Study on Reliability of Reinforced Concrete Beams Based on Crack Width

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Abstract: By studying the reliability of concrete components, the reliability of reinforced concrete beams based on crack width is studied. Firstly expounds the basic theory of structural reliability, and then, according to a second order moment method based on crack width limit state equation, and then according to the reliability index formula to solve the parameters in the formula, to calculate reliability index expression eventually. According to the reliability index of the calculate formula, you can see that there are many factors that affect the crack width and reliable indicators of the effective height of beam section, steel rebar diameter, root number, beam section width and variable load and the ratio of the permanent load effect.

Keywords: crack width; format; limit state equation; reliable indicator; reliability

1. Introduction

In the process of building and using, in addition to all kinds of load and natural load, the structure also needs to endure the physical and chemical actions of natural environment and use environment. These effects not only change the material composition and properties of the structure, but also deteriorate the mechanical properties of the structure to a certain degree, so this will affect the safety and normal use of the structure. The construction and use of the structure will mean the beginning of the deterioration of the structure, and the deterioration will increase over time. The structure is composed of many components, so the failure of the component may lead to the failure of the structure. Therefore, it is very important to study the reliability of concrete components.

2. Structural Reliability

Reliability of the structure is measured by reliability. Reliability refers to the probability that the structure completes the predetermined function under the prescribed time and the specified conditions, which is expressed in Pr. When the structure has only two basic variables of resistance and effect, the functional function of structural component can be written as Z=R-S. When Z>0, the structure is in a reliable state; When Z<0, the structure is in a state of failure; when Z=0, the structure is in the limit state. The main calculation methods of reliability are the exact method (also called the full probability method) and the fuzzy mathematics method. The most of the exact method is the first two order moment method (the center point method).

In the functional function of the structure, if R and S are normal distribution, the failure probability is as follows:

$$P_f = \int_{-\infty}^{0} f_Z(z) d_z = \int_{-\infty}^{0} \frac{1}{\sqrt{2p}\, s_Z} \exp\left[-\frac{1}{2}\left(\frac{z-u_z}{s_z}\right)^2\right] d_z$$

Command $b = \frac{u_z}{s_z}$ as a reliable indicator, we obtain: $P_f = f(-b)$, the relationship between the reliability index b and the reliability P_s is:

$P_r = 1 - P_f = 1 - f(-b) = f(b)$.

In order to express convenience, the standard value of the random variable is expressed with the symbol of the random variable itself and the standard k. The mean value of the random variable is expressed by u. d is used to represent the coefficient of variation of the random variable, which is the ratio of the standard deviation to the mean. The K is defined as the ratio of the average value to the standard value.

When R and S obey normal distribution and are independent of each other, mean value $u_z = u_R - u_S$, standard deviation $d_Z = \sqrt{d_R^2 + d_s^2}$, At this time the reliability index can be expressed as:

$$b = \frac{u_Z}{s_z} = \frac{u_R - u_S}{\sqrt{s_R^2 + s_S^2}}$$

$$\text{Or } b = \frac{k_R R_k - k_s S_k}{\sqrt{d_R^2 k_R^2 R_k^2 + d_S^2 k_S^2 S_k^2}}$$

When R and S obey lognormal distribution and are independent of each other, meal value

$$u_Z = In\frac{u_R\sqrt{1+d_S^2}}{u_S\sqrt{1+d_R^2}}$$

standard deviation

$$s_Z = \sqrt{In\left[\left(1+d_R^2\right)+\left(1+d_S^2\right)\right]}$$

At this point, the reliability index can be expressed as:

$$b_z = \frac{In\frac{u_R\sqrt{1+d_S^2}}{u_S\sqrt{1+d_R^2}}}{\sqrt{In\left[\left(1+d_R^2\right)+\left(1+d_S^2\right)\right]}}$$

Or $$b_z = \frac{In\frac{k_R R_k\sqrt{1+d_S^2}}{k_S S_k\sqrt{1+d_R^2}}}{\sqrt{In\left[\left(1+d_R^2\right)+\left(1+d_S^2\right)\right]}}$$

3. Reliability Based on Crack Width

3.1. The establishment of the limit equation

The limit value of the width of the crack width $W_{\lim}$ stipulated in the general specification for the design of highway bridge and culvert is used as the resistance of the component. The actual measurement of the crack value W_k^0 is used as the effect of the component. Establishing the limit state equation $Z = W_{\lim} - W_k^0 = 0$, effect $W_k^0 = pW_k$, p is the calculation mode indefinite coefficient (The ratio of the measured value to the calculated value of the maximum crack width of the component under the function of the standard combination of p), W_k is the maximum crack width calculated according to the crack width calculation formula of the general specification for the design of highway bridge and culvert. $W_k = C_1C_2C_3\frac{s_{ss}}{E_s}(\frac{30+d}{0.28+10r_{te}})$, And for the flexural member, the stress of the longitudinal tensile steel bar $s_{ss} = \frac{M_s}{0.87A_S h_0}$. The moment value of M_s is short term combination, According to the literature [2], M_s is a combination of the standard value of permanent action and the frequency of variable action. When the variable effect is only the car load $M_S = M_G + 0.7M_Q$.

