Research on Locally Quadratic Convergence Algorithm for Linear Hyperbolic Equation

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Abstract: With vast calculating amount, the conventional convergence algorithm for linear hyperbolic problem is quite complex, therefore, the quadratic convergence algorithm was proposed to resolve this problem. While the linear hyperbolic problem being proposed, the iterative sequences and algorithm complexity x being introduced; the algorithm complexity being analyzed by using the local region and locally quadratic convergence; the problem of ellipse linear hyperbolic equation being taken as the example to analyze the problems; and the numerical experiment being performed, the research on locally quadratic convergence algorithm for linear hyperbolic equation was completed. It can be proved by experiment that with less computation, the locally quadratic convergence algorithm is simpler than the conventional algorithm.

Keywords: Wide-neighborhood; Linear hyperbolic problem; Quadratic convergence;

1. Introduction

The linear hyperbolic problem plays an important role in any fields, the linear problem was proposed by R.W.Cottle in 1964 as a new mathematics model, which is closely associated with mathematical programming, variational inequality, problem of fixed points, generalized equation and theory of games and is widely used in engineering technology. To accelerate the computation of linear hyperbolic problem and reduce the complexity of computation of linear hyperbolic problem, the iterative sequences and algorithm complexity x are introduced to solve the local region and locally quadratic convergence. By performing the analysis of complexity and numerical experiment to linear hyperbolic equation, the linear hyperbolic equation was calculated and the complexity degree was verified [1].

2. Research on Locally Quadratic Convergence Algorithm for Linear Hyperbolic Equation

The linear hyperbolic problem (LHP) is discussed in this paper, it comes from the linear programming and quadratic programming.

A standard LHP problem can be described below:

$$\mathrm{LCP}\begin{cases} S = Mp+\mathrm{q} \\ p \ge 0, S \ge 0, p^T S = 0 \end{cases} \quad (1)$$

Where M represents a monotone matrix, and q a generalized constant, so that the algorithm complexity x is introduced [2].

The algorithm complexity analysis is quite important in the research on locally quadratic convergence algorithm. Generally, only the complexity of algorithm in worst case is analyzed. Many algorithms have the complexity in worst case of exponential order, but they are effective in practical application. One of the typical example of these algorithms is the simplex algorithm for solving the linear programming problem [3]. When analyzing an algorithm, the small stochastic disturbance is added to the input case, then the relationship between the algorithm complexity and input scale and disturbance is analyzed, the complexity obtained is the algorithm complexity. The formula is given by:

$$\mathrm{x} = \frac{D}{T} + \frac{D}{\frac{D}{B} + \mathrm{n}\Delta\mathrm{t}} = \frac{D}{\frac{D}{B} + \frac{\mathrm{nd}}{B}} = \frac{1}{\frac{1}{B} + \frac{\mathrm{nd}}{BD}} \quad (2)$$

Where D is the iterative sequences, and n the iterative numbers.

2.1. Algorithm complexity

Through the calculation of algorithm complexity, the locally quadratic convergence algorithm is compared with conventional convergence algorithm for linear hyperbolic problem, while the data is substituted into formula, the analysis obtained is shown in Table below:

Table 1. Comparison of algorithm complexity

Linear hyperbolic equation	x value of conventional algorithm	x value of locally quadratic convergence algorithm
Linear equation	0.4857	0.8456

Quadratic linear equation	0.4926	0.8735
Cubic linear equation	0.5493	0.8965
Quartic linear equation	0.5987	0.9126
Fifth-order linear equation	0.6154	0.9327

For comparison, the data is converted into cartogram.

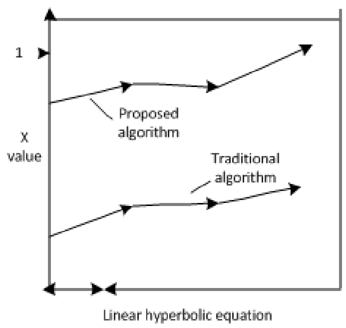

Figure 1. Comparison of algorithm complexity

Under the premise of verifying the correctness of the algorithm, the complexity of the algorithm is mainly reflected in the amount of computer resources required to execute the algorithm. The more resources required, the higher the complexity of the algorithm; Conversely, the less resources required, the lower the complexity of the algorithm [4]. The most important resources of the computer are the time resources and space resources. Therefore, the complexity can be divided into time complexity and space complexity. Since the time complexity is similar to the space complexity concept, the calculation method is similar, and its influence is negligible in the case of sufficient space, so the complexity of iteration of algorithm is mainly considered [5].

