	-14
2	3	44	34	44	-32
2	5	-3	-22	-3	15
3	5	-26	8	-26	-11
4	5	111	18	109	-17

The table above shows that the calculated values of Matlab are basically similar to those of Power world. This shows that the calculation of Matlab and Power World is correct.

8.3. Convergence error and iteration numbers

In the Power World, the iteration of program was continuous. This could be seen from Message Log. As the system runs, the iteration will start because it is dynamic which means that it could not be certain.

However, the number of iterations was fixed in the MATLAB. The set convergence error is 10^{-6} . In this program, the number of iterations is 34. The total iteration time is 0.0177s.

8.4. Comparison with other methods

The principle of Gauss-Seidel method is simple. Admittance matrix is a symmetrical and highly sparse matrix. Hence, it also occupies very little memory. As far as the time required for each iteration is concerned, the Gauss-Seidel method takes the least time.

The advantage of Newton-Raphson power flow algorithm is its fast convergence speed. If a better initial value is chosen, the algorithm will have a quadratic convergence property. In general, the Newton-Raphson method can converge to a very precise solution by iterating four to five times. Moreover, the number of iterations is basically independent of the size of the network. Newton-

Raphson method also has good convergence reliability. Newton-Raphson's method can also be used for systems that can't use Gauss-Seidel's method.

9. Acknowledgment

In this study, the results of the program showed a correct Y-bus matrix, a clear iteration numbers and time of Gauss-Seidel method and a small convergence error which is 10^{-6} . However, due to the accuracy of Power World, the comparison of value, voltage angle, active power and reactive power showed that a little difference between them. From the results, the voltage distribution

is pretty good, so it is not necessary to install capacitors to compensate the reactive power. but if the values voltages of this system is lower than normal values, it need compensation in that way to be improved.

References

- [1] Weedy B.M., Cory B.J., Jenkins N., Ekanayake J.B and Strbac G. Electric power systems, Noida: A John Wiley & Sons, Ltd, Publication. 2012.
- [2] Qiu X and Liu T. Computer algorithm for power system analysis. Beijing: China Electric Power Press. 2009.