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Design and Evaluation of the Tollbooth Plaza based on Mathematical Model

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Abstract: Nowadays, how to design a scientific tollbooth plaza on high way has been an issue of hot discussion. To find the solution about building the tollbooth plaza as reasonable as possible, we construct two models from two aspects to consider it, those are: Cost Model and Time Model. Cost model refers to the total cost of building a tollbooth plaza. It is connected to the area of the tollbooth plaza and the proportions of conventional tollbooths, exact-change tollbooths and electronic toll collection booths. In addition, the safety problems should also be considered during the process of building the tollbooth plaza. As a result, the Gradual Change Rate is put forward, describing the area of the gradual change section. Gradual change section refers to the area L lanes on highway change into B lanes on passageway, and vice versa. Besides, a widening passageway is built specially for freight vehicles on the right side of the plaza to reduce accidents. Time model refers to the expectation of total time spent on tollbooth plaza for one vehicle. It can be divided into three parts: The expectation of time spent before the barrier toll in the toll plaza, the expectation of time spent on service and the expectation of time spent after the barrier toll in the toll plaza. To determine this physical quantity, we use to Queuing Theory and Free Flow Model. Queuing theory show the time spent before the barrier toll and on service and Free flow model show the time spent in the gradual change sections. Similarly, the total time spent on tollbooth plaza is also influenced by the proportions of different kinds of tollbooths. Lastly, we discuss the effect of my solution in light and heavy traffic as well as the autonomous vehicles conditions and give the detailed solution to explain it with the Evaluation model that we construct.

Keywords: Cost model; Time model; Queuing theory; Free flow model; Free flow model; Genetic algorithm

1. Introduction

A toll plaza is the area of the highway needed to facilitate the barrier toll, consisting of the fan-out area before the barrier toll, the toll barrier itself, and the fan-in area after the tollbooth. Although the area of a single toll plaza is small, the quantity of them is quite a lot. They are the most common buildings distributed on highways. Meanwhile, they are the most important part of highways. As private vehicles popularize and the highway transportation develops, there is always phenomenon of heavy traffic and problem of accidents at the toll plaza because of unreasonable setting of toll plaza (lack of tollbooths, inefficient service of tollbooths). Therefore, there is a need to determine a better design of the toll plaza. In America, the ratio of toll highways is less than 10%, so there is only a small number of researches on toll plaza. Nico M.van Dijk discussed how to design a toll plaza, determine the type and quantity of tollbooths by Queuing simulation.

In our solution, to consider a better design for 'barrier tolls', we are supposed to establish models to determine if there are better solutions (shape, size, and merging pattern of the area following the toll barrier in which vehicles fan in from B tollbooth egress lanes down to L lanes of traffic) than any in common use. Meanwhile, we must consider accident prevention, throughput and cost in our models. In addition, how the influence brought by more autonomous (self-driving) vehicles added to the traffic mix changes our solutions should be taken into consideration. Finally, we would observe how the model affected by the proportions of conventional (human-staffed) tollbooths, exact-change (automated) tollbooths, and electronic toll collection tollbooths.

To put forward a better solution of the area to merge after toll, we make our work clear before establishing the models. Specifically, we divide our work into the following parts.

Establish M (L, B,1) Model to solve throughput.

Establish model to solve the cost of the solution.

Put forward a better solution of the area following the toll barrier (shape, size, and merging pattern), considering all factors including accident prevention, throughput and cost.

Observe the effect brought by more autonomous vehicles or proportions of different types of tollbooths to our model.

2. Overview of the Models

2.1. Model for cost

Before establishing our model for cost, we will explain the basic design of the toll plaza at first. The plaza consists of 3 parts, including two areas for fanning out and fanning in and an area of passageway of tollbooths for service. When considering the two areas for fanning out and fanning in, to prevent the happening of accidents, the gradual change rate and the longitudinal slope are taken into our consideration, which will be explained specifically in the following sections of our solution. When considering the area of passageway of tollbooths for service, we establish not only some normal passageway of tollbooths but also a widening passageway of a tollbooth for freight vehicles on the right side of the row of tollbooths, the width of which is D. To explain our model more clearly, we give the projected layout of the toll system in Figure 1.