There are maily two methods to calculate the complexity of iteration of algorithm, one is to calculate the sum of frequentness of all statements in the algorithm, which is the function of the problem scale solved by the algorithm, when n is used to represent the problem scale, x is used to represent the time complexity of algorithm, this method analyzes the iteration number complexity of the feature extraction algorithm, but the result is too subjective, and can not accurately analyze and compare the iteration number complexity of each algorithm. Another is to calculate the operation numbers of add, subtract, multiply, divide used when executing the algorithm, so that the time complexity of algorithm is analyzed. This method can quantitatively analyze the complexity of the algorithm, and then better analyze the performance of the algorithm.

2.2. Locally quadratic convergence

It should be pointed out that the indicators that measure the superiority of an algorithm are not only polynomial complexity, ie global convergence, and local convergence. That is to say, the fast and slow problem network of the dual gap formed by the algorithm generated by the algorithm in the local range close to the optimal value converges to 0. ZHANG, Tapia and Dennis discuss the conditions that the local convergence algorithm needs to satisfy. A great theoretical contribution to this is that YE et al. have proved that the wide-neighbor classical prediction correction algorithm for monotone LHP has quadratic convergence under the condition that the problem has a strictly complementary solution without the hypothesis problem non-degenerate [6].

Based on the local convergence of the neighborhood tracking algorithm, the linear hyperbolic problem is investigated. It is proved that in a more general case (that is, without hypothesis that the problem is not degraded), the linear programming neighborhood tracking algorithm has local quadratic convergence. Theoretically, the numerical convergence characteristics of the algorithm are mostly explained by the iterative method for solving nonlinear equations. The structural idea is either derived from an iterative method for solving nonlinear equations or from an iterative method for solving linear equations. Moreover, in the study of the nonlinear equations iterative method, the equation is often used as a model. When the function is more complicated, the calculation of the derivative is more complicated, which increases the calculation amount of the iterative method for the linear hyperbolic problem. So sum up the local quadratic convergence as follows:

In general, the non-internal point continuous method requires a line search to find the step size so that the step size is multiplied by the Newton direction plus the current point as the next iteration, which is an indispensable step in the non-internal point continuous algorithm. However, in the method we provide, each step uses the full Newton step size, eliminating the traditional line search step [7].

This algorithm is globally convergent. In particular, according to the nature of the M-matrix, it is proved that the iterative sequence of the algorithm is monotonic and globally convergent. Results like this have not appeared in previous non-internal point continuous algorithms [8].

This algorithm is globally linear and locally superlinearly convergent without other assumptions. Under some weak conditions, it can be concluded that this algorithm is locally squared.

A new non-internal point continuous method is proposed to solve the problem by using a smooth function. The proposed new method is different from the previous ones. The above points are the difference between the local quadratic convergence algorithm and the convergence algorithm of the conventional linear hyperbolic problem. The conventional algorithm uses a semi-smooth algorithm and proves the convergence of the algorithm, but this convergence is not completely convergent, which leads to a problem. It is very difficult to select the initial value. If you choose an initial value that is not suitable, the algorithm may not converge, which is a defect of this algorithm. Although this algorithm also has local superlinear convergence, it must rely on finding the initial value in the vicinity of the real solution. This selection is not easy to do because the real solution is unknown. For the case of such defects, the generalized Newton method can be studied using the dual active set strategy of the original problem under certain conditions. Similarly, we will use the active set method for this problem. But this optimal control problem has two different types of inequality constraints: one is the control constraint and the other is the hybrid control-state constraint. It is because of the different types of these two constraints that they are in trouble when dealing with this problem. It is difficult to apply general theory to solve this problem. To overcome this difficulty, we reconstructed the optimization conditions. The structure of the constraint of this problem is special, and its Lagrange multiplier exists in the function space. It is this point that the active sets of the two constraints are disjoint. However, we apply the global convergence of the local quadratic convergence algorithm. That is to say, for any initial value, the local quadratic convergence algorithm is convergent, which overcomes the shortcomings of the semi-smooth algorithm. This is also an improvement in the algorithm for this problem.

In the algorithm of this problem, the state equations that are relied on are linear. There is a linear relationship between the constant term of the equation (the term to the right of the equal sign) and the unknown function, so we can use the semismooth algorithm. If the equation of state is not linear, then the whole problem is no longer convex, and the study becomes complicated. To make up for this. We choose a local quadratic convergence algorithm whose optimization conditions are satisfied at the most advantageous point. Then similar results can be obtained by using the following proof method [9].

