# Optimal Pricing Strategy and Intermediary Selection of Airline Ticket Sales 

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#### Abstract

This paper researches the pricing strategy and intermediary selection of the decision makers in a dual channel supply chain of airline ticket sales with passenger preference. We establish the game model under fixed intermediary and common intermediary to analyze the equilibrium decision between airlines and intermediaries. By comparing the revenue of the two sales models, we get the intermediary selection threshold under different ticket supplies, and discuss the feasibility of the sales model from the perspective of airlines and intermediaries. The research shows that the two sales models can achieve market equilibrium, but passenger preference without affect on the intermediary selection; when the ticket supply is below the threshold, airlines choose the common intermediary sales model, otherwise, select fixed intermediary sales model.


Keywords: Fixed intermediary; Common intermediary; Pricing strategy; Passenger preference

## 1. Introduction

The airline ticket sales supply chain refers to the overall network chainstructure formed by airlines selling tickets to passengers through the sales network. For a long time, in the practice of airline ticket sales, dual channel supply chain of direct sales and intermediary sales have been formed, and the intermediary sales have dominated. With the increase in the proportion of intermediary sales, there are two types of intermediary sales models. One is that domestic intermediaries sell tickets for multiple airlines at the same time, satisfying the diversified demand of passengers. A typical intermediary is "Qunar". The other is that some foreign intermediaries only sell tickets for one airline, providing services for passengers with specific ticket demand. For example, "Kayak" sells tickets for "Virgin Australia" and accounts for more than $60 \%$ of the sales. In this context, choosing the right intermediary sales model has become the key to airlines' core competitiveness. What are the equilibrium prices and benefits of airlines and intermediaries in these two ticket sales models? What sales model should the airline use? It is the content that this paper needs to research.
The research related to this paper mainly focuses on two aspects: supply chain pricing strategy and vendor model selection. On the one hand, researches on the pricing strategies of manufacturers and retailers in the supply chain have been abundant, such as the pricing strategy of the supply chain dominated by manufacturers; equilibrium pricing of supply chains dominated by retailers. On the other hand, the selection of supply chain sales model has also been extensively studied, such as the
choice of sales models in the supply chain composed of manufacturers and single retailers; considering the competition among multiple retailers, study the choice of sales model in the multi-channel supply chain. None of the above literature has been analyzed in the context of airline ticket sales. As the delivery period of air ticket service is at the time of aircraft taking off, the pricing of airline ticket and the selection of sales model have certain particularity. Ovchinnikov and Milner (2012) studied the pricing strategies of individual airlines in different periods, and found that airlines provide some tickets at discounted prices in a certain period to stimulate demand and increase revenue. Bertsimas and Popescu (2003) assume that airlines have multiple resources under monopoly conditions. By establishing the revenue management model of airlines, they use price control and demand management to expand revenue. The above research does not take into account the competition between airlines, which is very fierce in real life. Koo , Mantin and Connor (2011) analyzed the influencing factors of airlines' sales model selection in the competitive market environment, and found that if airlines have a large loyal customer, they tend to adopt the network direct selling model. The study assumes that passengers have the same value judgment on a single channel, ignoring the impact of passenger preferences on supply chain decision makers. In fact, passengers have a certain preference for airline tickets, which affects the pricing strategy and model selection of the supply chain. Cheng, Lee and Klingenberg (2011) analyzed the selection of online and offline sales models of airlines when considering the preference of passengers. Jerath, Netessine and Veeraragha-
van (2010) studied the pricing strategy and sales model selection of airline direct selling and intermediary opaque selling, based on the influence of passenger preference on airline pricing, and concluded that when passengers have high valuations or the product is not different, direct sales are taken[16]. On the contrary, intermediary opaque sales can increase the revenue of airlines. Although these literatures considered the preferences of passengers, they did not analyze from the perspective of supply chain and did not involve the choice of intermediary sales model.
Considering two different intermediaries' sales models under the dualchannel supply chain, assuming that the passengers have a preference for the two airlines, this paper establish a game model of airlines and intermediaries under the ticket sales supply chain, analyzes the equilibrium decisionofthe makers, discussestheselection of theoptimal intermediary model for airlines under different ticket supplies. The research in this paper can provide a theoretical basis for airlines to choose the appropriate intermediary sales model under the dualchannel supply chain, and provide a theoretical basis for airlines to increase profits in the fierce market competition.