From the above, the limit state equation is as follows:

$$Z = W_{\lim} - pC_1C_2C_3\frac{M_G+0.7M_Q}{0.87E_s h_0 A_s}(\frac{30+d}{0.28+10r_{te}}) = 0$$

3.2. Reliable indicators

According to the data in document [4], we measured the crack width of 133 test beams, and concluded that the probability distribution of crack width obeys lognormal distribution. So, the reliable indicators

$$b_z = \frac{In\frac{k_R R_k\sqrt{1+d_S^2}}{k_S S_k\sqrt{1+d_R^2}}}{\sqrt{In\left[\left(1+d_R^2\right)+\left(1+d_S^2\right)\right]}}$$

. In the formula, R_k and S_k are calculated by the specification and formula, respectively, So only k_R, k_S, d_R, d_S a can be required to calculate the reliability index of the crack width.

The ratio of the mean of resistance R to the standard value of S is $k_R = \frac{u_R}{R_k}$, The resistance s R to the structural members is the maximum limit of the maximum crack width specified in the specification, it is a constant. So the average value of $R = W_{\lim}$ and the standard value are both $W_{\lim}$,so $k_R = 1$, And the coefficient of resistance variation $d_R = 0$.

The effect S is a nonlinear function with multiple variables, In order to find the mean of S, The S is expanded at the center point by Taylor series and only one term is taken, According to the literature [5], it is concluded that:

$$k_s = \frac{u_S}{s_k} = \frac{k_{MG}+0.7rk_{MQ}}{1+0.7r}$$

r is the ratio of variable effect to permanent effect. Finding the coefficient of variation of action effect d_S.

In the calculation formula of the effect S, the diameter of the steel bar is d, the ratio of reinforced bar area A_s and longitudinal tensile steel bar r_{te} is three independent random variables. In order to represent S as a function that is completely independent of random variables, so a single bar diameter d is used here, and it is a rectangular section of single row reinforcement. In this way, the area of the reinforced bar and the ratio of reinforcement can represent the function of the diameter of the steel bar. The expression $A_s = \frac{np\,d^2}{4}$ of the tensile area formula and the reinforcement ratio $r_{te} = \frac{A_s}{bh_0} = \frac{np\,d^2}{4bh_0}$ of reinforcing bar into the effect S, so we can get the effect of a completely independent random variable.

$$S = pC_1C_2C_3\frac{b(M_G+0.7M_Q)}{0.0544np\,d^2E_s}(\frac{30+d}{1.12bh_0+10np\,d^2})$$

Replace it into a functional function, at the center point, the Taylor formula is carried out at the center point and take only one item. The coefficient of variation is analyzed for the effect of S. And take

$$k_p = k_{E_s} = k_{h_0} = k_d = k_b = 1$$

It can be obtained:

$$d_S^2 = d_p^2 + l_M^2\left[d_{MG}^2 k_{MG}^2 + d_{MQ}^2 k_{MQ}^2 (0.7r)^2\right] + d_{E_s}^2 + l\, h_0^2 d_{h0}^2 + l_d^2 d_d^2 + l_b^2 d_b^2$$

Among them:

$$l = 1.12bh_{0+}10np\, d^2$$

$$l_M = \frac{1}{k_{MG} + 0.7r\, k_{MQ}},\ l_{h0} = \frac{1.12bh_0}{l}$$

$$l_d = 2 + \frac{20np\, d^2(30+d) - l\, d}{(30+d)\, l}$$

$$l_b = 1 - \frac{1.12bh_0}{l}$$

Finally, the formula for calculating the reliability index of the crack width in the normal use limit state is obtained.

$$b = \frac{In\dfrac{R_k\sqrt{1+d_p^2 + l_M^2\left[d_{MG}^2 k_{MG}^2 + d_{MQ}^2 k_{MQ}^2 (0.7r)^2\right] + d_{E_s}^2 + l\, h_0^2 d_{h0}^2 + l_d^2 d_d^2 + l_b^2 d_b^2}}{S_k \dfrac{k_{MG} + 0.7r\, k_{MQ}}{1+0.7r}}}{\sqrt{In\left\{1+d_p^2 + l_M^2\left[d_{MG}^2 k_{MG}^2 + d_{MQ}^2 k_{MQ}^2 (0.7r)^2\right] + d_{E_s}^2 + l\, h_0^2 d_{h0}^2 + l_d^2 d_d^2 + l_b^2 d_b^2\right\}}}$$

4. Analysis and Conclusion

It can be seen from the calculation formula of the reliability index of the crack width of the normal use limit state that the reliability index of the crack width is related to the following factors:

The standard values of resistance and action effect R_k and S_k, the resistance is the maximum crack width specified in the standard. It can be found out by the bridge rules, and the standard value of the effect can be calculated by the formula given by the standard.

(1)The coefficient of variation d_p 、 d_M 、 d_{MG} 、 d_{MQ} 、 d_{E_s} 、 d_b 、 d_{h_0} , The corresponding statistical values have been given in the literature [4,6].

(2)The ratio of the average value and the standard value of the permanent and variable roles to k_{MG} and k_{MQ}, the corresponding statistical values have been given in the literature [4].

(3)The effective height of cross section h_0、diameter of steel bar d 、rebar root number n, the ratio of cross section width to b and the effect of variable and permanent action r .

Since the above parameters (1) (2) (3) have been given in the literature or can be calculated by the formula, therefore, the main factors that affect the reliability index of crack width are only (4) factors.

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