2.3. Hyperbolic problem of ellipse

For the problem of elliptic linear hyperbolic equations, this paper provides a local quadratic convergence algorithm. The design of the algorithm is based on the properties of the M-matrix, and then a smoothing function is used to reconstruct an equivalent problem of the original problem, and the Newton equation is used to find the Newton direction. This paper removes the steps of line search in the smoothing algorithm, and obtains a step to correct the Newton direction by adjusting the M-matrix. After the correction step, the step of line search is not needed, so each iteration step uses the full step value [10].

This algorithm is valid for a large class of functions. The classical elliptic equation optimal control problem is that the objective function contains the L1 norm, which is non-differentiable. After transforming it into a non-smooth equation, it cannot be solved by using the first-order necessary and sufficient condition because the equation is non-smooth. This article introduces a smooth function:

$$F(u, v)=\frac{c(u)c(v)}{4}\sum_{x=0}^{n}\sum_{y=0}^{n} f(x, y) \cos\frac{(2x+1)u\pi}{16}\cos\frac{(2y+1)v\pi}{16} \tag{3}$$

This function smoothes the nonsmooth terms in the equation, however, the algorithm does not use smooth ideas, but uses the limits of this smooth function to design the algorithm. The ultimate design algorithm is the second part of this paper. The first part uses the deformation of the fixed point method. The first part is convergent for any initial value, so this guarantees global convergence. When the iterative sequence generated by the first part is close to the real solution, the iterative sequence enters the second part of the algorithm. The second part guarantees the local square convergence of the algorithm. For the problem studied in this paper, a semi-smooth algorithm is also used. Since the constraint in the condition is the constraint of the uniform state, the theory of directly using the smoothing algorithm is not acceptable. In this paper, the unknown function and the constant term part of the equation of state are linear and complementary, and the original problem is transformed into a non-smooth equation system. Then the generalized Newtonian theory is used to find the Slant function of the nonsmooth term in the nonsmooth equation, so the semismooth algorithm can be used. The convergence of the algorithm is studied using the strategies of active sets and inactive sets [11].

Another solution to the local quadratic convergence algorithm is to use the decomposition function form:

$$\begin{cases} E_1 = J_{pbr} J_m J_p E_{exp} J_{pbr}\left(E_r + E_m\right) \\ E_2 = J_{pbr} J_{NIHO} J_p E_{exp} J_{HW}\left(E_r + E_m\right) \\ E_3 = J_{pbr} J_{qw} J_p E_{das} J_{pbr} J_{pbr}\left(E_r + E_m\right) \\ E_4 = J_{pbr} J_m J_p E_{exp} J_{fg} J_{pbr} J_{fdg}\left(E_r + E_m\right) \end{cases} \tag{4}$$

This function rewrites the linear hyperbolic equation into a first-order equations by using the divergence form of the higher-order equation. Then, the elliptic first-order linear hyperbolic equation is successfully decomposed into third-order, fourth-order and fifth-order partial differential equations by using the idea of the first-order equation one by one. Later, it was extended to a series of wider high-order partial differential equations, and it was successfully applied to higher-order geometric evolution equations [12]. The method does not impose too high requirement on the mesh and can be applied to any regularity mesh. The design of the numerical flux function makes it possible to finally solve the matrix partial block, which is very easy to parallelize the calculation. Moreover, the method does not require the finite element space basis function, and any high-order polynomial can be used to approximate the highly accurate numerical format, which is a great advantage for the traditional continuous finite element [13].

The algorithm for the problem involving elliptic operators has been studied for many years. Based on the theory provided by the existing literature, this paper proposes two numerical solutions to the elliptic operator problem. Using the characteristics of the problem itself, the algorithm is designed by using new methods and techniques, and their convergence properties and local fast convergence properties are proved. Both theoretical research and numerical calculation have certain scientific research value [14-15].

3. Numerical Experiment

This experiment is a verification of the full text overview, confirming the accuracy and rigor of the article, and the partial quadratic convergence algorithm is simpler than the convergence algorithm of the conventional linear hyperbolic problem.

This part will illustrate the numerical effects produced by combining the two directions through numerical experiments. An integer n-order matrix A is generated by MIatlab. Let D = Ad, q = ep, then an LHP problem will be generated and the initial feasible point is (0, 0). For such problems, the numerical results of programming implementation of Equation 1 are as follows, where the number of iterations and time are ten times, and the mean of the same n is solved.