## 2. Model Descriptions

Consider two competing airlines A and B , each hold a quantity $S / 2$ of inventory. We assume that there is no vertical differentiation between products ofthe two firms, and one product is not inherently superior to the other. The products are perishable inthe sense that they have to be sold before a certain time and have no value if they remain unsold (products have no value after the fightstake off). The total number of tickets for the two airlines is $S$, and the total market demand is $D$. We consider cases of deterministic low demand ( $D<S$ ), deterministic highdemand ( $D>S$ ). Airlines have two sales channels: one is direct selling through official websites or telephone sales, and the price of products is determined by the airlines. Another channel is to sell tickets through intermediaries, who decide the price of the product.
Consumers have different preferences between firms. The reason might be loyalty to the firm, preference for a brand or simply an established relationship with the company. We assume that the two competing firms A and $B$ are located at each end of a "Hotelling" line of length 1 and a continuum of consumers is spread on the horizontal line over the interval $[0,1]$ with uniform density. A population of $D$ consumers is spread uniformly over the entire line. Each consumer has a valuation $v$ for the product and purchases at most one unit. The brand preference of every consumer is completely characterized by his location $x \in[0,1]$ on the line. Thus, although all the consumers have the same valuation $v$ for the product, they have varying preferences towards the competing firms, which influences the utility a consumer derives when he
purchases a product from a firm. The parameter $t$ denotes the strength of brand preference in the market. A consumer at $x$ incurs a disutility $t x$ when buying a product from firm A and a disutility $t(1-x)$ when buying a product from firm B. If firms A and B charge prices $p_{A}$ and $p_{B}$, respectively, then aconsumer located at $x$ receives net utility $V-t x-p_{A}$ when purchasing a product from firm A and receivesnet utility $V-t(1-x)-p_{B}$ when purchasing a productfrom firm B.
The following notation is used throughout the paper:
$\lambda$ : supply and demand ratio, $\lambda=S / D, 0<\lambda<1$ is highdemand, $\lambda>1$ is low demand.
$p_{i j}$ : the price of direct sales channel. $(i=A, B$;
$j=L, H ; L$ is low demand, $H$ is highdemand).
$p_{i j}^{l}$ : the price of intermediary sales channel.
$\pi_{i j}$ : the expected total revenue of the airline.
$\pi_{i j}^{I}$ : the expected revenue of the intermediary.
$\pi_{j}$ : the revenue of the supply chain under different airline ticket supplies.
$\beta_{j}$ : the revenue distribution ratio of airlines and intermediaries under different ticket supply, $\beta_{j} \in(0,1)$. The intermediary keeps a fraction $1-\beta_{j}$ of the revenues from the intermediary sales channel. The remaining frac$\operatorname{tion} \beta_{j}$ is distributed between firms A and B in proportion to the products sold for each firm.
Let us formally make some assumption for our model:
The ratio $V / t$ can be interpreted as a "brand preference adjusted valuation" for a product and it reflects the degree of competition between the firms. If $V$ is large, the valuation for a product in the market is high and the market will be competitive, and vice versa. Further, if $t$ is small, the consumers do not care about the firm that they buy from and competition will be high, and vice versa. Overall, as $V / t$ increases, the market becomes more competitive. We assume that $V / t \geq 1 / 2$ so that every consumer would receive non-negative utility from the product from at least one of the firms if it were offered for free by both firms (Jerath, 2010).
Airlines and passengers have access to all market information.
We establish the rational expectations equilibrium (Muth, 1961). A rational expectations equilibrium ensures that the expectations of all the players are consistent with the equilibrium outcomes.
It is assumed that there are two models of intermediary sales under the dual channel supply chain. In the first model, airlines A and B each have a fixed intermediary, as shown in figure 1, which is referred to as the fixed intermediary sales model. This model is common in reality. For example, "Kayak" website only sells tickets for
"Virgin Australia". In the second model, the intermediary simultaneously sells tickets of airlines A and B, as shown in figure 2, which is called the common intermediary sales model. This model is also widely used in reality. For example, "Qunar" sells tickets of several airlines including "China Eastern Airlines", "Hainan Airlines" and "Air China" at the same time. Under the two sales models, airlines and intermediaries maximize revenue through the pricing strategy


Figure1. Fixedintermediary sales model.


Figure 2. Common intermediary sales model.

To differentiate from the fixed intermediary sales model, we represent the common intermediary sales model by using the superscript "-". Due to the symmetry of airlines A and B in "Hotelling" model, this paper takes airline A as an example for analysis.

## 3. The Model

In this section, we explore the equilibrium price and revenue of airlines and intermediaries in two sales models:
(i)the airlines can sell through their own channels and fixed intermediary channels, and have the option of offering different prices in the two sales channels; (ii) the airlines can sell through their own channels and common intermediary channels, and have the option of offering different prices in the two sales channels. We consider two possible scenarios for each model: low demand scenario ( $D<S$ ) and high demand scenario ( $D>S$ ). The deterministic demand model helps us gain insights into the players' decisions when demand is lower or higher than capacity, and it serves as a logical building block for the more complex model.

