Adaptive Projective Synchronization of Delay Neural Networks with Unknown Parameters and Noise Perturbation

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Abstract: In this paper, we investigate the adaptive projective synchronization problem of delay neural networks with unknown parameters and noise perturbation. An adaptive updating law for unknown parameters is designed based on the lapunov stability method. Several criteria are established to guarantee the synchronization of master and slave delay neural networks system with adaptive update parameters and stochastic noise. In the end, simulation result is given to show the effectiveness of built theory in this paper.

Keywords: adaptive projective synchronization; neural networks; time delay; noise perturbation

1. Introduction

It is well known that neural networks have been widely used in many fields, such as image restoration[1], optimization problem[2], secure communication[3,4] and so on, which have attracted more and more attention. The drive-response synchronization method is first proposed by Popar and Calk [5], which can effectively control chaos system. Therefore, the synchronization of chaotic neural networks has been developing rapidly. At present, a number of synchronous definitions of different types of neural networks have been proposed (see [6-10]).

However, when the response system is constructed according to the drive system, the system is described as an uncertain system because the parameters of the drive system are un-measurable or measurement error. Besides, noise disturbance and time delay are unavoidable effects in nature, which may cause some instability of neural networks. Therefore, the study of neural networks with parameter uncertainties, noise perturbations and time delays has high application value and research prospects.

At the same time, the stability analysis of Neural Networks with unknown parameters, noise disturbance and time delay has not been given enough attention. Therefore, we use special Lyapunov function to study the adaptive projective synchronization of neural networks. Then we use the LASHALL invariance principle to transform the synchronization problem into an optimization problem which is easy to solve [11]. Finally, a numerical example is given to illustrate the effectiveness and effectiveness of the stability condition.

2. Problem Formation

Consider the delay neural networks as follows:

$$dx(t) = [-Cx(t) + Af(x(t)) + Bf(x_t(t))]dt \qquad (1)$$

where $x(t) = [x_1(t), x_2(t), ..., x_n(t)]^T \in R^n$ represents the state vector of the neural network; n is the total number of neurons; $C = \mathrm{diag}(c_1, c_2, ..., c_n)$ is positive matix; $A = (a_{ij})_{n\times n}$, $B = (b_{ij})_{n\times n} \in R^{n\times n}$ is the connect weight and delay connect weight matrix; f represents activation functions, and satisfy

$$f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), ..., f_n(x_n(t))]^T \in R^n,$$
$$f(x_t(t)) = [f_1(x_1(t-t(t))), f_2(x_2(t-t(t))), ..., f_n(x_n(t-t(t)))]^T \in R^n,$$

Where $t(t)$ is the transmission delay and satisfy that $0 < t(t) < t$, $\dot{t}(t) < \bar{t}$.

The slave system with unknown parameters, noise perturbation and delay is given as follow:

$$\begin{aligned} dy(t) &= [-\hat{C}y(t) + \hat{A}f(y(t)) + \hat{B}f(y_t(t)) + U]dt \\ &+ H(t, y(t) - hx(t), y_t(t) - hx_t(t))dw(t), \end{aligned} \qquad (2)$$

where $\hat{C} = C + \Delta C$, $\hat{A} = A + \Delta A$ and $\hat{B} = B + \Delta B$, are the estimations of unknown matrices C , A and B; $w(t)$ is an n-dimensional Brown motion satisfied $E\{dw(t)\} = 0$ and $E\{[dw(t)]^2\} = dt$, U is a controller.

In order to develop our main results, we give the following assumptions and lemma.

Assumption 1. There is a constant l_i to make the neuron activation functions $f(x)$ in (1) and (2) meet following conditions:

$|f_i(y) - f_i(hx)| \le l_i |y - hx|$, $x, y \in R$ and l >0.

Assumption 2. For the function H(t,x,y) , there always exist appropriate constant matrices G_1, G_2 satisfying

$trace[H^T(t,x,y)H(t,x,y)] \le \|G_1 x\|^2 + \|G_2 y\|^2, (t,x,y) \in R^n \times R^n \times R^n$.

Assumption 3. The initial data of system (1) and (2) are $f(0) \equiv 0, t(t,0,0) \equiv 0$.

We define the synchronization error as e(t)=y(t)-hx(t), where $h=(h_1,h_2,...,h_n)$ is a scaling vector which represents the proportion of projection, x(t), y(t) are state variables of drive system (1) and response system (2). Then the error system can be derived as

$$\begin{aligned} de(t) = [&-Ce(t) + A(f(y(t)) - hf(x(t))) + B(f(y_t(t)) - hf(x_t(t))) \\ &-\Delta Cy(t) + \Delta Af(y(t)) + \Delta Bf(y_t(t)) + U]dt \\ &+H(t,e(t),e_t(t))dw(t) \end{aligned} \quad (3)$$

The controller U is chosen as

$$U = A(hf(x) - f(hx)) + B(hf(x_t(t)) - f(hx_t(t))) - K_1 e(t) \quad (4)$$

where K_1 is the feedback gain matrix.

3. Main Results

In this part, the main goal is to design a controller to realize projective synchronization between the master system (1) and the system (2). We have the following result.