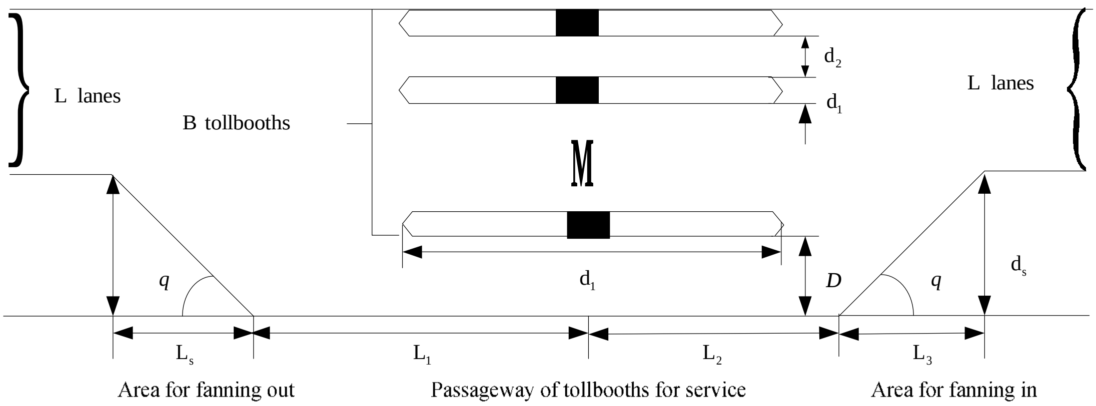

Figure 1. A projected layout of toll system

To explain our model more clearly, we give the projected layout of the toll system in Figure 1. According to Figure 1, with the knowledge of geometry, we want to determine the voltage of the area. The voltage of the area can be divided into two parts: a rectangular section for waiting for the service and two gradual change sections with longitudinal slope. Apparently, since the rectangular area is parallel to the ground, the voltage of it can be determined easily, so we find the value of the voltage of the gradual change sections firstly.

First, we get the sketch of the land of the toll area for fanning out in the three-dimensional coordinates shown in Figure 2, of which the coordinate axises are $h(x)$, $w(x)$ and x.

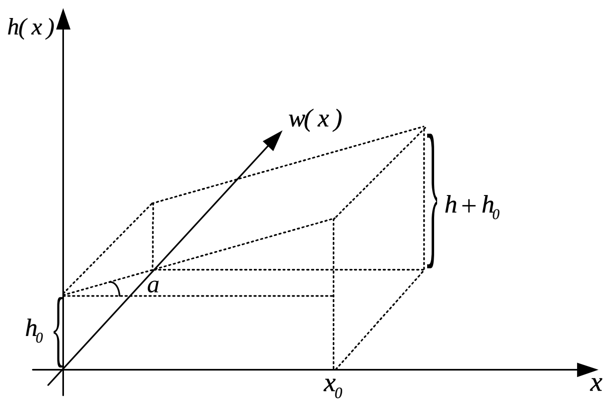

Figure 2. The 3D sketch of the land of the toll area for fanning out

From research, we determine the longitudinal slope of the area to be 1.5%, which means $tana = 1.5\%$.

After differentiating this area, we have:

$$dV_{gc} = w(x)h(x)dx$$

where $w(x)$ is the width at a point of the area, $h(x)$ is the height at a point of the area and x means the length at a point of the area.

To get the formula of $h(x)$, from the sketch of the toll plaza, we can easily get the formula of $tan a$ at first, shown in:

$$tan a = \frac{h(x) - h(0)}{x}$$

After deforming the formula, we have the formula of $h(x)$.

$$h(x) = x tan x + h(0)$$

To get the formula of $w(x)$, we have the vertical view of the area.

According to the geometrical relation shown in the figure, we get the formula of $tanq$, then $w(x)$.

$$tanq = \frac{w(x) - w(0)}{x} \Rightarrow w(x) = x tanq + w(0)$$

According to all the above formulas, we obtain the final formula of V_{gc}.

$$V_{gc} = \int_0^{x_0} dV_{gc} = \int_0^{x_0} (x tanq + w_0)(x tan a + h_0)dx$$

$$\Rightarrow V_{gc} = \frac{1}{3} tanq \, tan a \cdot x_0^3 + \frac{1}{2}(h_0 tanq + w_0 tan a)x_0^2 + h_0 w_0 x_0$$

Here, we divide the total cost into two parts. The first part is the concrete material cost for building the toll plaza and the second part is the material cost for building different types of tollbooths. We assume that b_1 means cost of one conventional tollbooth, b_2 means cost of one exact-change tollbooth, b_3 means cost of one electronic toll collection booths, b_4 means the unit price per cubic meters of concrete. The formula of total cost C is:

$$C = n_1 b_1 + n_2 b_2 + n_3 b_3 + V b_4 \#(2-2)$$

in which: $n_1 + n_2 + n_3 = B$

n_1, n_2 and n_3 represents the number of conventional tollbooths, exact-change tollbooth and electronic toll collection booths respectively.