### 3.1. Fixed intermediary sales model

In the fixed intermediary sales model, the passenger who purchases the ticket from the intermediary can be considered to have no preference cost because the intermediary sells the ticket of a certain airline separately. Based on the above assumptions, the equilibrium between airlines and intermediaries is shown in theorem 1 and theorem 2.
Theorem1. At low demand, the equilibrium decisions of airlines and intermediaries under the fixed intermediary sales model are as follows:
Equilibrium price: $p_{A L}^{*}=p_{B L}^{*}=\frac{4 V-t}{4}, p_{A L}^{I^{*}}=p_{B L}^{I^{*}}=V$;
The intermediary determines the optimal revenue distribution ratio: $\beta_{L}^{*}=1-\frac{t}{2 V}$;
Equilibrium revenue: $\pi_{A L}^{*}=\pi_{B L}^{*}=\frac{8 V-3 t}{16} D, \pi_{A L}^{I^{*}}=\pi_{B L}^{I^{*}}=\frac{t}{8} D$, $\pi_{L}^{*}=\frac{8 V-t}{16} D$.
Proof: We prove the proposition for low demand. Note that the total capacity of the two airlines $(\mathrm{S})$ is more than the total demand (D). Let $V / t \geq 1 / 2$ as described in the paper.
We consider the decision of airline $A$ in detail, and the analysis will be identical for airline B . Assuming that passengers on the hotelling line $\left(0, x_{A}\right]$ purchase tickets at airline A, the number of tickets sold by airline A is $x_{A} D$, and the number of tickets sold by intermediary is $\left(1 / 2-x_{A}\right) D$. If airline and intermediary choose price $p_{A L}$ and $p_{A L}^{I}$, the right-most consumer to buy from airline or intermediary will be at $x_{A}$ such that $V-p_{A L}^{I}=V-p_{A L}-t x_{A}=0$, i.e., the utility of the consumer at ${ }_{X_{A}}$ is zero. The price charged by airline and intermediary to all consumers will then be $p_{A L}=V-t x_{A}$ and $p_{A L}^{I}=V$. Thus, the revenue of the airline will be $\pi_{A L}=p_{A L} x_{A} D+\beta_{L} p_{A L}^{I}\left(1 / 2-x_{A}\right) D$. This revenue is maximized at $x_{A}^{*}=V\left(1-\beta_{L}\right) /(2 t)$, and the maximized revenue is:

$$
\begin{equation*}
\pi_{A L}=\frac{\left(1-\beta_{L}^{2}\right) V^{2}}{4 t} D+\frac{\left(t-V+\beta_{L} V\right) \beta_{L} V}{2 t} D \tag{1}
\end{equation*}
$$

Then, the revenue of the intermediary will be $\pi_{A L}^{I}=\left(1-\beta_{L}\right) p_{A L}^{I}\left(1 / 2-x_{A}\right) D$, and the maximized revenue is:

$$
\begin{equation*}
\pi_{A L}^{I}=\frac{V t-V^{2}}{2 t} D+\frac{2 V^{2}-V t}{2 t} \beta_{L} D-\frac{V^{2}}{2 t} \beta_{L}^{2} D \tag{2}
\end{equation*}
$$

Next, we discuss the effectiveness of the sales model and the revenue distribution determinants. If the airline decides the revenue distribution ratio, take the derivative of equation (1), and let $\partial \pi_{A L} / \partial \beta_{L}=0$, get $\beta_{L}=1-t / V$. As a result of $\partial^{2} \pi_{A L} / \partial \beta_{L}^{2}=V^{2} D /(2 t)>0$, the airline can only take the maximum value when $\beta_{L}$ is 0 or 1 , that is, the airline has no revenue or obtains all revenue. It can be known that the intermediary sales channel is invalid in this case. If the revenue distribution ratio is determined by the intermediary, take the derivative of equation (2), and let $\partial \pi_{A L}^{I} / \partial \beta_{L}=0$,get $\beta_{L}^{*}=1-t /(2 V)$. Because $\partial^{2} \pi_{A L}^{I} / \partial \beta_{L}^{2}=-V^{2} D / t<0$, it indicates that the intermediary decides the distribution ratio to achieve the maximization of revenue. Therefore, the intermediary is the determinant of revenue distribution, and the optimal revenue distribution ratio is $\beta_{L}^{*}=1-t /(2 V)$. Because of $V / t \geq 1 / 2$, then $\beta_{L}^{*} \in(0,1)$.
From $\beta_{L}^{*}$ we can get $x_{A}^{*}$, and $x_{A}^{*}<1 / 2$. The revenue of the supply chain is $\pi_{L}^{*}=\pi_{A L}^{*}+\pi_{A L}^{I^{*}}$, so in the case of low demand, the equilibrium decisions are shown in theorem 1. Theorem 2. At high demand, the equilibrium decisions of airlines and intermediaries under the fixed intermediary sales model are as follows:
Equilibrium price: $p_{A H}^{*}=p_{B H}^{*}=\frac{4 V-\lambda t}{4}, p_{A H}^{I^{*}}=p_{B H}^{I^{*}}=V$;
The intermediary determines the optimal revenue distribution ratio: $\beta_{H}^{*}=1-\frac{\lambda t}{2 V}$;
Equilibrium revenue: $\pi_{A H}^{*}=\pi_{B H}^{*}=\frac{8 V \lambda-3 t \lambda^{2}}{16} D$, $\pi_{A H}^{I^{*}}=\pi_{B H}^{I^{*}}=\frac{t \lambda^{2}}{8} D, \pi_{H}^{*}=\frac{8 V \lambda-t \lambda^{2}}{16} D$.
Proof: In this proposition, we analyze the high demand case. The total capacity of the two airlines $(\mathrm{S})$ is less than the total demand (D), some passengers will not get the tickets. Use $x^{\prime}$ indicate the farthest position of the passenger who can buy the ticket, and get $x^{\prime}=S /(2 D)=\lambda / 2$ from $x^{\prime} D=S / 2$. However, to ensure that the airlines do not stock out, we need to ensure that at the optimum $x_{A} \leq \lambda / 2$. Then, the number of tickets sold by airline A is $x_{A} D$, and the number of tickets sold by intermediary is $\left(\lambda / 2-x_{A}\right) D$. If airline and intermediary choose price $p_{A H}$ and $p_{A H}^{I}$, the right-most consum-
er to buy from airline or intermediary will be at $x_{A}$ such that $V-p_{A H}^{I}=V-p_{A H}-t x_{A}=0$, i.e., the utility of the consumer at $X_{A}$ is zero. The price charged by airline and intermediary to all consumers will then be $p_{A H}=V-t x_{A}$ and $p_{A H}^{I}=V$. Thus, the revenue of the airline will be $\pi_{A H}=p_{A H} x_{A} D+\beta_{H} p_{A H}^{I}\left(\lambda / 2-x_{A}\right) D$. This revenue is maximized at $x_{A}^{*}=V\left(1-\beta_{H}\right) /(2 t)$, and the maximized revenue is:

$$
\begin{equation*}
\pi_{A H}=\frac{\left(1-\beta_{H}^{2}\right) V}{4 t} D+\frac{\left(\lambda t-V+\beta_{H} V\right) \beta_{H} V}{2 t} D \tag{3}
\end{equation*}
$$

The revenue of the intermediary will be $\pi_{A H}^{I}=\left(1-\beta_{H}\right) p_{A H}^{I}\left(\lambda / 2-x_{A}\right) D$, and the maximized revenue is:

$$
\begin{equation*}
\pi_{A H}^{I}=\frac{\lambda V t-V^{2}}{2 t} D+\frac{2 V^{2}-\lambda V t}{2 t} \beta_{H} D-\frac{V^{2}}{2 t} \beta_{H}^{2} D \tag{4}
\end{equation*}
$$

Secondly, analyze the validity of the sales model and determine the revenue distribution. If the airline decides the revenue distribution ratio, take the derivative of equation (3), and let $\partial \pi_{A H} / \partial \beta_{H}=0$, get $\beta_{H}=1-\lambda t / V$. As a result of $\partial^{2} \pi_{A H} / \partial \beta_{H}^{2}=V^{2} D /(2 t)>0$, indicates that the sales model is invalid. If the revenue distribution ratio is determined by the intermediary, take the derivative of equation (4), and let $\partial \pi_{A H}^{I} / \partial \beta_{H}=0$, get $\beta_{H}^{*}=1-\lambda t /(2 V)$. Because $\partial^{2} \pi_{A H}^{I} / \partial \beta_{H}^{2}=-V^{2} D / t<0$, the intermediary determines the distribution ratio to maximize the revenue. Therefore, the intermediary determines the optimal revenue distribution ratio is $\beta_{H}^{*}=1-\lambda t /(2 V)$. Because of $V / t \geq 1 / 2$, then $\beta_{H}^{*} \in(0,1)$.
Finally, the revenue of the supply chain is $\pi_{H}^{*}=\pi_{A H}^{*}+\pi_{A H}^{I^{*}}$, so theorem 2 can be obtained for high demand.

### 3.2. Common intermediary sales model

Under the common intermediary sales model, the intermediary sell the tickets of airline $A$ and $B$ at the same time. Assuming that the number of tickets sold by airlines A and B to the intermediary is $l_{A}$ and $l_{B}$, the probability that passengers buy tickets from airlines A and B is $r_{A}$ and $r_{B}$, then $r_{A}=l_{A} /\left(l_{A}+l_{B}\right), r_{B}=l_{B} /\left(l_{A}+l_{B}\right)$. Since both airlines have the same number of tickets, then $l_{A}=1_{B}$, so $r_{A}=r_{B}=1 / 2$. Under the assumption of rational expectation equilibrium, the probability that the ticket comes from airline A and B is $1 / 2$. Therefore, the equilibrium between airlines and intermediary is shown in theorems 3 and 4.
Theorem 3. At low demand, the equilibrium decisions of airlines and intermediary under the common intermediary sales model are as follows:
Equilibrium price: $\stackrel{-}{p}_{A L}^{*}=\bar{p}_{B L}^{-*}=\frac{8 V-3 t}{8}, \stackrel{-p_{L}^{I}}{p_{L}}=V-\frac{t}{2}$;