Theorem 1. Let Assumptions 1 to 3 hold. If there exist arbitrary positive constants r_1, r_2 and positive definite matrix Q such that the following conditions are satisfied:

(1) $\begin{pmatrix} \Xi_{11} & mA & mB & m\Lambda \\ mA & 2mI & 0 & 0 \\ mB & 0 & 2mI & 0 \\ m\Lambda & 0 & 0 & 2mI \end{pmatrix} < 0$, where

$\Xi_{11} = G_1^T G_1 - K_1 - C + Q + r_1 I_N$,

(2) $\frac{1}{2} m^{-1} \Lambda^T \Lambda + G_2^T G_2 - (1-\dot{t})Q < r_2 I_N$

(3) $r_1 > r_2$

and the adaptive law of unknown parameters are designed as

$$\begin{cases} \Delta \dot{c}_i = n_i e_i(t) y_i(t), \\ \Delta \dot{a}_{ij} = -v_{ij} e_i(t) f_j(y_j(t)), \qquad i,j = 1,2,\ldots,n, \\ \Delta b_{ij} = -q_{ij} e_i(t) f_j(y_j(t-t(t))), \end{cases} \quad (5)$$

Then the systems (1) and (2) are achieve projective synchronization.

Remark 1. In this paper, when h=(1,1,...,1), we can achieve complete synchronization for delay neural networks with noise perturbation and unknown parameters. When $h=(a,a,...,a)$ and $a<0$ it converts to general anti-projective synchronization for neural networks; When $a=-1$, it becomes anti-synchronization for neural networks.

Remark 2. The function adopted in this paper can also be applied in many complex systems which are related to synchronization of known parameters master and slave system. We can derive some corollaries as follows.

Corollary 1. Under Assumption 1-3, system (1) and system (2) is projective synchronization, without time-delay, if there exist arbitrary positive constants n_i, v_{ij} *and* q_{ij}, $(i,j=1,2,...,n)$ satisfying(5), and the feedback gains K_1 of controller U satisfying the following condition:

$\frac{1}{2} mA^T A + \frac{1}{2} m\Lambda^T \Lambda + G_1^T G_1 - K_1 - C < 0$.

Corollary 2. Under above assumptions, system (1) and (2) is projective synchronization without noise perturbation, if there exist arbitrary positive constants n_i, v_{ij} *and* q_{ij}, $(i,j=1,2,...,n)$ satisfying (5), and the feedback gains K_1 of controller U satisfying the following condition.

$\frac{1}{2} mA^T A + \frac{1}{2} m\Lambda^T \Lambda + G_1^T G_1 - K_1 - C + Q < 0$.

4. Illustrative Example

In this section, an example is present to show the effectiveness of our main results. Consider about the systems (1) and (2) with the following parameters:

$$C = \begin{pmatrix} 1.1 & 0 \\ 0 & 1.1 \end{pmatrix}, A = \begin{pmatrix} 1.8 & -0.12 \\ -5.0 & 2.9 \end{pmatrix}, B = \begin{pmatrix} -1.4 & -0.1 \\ -0.28 & -2.5 \end{pmatrix},$$

$$\hat{C} = \begin{pmatrix} \hat{c}_1 & 0 \\ 0 & \hat{c}_2 \end{pmatrix}, \hat{A} = \begin{pmatrix} \hat{a}_{11} & \hat{a}_{12} \\ \hat{a}_{21} & \hat{a}_{22} \end{pmatrix}, \hat{B} = \begin{pmatrix} \hat{b}_{11} & \hat{b}_{12} \\ \hat{b}_{21} & \hat{b}_{22} \end{pmatrix},$$

$f(x) = \tanh(x(t))$ and $t = 1$. The parameters of simulation are:

$n_1 = n_2 = 5$, $v_{11} = v_{12} = v_{21} = v_{22} = 5$, $q_{11} = q_{12} = q_{21} = q_{22} = 5$,

T=[0,500] and $h = [h_1,\ h_2]^T = [1.2,\ 1.3]$

The initial conditions of simulation are:

$[x_1(s)\ x_2(s)]^T = [0.4\ \ 0.6]^T, [y_1(s)\ \ y_2(s)]^T = [-0.8\ \ 0.5]^T$, for $s \in [-1,0]$.

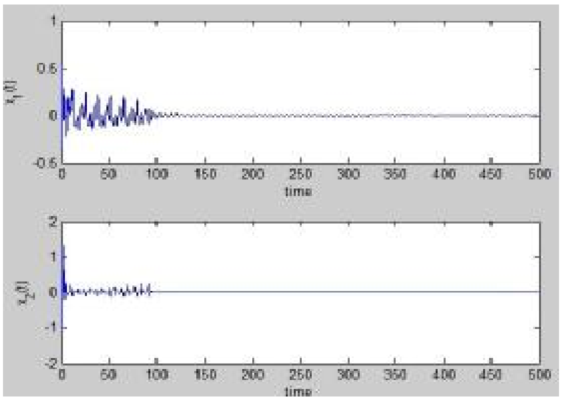

Figure 1. Projective synchronization error of system (1) and (2)

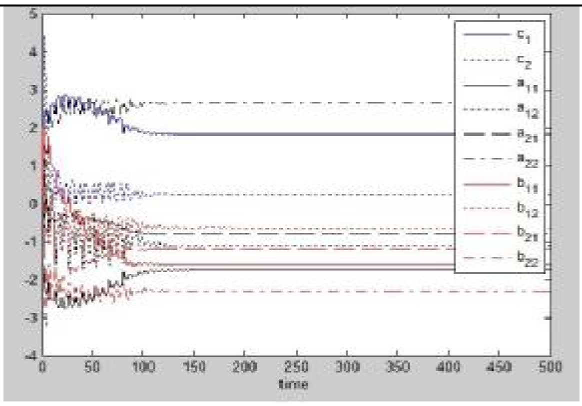

Figure 2. Parameters variation in the slave system

The simulation results are shown as Fig.1-Fig.2. Fig.1 is the trajectories of error system (3); Fig.2 represents the variation of unknown parameters in the slave system.

5. Conclusion

Throughout this paper, we study the adaptive projective synchronization of delay neural networks with unknown parameters and noise perturbation. By using the Lyapunov stability theory a sufficient conditions has been proposed to guarantee the projective synchronization of the master and slave systems.

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