2.2. Model for throughput

To establish the model for throughput, we are supposed to consider the whole toll plaza to pass through a barrier toll as a toll system shown in Figure 3. The toll system consists of four areas, the fan-out area before the tollbooth, the waiting area before the tollbooth, the tollbooth itself, and the fan-in area after the toll barrier.

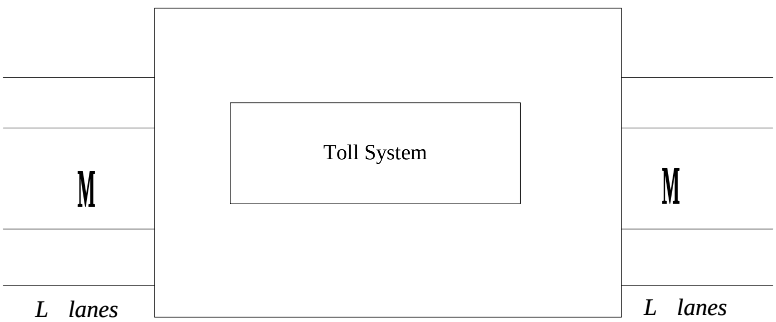

Figure 3. A sketch of a toll system

What is more, we assume a variable to describe the capacity of through put of a toll system. Here we use TP to determine the capacity of throughput of a toll system. Now, we can get the following formula.

$$TP = \frac{1}{E(T)} \cdot B$$

where B is the number of tollbooths in each direction and $E(T)$ is the expectation of total time spent on one tollbooth for one vehicle (unit: h), which can be determined by the following formula.

$$E(T) = E(W) + E(S) + 2E(G)$$

in which $E(W)$ means the expectation of time spent before the barrier toll in the toll system for one vehicle (the time from finishing the process of fanning out to beginning getting service at the tollbooth), $E(S)$ means the expectation of time spent on service, $E(G)$ means the expectation of time spent on the process of fanning in (according to our assumption, the vehicles begin merging (fan in) immediately after the service of tollbooth, so the time is from the end of getting service to leaving the toll system). Because of symmetry, the time spent on fanning out is $E(G)$ as well. Then we get the above formula.

In our report, we consider a toll system with more than one tollbooths. To get the final formula of TP, as L, B are constants, the main task for us is to observe the formula of $E(W)$, $E(S)$ and $E(G)$.

We construct the free model to solve the problem. Because of the characteristic of free flow, we know that the traffic flow on each line on highway is generally equal. For this reason, model of merging pattern of cars is put forward. This model can guarantee the traffic flow on each line on high way is generally equal, satisfying the assumption fluid dynamics simulation theory perfectly.

As a result, the government can design the merging pattern from the formula. The detailed description is that: There are B lanes on passageway and L lanes on highway. Passing through the gradual change section, cars fan in from B lanes on passageway to L lanes on highway. In addition, there are p lanes consisting the a lanes on passageway and q lanes consisting the b lanes on passageway after the gradual change section. It should be noted that: p , q , a , b are specific value, representing some specific number depended on a , b .Some specific B and L to explain the merging pattern shown in Figure 4 and 5.