The intermediary determines the optimal revenue distribution ratio: $\bar{\beta}_{L}^{*}=\frac{4 V-3 t}{4 V-2 t}$;
Equilibrium revenue: $\pi_{A L}^{-*}=\pi_{B L}^{-*}=\frac{32 V-15 t}{64} D \quad$, $\pi_{L}^{-*}=\frac{t}{32} D, \pi_{L}^{-*}=\frac{32 V-13 t}{64} D$.
Proof: We prove the proposition for low demand.
Assuming that passengers on the hotelling line $\left(0, \bar{x}_{A}\right]$ purchase tickets at airline A, the number of tickets sold by airline A is $\bar{x}_{A} D$. Since both airline A and B sell tickets to the common intermediary, the number of tickets sold by the intermediary is $\left(1-2 \bar{x}_{A}\right) D$. If airline and intermediary choose price $\overline{p_{A L}}$ and $\overline{p_{L}^{I}}$, the right-most consumer to buy from airline or intermediary will be at $\bar{x}_{A}$ such that
$V-\overline{p_{L}^{I}}-r_{A} t \overline{x_{A}}-r_{B} t\left(1-\overline{x_{A}}\right)=V-\overline{p_{A L}}-t \bar{x}_{A}=0$, i.e., the utility of the consumer at $\bar{x}_{A}$ is zero. The price charged by airline and intermediary to all consumers will then be $\overline{p_{A L}}=V-t \bar{x}_{A}$ and $\overline{p_{L}^{I}}=V-t / 2$.Thus, the revenue of the airlinewill be $\overline{\pi_{A L}}=\overline{p_{A L}} \overline{x_{A}} D+\overline{\beta_{L}} \overline{p_{L}^{I}}\left(1 / 2-\overline{x_{A}}\right) D$. This revenue is maximized at $\bar{x}_{A}^{*}=\left(V-\overline{\beta_{L}} \overline{p_{L}^{I}}\right) /(2 t)$, and the maximized revenue is:

$$
\begin{equation*}
\pi_{A L}^{-}=\frac{4 V^{2}-\bar{\beta}_{L}^{2}(2 V-t)^{2}}{16 t} D+\frac{2 \bar{\beta}_{L}(2 V-t)(t-V)+\bar{\beta}_{L}^{2}(2 V-t)^{2}}{8 t} D(5) \tag{5}
\end{equation*}
$$

Similarly, the revenue of the intermediary will be $\overline{\pi_{L}^{I}}=\left(1-\overline{\beta_{L}}\right) \overline{p_{L}^{I}}\left(1-2 \overline{x_{A}}\right) D$, and the maximized revenue is:

$$
\begin{equation*}
\overline{\pi_{L}^{I}}=\left(1-\overline{\beta_{L}}\right) \frac{(2 V-t)\left[2 t-2 V+(2 V-t) \overline{\beta_{L}}\right]}{4 t} D \tag{6}
\end{equation*}
$$

Then analyze the validity of the sales model and the decision maker of the revenue distribution. If the airline decides the revenue distribution ratio, take the derivative of equation (5), and let $\partial \pi_{A L}^{-} / \partial \bar{\beta}_{L}=0$, get $\bar{\beta}_{L}=2\left(2 V^{2}+t^{2}-3 V t\right) /(2 V-t)^{2}$.
However $\partial^{2} \pi_{A L}^{-} / \partial \bar{\beta}_{L}^{2}=(2 V-t)^{2} D /(8 t)>0$, so this sales model is not valid. If the revenue distribution ratio is determined by the intermediary, take the derivative of equation (6), and let $\partial \pi_{A L}^{\bar{I}} / \partial \overline{\beta_{L}}=0$, get $\bar{\beta}_{L}^{*}=(4 V-3 t) /(4 V-2 t)$ Because
$\partial^{2} \bar{\pi}_{L}^{I} / \partial \bar{\beta}_{L}^{2}=-(2 V-t)^{2} D /(2 t)<0$, it means that the intermediary decides the distribution ratio to achieve the maximization of revenue. Therefore, the intermediary decides the optimal revenue distribution ratio is $\bar{\beta}_{L}^{-*}=(4 V-3 t) /(4 V-2 t)$. Because of $V / t \geq 1 / 2$, then $\beta_{L}^{*} \in(0,1)$.
From $\bar{\beta}_{L}^{-*}$ we can get $\bar{x}_{A}^{-*}$, and $\bar{x}_{A}^{-*}<1 / 2$. The revenue of the supply chain is $\pi_{L}^{-*}=\pi_{A L}^{-*}+\pi_{L}^{-*}$. Therefore, in the common intermediary sales model, theorem 3 can be obtained atlow demand.
Theorem 4. At high demand, the equilibrium decisions of airlines and intermediary under the common intermediary sales model are as follows:
Equilibrium price: $\bar{p}_{A H}^{-*}=\bar{p}_{B H}^{-*}=\frac{8 V-(2 \lambda+1) t}{8}, \stackrel{-}{p}_{H}^{I}=V-\frac{t}{2}$; The intermediary determines the optimal revenue distribution ratio: $\beta_{H}^{-*}=\frac{4 V-(2 \lambda+1) t}{4 V-2 t}$;
Equilibrium
revenue:
$\pi_{A H}^{-*}=\pi_{B H}^{-*}=\frac{32 V \lambda-(2 \lambda+1)(6 \lambda-1) t}{64} D$
$\bar{\pi}_{H}^{-*}=\frac{(2 \lambda-1)^{2} t}{32} D, \pi_{H}^{-*}=\frac{32 V \lambda-\left(4 \lambda^{2}+12 \lambda-3\right) t}{64} D$.
Proof: The following analysis of high demand situation. The farthest position of the passenger who can buy the ticket on the hotelling line is $\lambda / 2$. Similarly, the number of tickets sold by airline A is $x_{A} D$, and the number of tickets sold by intermediary is $\left(\lambda-2 x_{A}\right) D$, and $\bar{x}_{A}<\lambda / 2$. If airline and intermediary choose price $\overline{p_{A H}}$ and $\overline{p_{H}^{I}}$, the right-most consumer to buy from airline or intermediary will be at $x_{A}$ such that $V-\bar{p}_{H}^{I}-r_{A} t \bar{x}_{A}-r_{B} t\left(1-\bar{x}_{A}\right)=V-\bar{p}_{A H}-t \bar{x}_{A}=0$, i.e., the utility of the consumer at $\bar{x}_{A}$ is zero. The price charged by airline and intermediary to all consumers will then be $\overline{p_{A H}}=V-t \overline{x_{A}}$ and $\overline{p_{H}^{I}}=V-t / 2$. Thus, the revenue of the airline will be $\pi_{A H}^{-}=\bar{p}_{A H}^{-} \bar{x}_{A} D+\beta_{H}^{-} \overline{p_{H}^{I}}\left(\lambda / 2-\bar{x}_{A}\right) D$. This revenue is maximized at $\overline{x_{A}^{*}}=\left(V-\overline{\beta_{H}} \overline{p_{H}^{I}}\right) /(2 t)$, and the maximized revenue is:

$$
\begin{equation*}
\pi_{A H}^{-}=\frac{4 V^{2}-\bar{\beta}_{H}^{2}(2 V-t)^{2}}{16 t} D+\bar{\beta}_{H}^{-}\left(V-\frac{t}{2}\right)\left[\frac{2 \lambda t-2 V+\bar{\beta}_{H}^{-}(2 V-t)}{4 t}\right] D \tag{7}
\end{equation*}
$$

Then, the revenue of the intermediary will be $\overline{\pi_{H}^{I}}=\left(1-\overline{\beta_{H}}\right) \overline{p_{H}^{I}}\left(\lambda-2 \bar{x}_{A}^{*}\right) D$, and the maximized revenue is:

$$
\begin{equation*}
\overline{\pi_{H}^{I}}=\left(1-\beta_{H}\right)\left(V-\frac{t}{2}\right)\left[\lambda-\frac{V}{t}+\overline{\beta_{H}}\left(\frac{V}{t}-\frac{1}{2}\right)\right] D \tag{8}
\end{equation*}
$$

Next, we discuss the effectiveness of the sales model and the revenue distribution determinants. If the airline decides the revenue distribution ratio, take the derivative of
equation (7), and let $\partial \pi_{A H}^{-} / \partial \beta_{H}=0$, get $\overline{\beta_{H}}=2\left[2 V^{2}+\lambda t^{2}-(2 \lambda+1) V t\right] /(2 V-t)^{2}$. This sales model is invalid due to $\partial^{2} \pi_{A H}^{-} / \partial \bar{\beta}_{H}^{2}=(2 V-t)^{2} D /(8 t)>0$. If the revenue distribution ratio is determined by the intermediary, take the derivative of equation (8), and let $\partial \pi_{A H}^{\bar{I}} / \partial \beta_{H}^{-}=0$, get $\beta_{H}^{-*}=[4 V-(2 \lambda+1) t] /(4 V-2 t)$. Because $\partial^{2} \pi_{H}^{I} / \partial \bar{\beta}_{H}^{2}=-(2 V-t)^{2} D /(2 t)<0$, the intermediary determines the distribution ratio to maximize the revenue. Therefore, the intermediary determines the optimal revenue distribution ratio is $\bar{\beta}_{H}^{-*}=[4 V-(2 \lambda+1) t] /(4 V-2 t)$. Because of $V / t \geq 1 / 2$, then $\beta_{H}^{-*} \in(0,1)$.
Finally, the revenue of the supply chain is $\pi_{H}^{-*}=\pi_{A H}^{-*}+\pi_{H}^{-*}$. Under the common intermediary sales model, when the demand is high, the equilibrium decisions are shown in theorem 4.
The following corollary can be obtained directly from theorem 1-4:
Corollary 1. Passenger preference influences the equilibrium of decision makers, and the greater the preference, the higher the equilibrium price and revenue.
According to corollary 1, airlines and intermediaries can raise revenue by charging higher prices to passengers with higher preferences; at the same time, lower prices for passengers with lower preferences will increase sales and revenue.