Table 2. Iterative value table

n	Iterative number	Time
100	11.4	0.1093
200	13.0	0.7048
300	12.5	2.2219
400	12.6	4.9125
500	13.5	9.8592
600	13.7	18.2800
700	14.4	29.8049

Numerical experiments show that these two algorithms are very effective for these functions. However, by comparing the results, we find that the local quadratic convergence algorithm is very effective and simple for these functions. Both time and loop iterations are superior to conventional algorithms, and for some functions our algorithm shows considerable advantages. This shows that this algorithm has extremely good arithmetic results for such problems.

3.1. Summary experiment

Firstly, the local convergence of the local quadratic equation is studied by decomposing the linear hyperbolic equation into the associated fourth-order time-dependent partial differential equation. By constructing a special global projection, it is proved that when using interlaced numerical fluxes, the local quadratic convergence algorithm solves a special projection with quadratic convergence to the true solution. Where k ≥ 1 is the number of pieces of the polynomial in the finite element space. The research results extend the superconvergence work of Cheng and Shu on the local discontinuous finite element method for solving one-dimensional linear convection-diffusion equations. In addition, the optimal convergence results of numerical solutions and their spatial derivatives and their time derivatives are obtained. A large number of numerical examples, such as linear problems, initial boundary value problems, nonlinear equations and singular solutions, verifies the superconvergence of the method and the long-term morphology of the solution.

Then, for the one-dimensional linearly correlated hyperbolic conservation law, the quadratic convergence of the local quadratic convergence algorithm is proposed and analyzed. Using Taylor's priori hypothesis of linearization and numerical solutions, it is proved that when using the local quadratic convergence algorithm, the discontinuous finite element solution is a special projection of the order superconvergence to the true solution, and the superconvergence analysis of the linear problem discontinuous finite element method is extended to the nonlinear problem. In addition, the optimal error estimate map of the numerical solution and its optimal method is obtained.

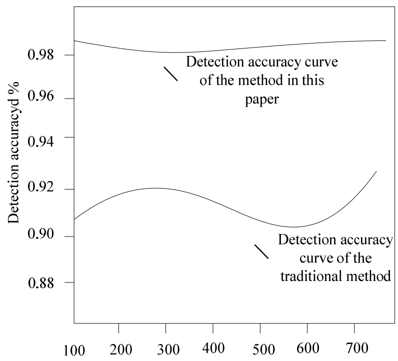

Figure 2. Optimal error accuracy

Finally, for the linear hyperbolic equation, the error analysis of the local quadratic convergence algorithm is given. Using the quadratic convergence properties of multi-dimensional special projections, the optimal error estimate of the algorithm complexity when using the local quadratic convergence algorithm is proved. And pointed out that for the general linear hyperbolic equation, this method can be the easiest and quickest method.

4. Conclusion

The locally quadratic convergence algorithm is a numerical method with high-order accuracy and high distinguishability for linear hyperbolic equation. In this paper, the quadratic convergence and estimation of algorithm complexity of locally quadratic convergence algorithm and conventional method are paid much attention. And the solid theorectical foundation which proved that the method is effective while used in numerical simulation is also given, which further proved the high-order accuracy of the algorithm. The error analysis and numerical experiment showed that the computational effectiveness and advantages of locally quadratic convergence algorithm and conventional method while used in solving linear equation, nonlinear equation, one-dimensional and multi-dimensional problem.

The difference between the theory and practice of interior point method and conventional algorithm does not exist in the locally quadratic convergence algorithm. Based on its advantages, the algorithm is extended from the linear hyperbolic problem to linear complementary problem. During the ieration, the new algorithm takes the linear combination of two directions as new direction to reach the next point at full step. It can be proved that this algorithm is less complex in theory and is the best complexity results so far. Meanwhile, based on the premise that there is strict complementarity solution for linear hyperbolic problem, it can be proved that the algorithm has optimal locally quadratic convergence. In this paper, the iterative sequences and algorithm complexity x is introduced, the linear hyperbolic equation is analyzed by using the locally adjacent region and locally quadratic convergence to analyze the algorithm complexity. Then, the value problem of elliptical linear hyperbolic equation is taken as example, and the comparative analysis of linear hyperbolic problem is performed by using the locally quadratic convergence algorithm and semi-circular function. The numerical experiment is conducted to further prove the accuracy and simplicity of locally quadratic convergence algorithm, so that the research on locally quadratic convergence algorithm for linear hyperbolic equation is completed. Finally, the numerical experiment shows that the algorithm is effective.

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