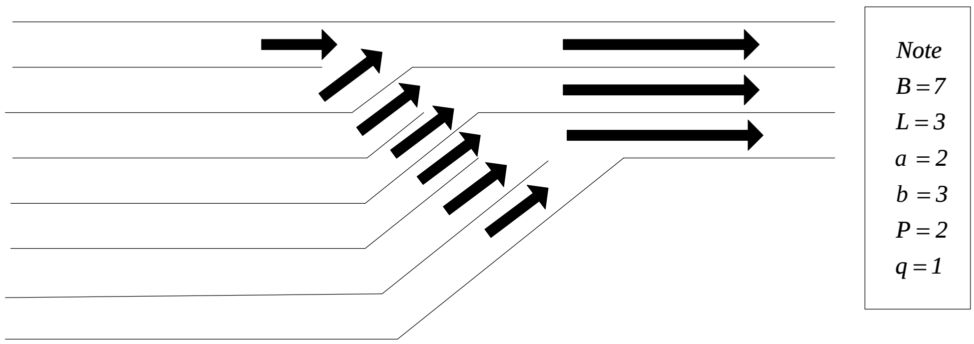

Figure 4. The merging pattern when B=7 and L=3

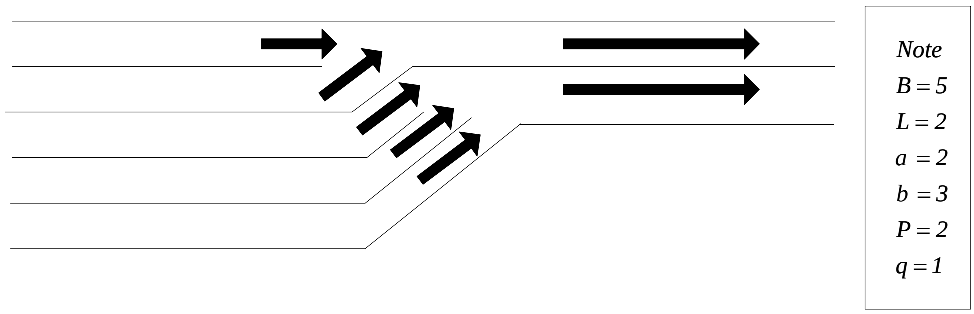

Figure 5. The merging pattern when B=5 and L=2

3. The Performance of the Solution

When considering the performance of our solution, we consider different conditions, including the toll plaza in usual, light or heavy traffic. What's more, we consider the situation that more autonomous vehicles join into the traffic mix of the toll area.

3.1. In usual traffic

When considering the performance of our solution in usual traffic, we determine the value of DHV per lane to be 680 (pcu/h), and we get our performance of solution is shown in the following tables (Table 1). We consider the situation of different quantity of lanes on highway and tollbooths. In addition, whether the government emphasize cost or throughput is also be considered.

For example, Table 1 shows the traffic flow performance when L equals 3 and B equals 8. $EVA1$ and $EVA2$ represents we emphasize time spent and cost respectively.

Table 1. The performance when L is 3 and B is 8

	The min value of S	The min value of C	The min value of E(T)	EVA1 $(n_1 : n_2 : n_3)$	EVA2 $(n_1 : n_2 : n_3)$
Value	1.4071×104	1.6075×106	1.1273×10-3	6.9519 0.0000 1.0481	4.2767 0.0000 3.7233

From those results shown in the tables above, we could obtain some useful information.

First, we get the min value of the total area of toll plaza the cost and the expectation of total time spent on one tollbooth for one vehicle, of which the order of magnitude all meets the reality. What's more, according to the value of EVA , we observe that the exact-change tollbooth is not used in every situation, which also meets the reality. Finally, the performance of our solution is stable in every situation above, which shows that our model is flexible, for it's suitable for the situation of different quantity of lanes on the highway and tollbooths. Meanwhile, whether the government emphasizes the cost or the throughput, our solution could give the most reasonable proportion of different types of tollbooths.

3.2. In light and heavy traffic

When considering the situation that the toll plaza is in light and heavy traffic, we determine the value of D per lane to be 500 and 850 in the range (pcu/h) to describe the light and heavy traffic. The performance is shown in Table 2 and 3.

Table 2. The performance when L is 3 and B is 8 in light traffic

	The min value of S	The min value of C	The min value of E(T)	EVA1 $(n_1:n_2:n_3)$	EVA2 $(n_1:n_2:n_3)$
Value	14085.20	1.6421×106	1.2578×10-3	6.7882 0.0000 1.2118	5.9219 0.0000 2.0781

Table 3. The performance when L is 3 and B is 8 in heavy traffic

	The min value of S	The min value of C	The min value of E(T)	EVA1 $(n_1:n_2:n_3)$	EVA2 $(n_1:n_2:n_3)$
Value	14069.80	1.5791×106	1.0556×10-3	6.9547 0.0000 1.0453	3.7908 0.0000 4.2092

From Table 3 and 4 above, we conclude that the performance of our solution is stable in light and heavy traffic. The conclusion of the order of magnitude of the values, the flexibility of our solution is similar to that of the situation in usual traffic.

4. Proposal to the New Jersey Turnpike Authority

After our several-days' work and calculation, we successfully put forward our better solution of the toll plaza, eventually the area for vehicles to fan in after toll.
In our solution, our toll plaza is the consists 3 parts, including two areas for fanning out and fanning in (1 and 3 in the following figure) and an area of passageway of tollbooths for service (1 in the following figure). The layout of the toll plaza is shown in the following figure.