## 4. Fixed Intermediary vs. Common Intermediary

According to the above analysis, in the dual-channel supply chain of airline ticket sales, both fixed intermediary and common intermediary sales models can achieve market equilibrium. In this section, we discuss how airlines choose the optimal sales model under different ticket supplies.
Theorem 5. let $\lambda^{*}=1 / 4$, when $\lambda<\lambda^{*}$, the airline's optimal choice is the common intermediary sales model; when $\lambda>\lambda^{*}$, the optimal choice of the airline is the fixed intermediary sales model.

Proof: In the case of high demand $(0<\lambda<1), \Delta \pi_{H}$ is the revenue of the fixed intermediary sales model minus the common intermediary sales model, and $\Delta \pi_{H}=\pi_{H}^{*}-\pi_{H}^{*}=(12 \lambda-3) t D / 64$. When the revenue of the two sales models are equal, we get $\lambda^{*}=1 / 4$. Therefore, when the ticket supply is $0<\lambda<\lambda^{*}$, the revenue of the common intermediary sales model is greater than the fixed intermediary; when $\lambda^{*}<\lambda<1$, the revenue of the fixed intermediary sales model is greater than the common intermediary.
At low demand $(\lambda>1)$, let $\Delta \pi_{L}$ be the difference between the revenue of two sales models, and $\Delta \pi_{L}=\pi_{L}^{*}-\pi_{L}^{*}=9 t D / 64$. Because $\Delta \pi_{L}$ is always greater than zero, that is, the revenue of the fixed intermediary sales model is always greater than the common intermediary.
The following analyzes the feasibility of the sales model from the perspective of airlines and intermediaries. Firstly, compare airline revenues in two sales models. At high demand $(0<\lambda<1)$, let $\Delta \pi_{A H}$ be the revenue of the airline in the fixed intermediary sales model minus the revenue under the common intermediary, and $\Delta \pi_{A H}=\pi_{A H}^{*}-\pi_{A H}^{*}=(4 \lambda-1) t D / 64$ can be obtained. When $0<\lambda<\lambda^{*}$, we get $\Delta \pi_{A H}<0$, indicating that the airline's optimal choice is the common intermediary sales model; when $\lambda^{*}<\lambda<1$, get $\Delta \pi_{A H}>0$, the optimal choice of airlines is the fixed intermediary sales model. Similarly, at low demand $(\lambda>1)$, we can get $\Delta \pi_{A L}=\pi_{A L}^{*}-\pi_{A L}^{\bar{*}}=3 t D / 64$. As $\Delta \pi_{A L}$ is always greater than zero, the airlines choose the fixed intermediary sales model. Secondly, compare the revenue of the intermediaries under the two sales models. In the case of high demand, let $\Delta \pi_{I H}$ be the revenue difference between the intermediaries in the two sales models, and get $\Delta \pi_{I H}=\pi_{A H}^{I^{*}}-\bar{\pi}_{H}^{l}=(4 \lambda-1) t D / 32$. When $0<\lambda<\lambda^{*}$, $\Delta \pi_{I H}<0$, it indicates that the common intermediary's revenue is greater than the fixed intermediary; when $\lambda^{*}<\lambda<1$, the $\Delta \pi_{I H}>0$, indicating that the revenue of the fixed intermediary is greater than the common intermediary. In the same way, get $\Delta \pi_{I L}=\pi_{A L}^{I^{*}}-\pi_{L}^{-^{\prime}}=3 t D / 32$. Since $\Delta \pi_{I L}$ is always greater than 0 , the fixed intermediary's revenue is always greater than the common intermediary.
Theorem 5 shows that in the dual-channel supply chain of ticket sales, the choice of airline's sales model has nothing to do with passenger preferences, only related to ticket supplies. When the ticket supply is low, choose the
common intermediary sales model; when the ticket supply is high, select the fixed intermediary sales model. Finally, by numerical simulation, the airline's optimal choice is intuitively reflected. Assume that the demand for the ticket is $1(D=1)$ and the unit preference cost is 1 $(t=1)$. The selection of the optimal sales model of airlines is shown in figure 3:


Figure 3. Optimal model selection of airlines under different ticket supplies.

It can be seen from the figure that when the ticket supply and demand ratio $\lambda<1 / 4$, get $\Delta \pi<0$, indicating that the revenue of the common intermediary sales model is greater than the fixed intermediary sales model; when $\lambda>1 / 4$, and $\Delta \pi>0$, the revenue of the fixed intermediary sales model is greater than the common intermediary sales model. The result of theorem 5 is verified .