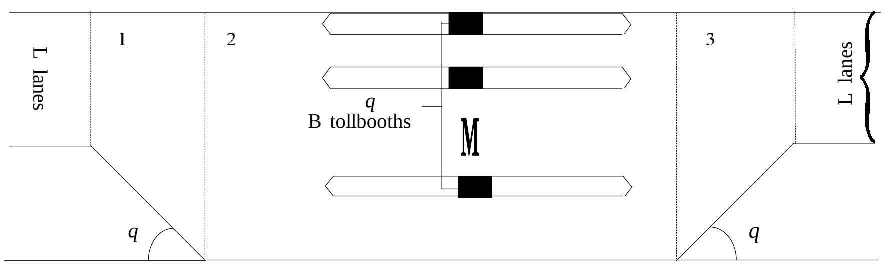

Figure 6. The layout of the toll plaza

4.1. Accident prevention

In our solution, we take accident prevention to our consideration.
When considering the two areas for fanning out and fanning in, to prevent the happening of accidents, the gradual change rate is taken into our consideration. As the frequency of traffic accidents at the toll plaza is in direct proportion to the gradual change rate, we specifically limit the value of the inflow, outflow angle (which could determine the gradual change rate) to a safe range (θ)in the above figure).
When considering the area of passageway of tollbooths for service, we establish not only some normal passageway of tollbooths but also a widening passageway of a tollbooth for freight vehicles on the right side of the row of tollbooths. When there is traffic peak or some unpredictable things in the toll plaza, it can also be used as an emergency passageway to release the traffic, reduce the lane change of the vehicles (lead to accidents) caused by heavy traffic.
Generally speaking, our solution is effective to prevent accidents.

4.2. Cost and throughput

In our solution, we also consider both the cost and throughput affected by the proportion of the type of tollbooths. What's more, we have verified the performance of our solution in the situation of different quantity of lanes on the highway and tollbooths. Whether the government emphasizes the cost or the throughput, our solution could give the most reasonable proportion of different types of tollbooths. Therefore, our solution have the ability to help you plan the toll plaza to balance the cost and throughput.

4.3. Merging pattern

After our consideration, we know that the traffic flow on each line on highway is generally equal. For this reason, the merging pattern of vehicles is put forward. Here we take one figure as an example. The specific merging pattern is determined by the number of lanes on highway and tollbooths.

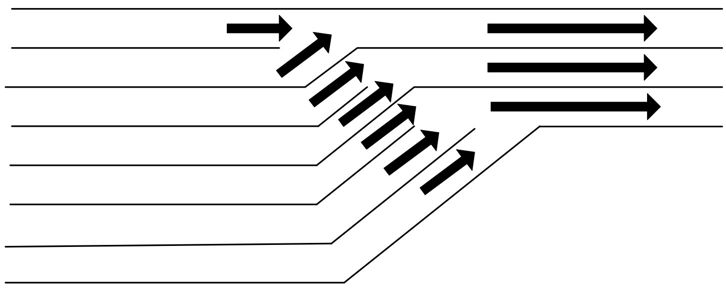

Figure 7. The number of lanes on highway and tollbooths

With our merging pattern, the process of changing lanes of vehicles is limited, while the number of lanes that a vehicle need to change is reduced, which makes the process of merging easier. What's more, the traffic flow on each way is generally equal. In this situation, the happening of accidents is prevented.

Therefore, our merging pattern could be considered as a better solution of the area following the tollbooths.

In conclusion, our solution of the toll plaza is regarded as an efficient, cheap and safe solution. Our consideration of it is comprehensive while the accident prevention, cost and throughput are considered. What's more, the solution is suitable for different types of toll plazas with different quantity of tollbooths and lanes of the highway. So, we believed that our solution could definitely be used in toll plaza construction and be taken into your consideration.

5. Conclusion

Generally speaking, we have determined a better design of the toll plaza. In our solution, the shape of the toll plaza is a certain trapezoid, including two areas for fanning out and fanning in and an area of passageway of tollbooths for service. Our merging pattern is put forward by us based on free flow model, which is effective to release traffic jam in the toll plaza. Considering accident prevention, the throughput and cost are determined by the number of lanes tollbooths, and the ratio of different tollbooths. For example, when there are 8 tollbooths and 3 lanes, the minimum value of the cost is 1.6075×106 yuan while the minimum value of size is 1.4071×104 , which could reflect the throughput. Finally, after testing our solution in light and heavy traffic, we conclude that our solution is stable in different situations.

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