## 5. Conclusions

In the practice of airline ticket sales at domestic and foreign, the sales model of fixed intermediary and common intermediary widely exists, and choosing the appropriate intermediary sales model is of great significance for airlines to increase revenue. Based on the dual-channel supply chain of airline ticket sales, this paper builds a game model between airlines and intermediaries under the fixed intermediary and common intermediary model, considering the passengers' preference. This paper studies the equilibrium price and revenue of airlines and intermediaries, analyzes the revenue distribution determinants of fixed intermediaries and common intermediary supply chains, obtains the optimal revenue distribution ratio. By comparing the revenue of the two sales models, the thresholds of choosing intermediaries under different ticket supplies are obtained, and the feasibility of the sales model is discussed from the perspective of airlines and intermediaries. The main conclusions are as follows:

1) Whether it is a fixed intermediary or a common intermediary sales model, the revenue distribution ratio of the supply chain is determined by intermediaries, which has
certain reference significance for airlines to improve the traditional revenue distribution mechanism;
2) Passenger preference influences the equilibrium decision of airlines and intermediaries, and the greater the preference, the higher the passenger's valuation of the ticket, the higher the equilibrium price and revenue. For passengers with different preferences, this conclusion provides a theoretical basis for airlines to develop optimal pricing strategies.
3) There is a threshold based on the supply level, which is not related to passenger preference. When the ticket supply is less than it, the airline's revenue in the common intermediary sales model is higher than the fixed intermediary; otherwise, the airline can get higher revenue in the fixed intermediary sales model. When the ticket supply increases to a certain extent, the airline's revenue in the fixed intermediary sales model is always greater than the common intermediary. Therefore, airlines can choose the appropriate intermediary sales model according to different market conditions.
This paper conducts analysis under the assumption that the market information is complete and the price remains unchanged. In fact, the airline's information is incomplete, and the ticket price will change with time. The research directions that can be considered in the future are as follows: first, the pricing strategy of airlines in different numbers of tickets and service levels; second, in real life, airlines often adopt dynamic pricing, how to choose the optimal sales model in the case of dynamic pricing requires further research.

## References

[1] H.L. Lee., M.J. Rosenblatt, A generalized quantity discount pricing model to increase supplier's profits, J. Management Science. 1986,32, 1177-1185.
[2] S.Y. Park., H.T. Keh,. Modeling hybrid distribution channels: A game-theoretic analysis, J. Journal of Retailing and Consumer Service. 2003, 10, 155-167.
[3] Zhao L.X., Cheng M.B., Pricing strategy of supply chain based on manufacturer's marketing channel selection, J. Systems Engineering - Theory\&Practice. 2016, 36, 2310-2319
[4] J.C. Lu, Y.C. Tsao,C. Charoensiriwath, Competition under manufacturer service and retail price, J. Economic Modelling. 28, 011, 1256-1264.
[5] B. Pal,S.S. Sana,K. Chaudhuri, Two-echelon manufacturer retailer supply chain strategies with price, quality and promotional effort sensitive demand, J. International Transactions in Operational Research. 2015, 22, 1071-1095.
[6] G. Reinhardt, M. Levesque,A new entrant's decision on virtual versus bricks-and-mortar retailing, J. Journal of Electronic Commerce Research. 2004, 5, 136-149.
[7] A.A. Tsay, N. Agrawal, Channel conflict and coordination in the e-commerce age, J. Production and Operations Management. 2010, 13, 93-110.
[8] W.Y.K. Chiang, D. Chhajed, J.D. Hess, Direct marketing, indirect profits: a strategic analysis of dual-channel supply-chain design, J. Management Science. 2003, 49, 1-20.
[9] E. Brynjolfsson, Y. Hu, M.S. Rahman, Battle of the retail channels: how product selection and geography drive crosschannel competition, J. Management Science. 2009, 55, 17551765.
[10] R.C. King, R. Sen, M. Xia, Impact of web-based e-commerce on channel strategy in retailing, J. International Journal of Electronic Commerce. 2004, 8, 109-127.
[11] N.A.H. Agatz, M. Fleischmann, J. Nunen, E-fulfillment and multi-channel distribution-a review, J. European Journal of Operational Research. 2008,187, 339-356.
[12] A. Ovchinnikov, J.M. Milner, Revenue management with end of period discounts in the presence of customer learning, J. Production \& Operations Management. 2012, 21, 69-84.
[13] D. Bertsimas, I. Popescu, Revenue management in a dynamic network environment, J. Transportation Science. 2003, 37, 257278.
[14] B. Koo, B. Mantin, P. Connor, Online distribution of airline tickets: should airlines adopt a single or a multi-channel approach, J. Tourism Management. 2011,32, 69-74.
[15] K. Cheng, Z.H. Lee, B. Klingenberg, Consumers' perspectives on the selection of air ticket purchasing channel, J. International Journal of Business Environment. 2011, 4, 63-91.
[16] K. Jerath, S. Netessine, S.K. Veeraraghavan, Revenue management with strategic customers: last-minute selling and opaque selling, J. Management Science. 2010, 56, 430-448.
[17] J.F. Muth, Rational expectations and the theory of price movements, J. Journal of the Econometric Society. 1961, 29, 315-335.

